PLANE TRIGONOMETRY

WITH TABLES

ADDISON-WESLEY MATHEMATICS SERIES

ERIC REISSNER, *Consulting Editor*

Bardell and Spitzbart—COLLEGE ALGEBRA

Dadourian—PLANE TRIGONOMETRY

Davis—MODERN COLLEGE GEOMETRY

Davis—THE TEACHING OF MATHEMATICS

Fuller—ANALYTIC GEOMETRY

Gnedenko and Kolmogorov—LIMIT DISTRIBUTIONS FOR SUMS OF INDEPENDENT RANDOM VARIABLES

Kaplan—ADVANCED CALCULUS

Kaplan—A FIRST COURSE IN FUNCTIONS OF A COMPLEX VARIABLE

LeVeque—TOPICS IN NUMBER THEORY, VOLS. I AND II

Martin and Reissner—ELEMENTARY DIFFERENTIAL EQUATIONS

Meserve—FUNDAMENTAL CONCEPTS OF ALGEBRA

Meserve—FUNDAMENTAL CONCEPTS OF GEOMETRY

Munroe—INTRODUCTION TO MEASURE AND INTEGRATION

Perlis—THEORY OF MATRICES

Spitzbart and Bardell—COLLEGE ALGEBRA AND PLANE TRIGONOMETRY

Spitzbart and Bardell—PLANE TRIGONOMETRY

Stabler—AN INTRODUCTION TO MATHEMATICAL THOUGHT

Struik—DIFFERENTIAL GEOMETRY

Struik—ELEMENTARY ANALYTIC AND PROJECTIVE GEOMETRY

Thomas—CALCULUS

Thomas—CALCULUS AND ANALYTIC GEOMETRY

Vance—TRIGONOMETRY

Vance—UNIFIED ALGEBRA AND TRIGONOMETRY

Wade—THE ALGEBRA OF VECTORS AND MATRICES

Wilkes, Wheeler, and Gill—THE PREPARATION OF PROGRAMS FOR AN ELECTRONIC DIGITAL COMPUTER

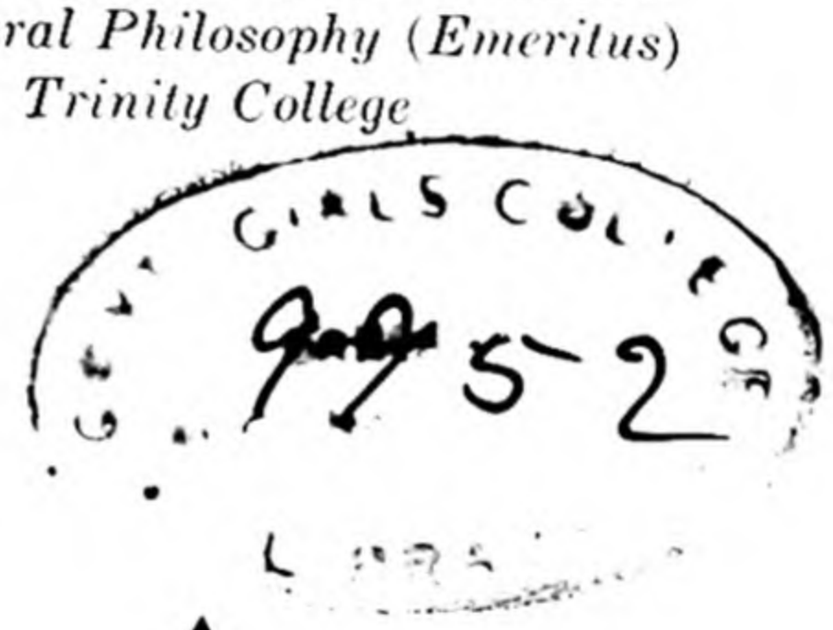
PLANE TRIGONOMETRY

WITH TABLES

by

H. M. DADOURIAN

Seabury Professor of Mathematics
and Natural Philosophy (Emeritus)
Trinity College



ADDISON-WESLEY PUBLISHING COMPANY, INC.
READING, MASSACHUSETTS

Copyright © 1950

ADDISON-WESLEY PUBLISHING COMPANY, INC.

Printed in the United States of America

ALL RIGHTS RESERVED. THIS BOOK, OR PARTS THEREOF, MAY NOT BE REPRODUCED IN ANY FORM WITHOUT WRITTEN PERMISSION OF THE PUBLISHERS.

Library of Congress Catalog No. 50-5925

Fifth printing—November 1957

516.042
D12p
Cat

PREFACE

In writing this book the author has had the following objectives:

1. To stimulate the student's interest and motivation, and to deepen his comprehension of the subject. To these ends, emphasis is laid on concepts, principles, and general methods; trigonometric functions are applied to simple problems of mensuration, mechanics, engineering, and surveying; and the application of the functions to other fields is pointed out. Whenever a new concept or method is introduced, the reason for introducing it is explained. In short, the book is given more of the character of a scientific book and less that of an engineer's handbook.

2. To reduce the need for memory work to a minimum by omitting the superfluous formulas and replacing some others with simple rules of procedure which are easy to remember and to apply. In referring to the rules which replace the forty-eight or more reduction formulas, a reviewer of the manuscript says: "The approach to the functions of negative angles and angles other than those in the first quadrant is both shorter and superior to the manifold formulas in some texts."

3. To present proofs and solved problems in such a way as to reduce the amount of necessary verbal explanation to a minimum, to make the analytical work orderly, concise and lucid; and thus to familiarize the student with a general method of procedure which is conducive to clear thinking, to greater freedom from blunders, and to economy of time and effort. (See the author's booklet, *How to Study—How to Solve*, Addison-Wesley Publishing Company, Inc., 1949.)

The following features of the book are worth mentioning here: Numerical problems are solved as literal problems first, and the expressions of the required magnitudes are discussed with the object of checking their validity and of obtaining interesting special cases from them. The equilibrium of a particle, introduced in Chapter 3, affords interesting yet simple applications of the func-

tions of positive acute angles. The variations, the ranges, and the graphs of the sine, the cosine, and the tangent of a variable angle are fully discussed. The meaning of an inverse function and the manner in which its graph may be obtained from the graph of the corresponding direct function are fully explained and illustrated.

The book is relatively brief, yet no essential topic is omitted. The brevity is largely due to the manner of presentation of the subject. The logarithmic and trigonometric tables, reproduced with the permission of Ginn and Company, are the most satisfactory tables from the point of view of easy legibility known to this author.

H. M. D.

December, 1949
Hartford, Connecticut.

CONTENTS

PAGE

Symbols x

CHAPTER 1

TRIGONOMETRIC FUNCTIONS

SECTION

1–1 Trigonometry 1
1–2 Angles 1
1–3 Units of angle 2
1–4 Rectangular coordinate system 2
1–5 Standard position of an angle 4
1–6 Trigonometric functions 4
1–7 Functions of a positive acute angle 8
1–8 Functions of complementary angles 9
1–9 Functions of 45° 11
1–10 Functions of 30° and 60° 11
1–11 Functions of 90° and 0° 12
1–12 Tables of trigonometric functions 15

CHAPTER 2

RIGHT TRIANGLES

2–1 Solution of right triangles 17
2–2 Application to surveying and navigation 18

CHAPTER 3

APPLICATION TO PHYSICS

3–1 Vectors 22
3–2 Geometric addition of vectors 22
3–3 Projection. Components of a vector 23
3–4 Resolution of vectors 24
3–5 Analytical addition of vectors 24
3–6 Friction 27
3–7 Condition of the equilibrium of a particle 27

CHAPTER 4

REDUCTION TO ACUTE ANGLES—CIRCULAR MEASURE

SECTION		PAGE
4–1	Absolute and numerical values	34
4–2	Reduction to acute angles	34
4–3	Functions of $(90° \pm \theta)$, $(180° \pm \theta)$, and $(360° \pm \theta)$	36
4–4	Circular measure of angles	37
4–5	Area of a sector of a circle	38
4–6	Relation between linear and angular velocities	39

CHAPTER 5

VARIATIONS AND GRAPHS OF TRIGONOMETRIC FUNCTIONS

5–1	Variation, range, and graph of $\sin x$	42
5–2	Variation, range, and graph of $\cos x$	44
5–3	Variation, range, and graph of $\tan x$	46
5–4	Graphs of $\cot x$, $\sec x$, and $\csc x$	48
5–5	Geometric addition of curves	48
5–6	Application to periodic phenomena	50
5–7	Function	52
5–8	Inverse function	52
5–9	Inverse trigonometric functions	53
5–10	Principal values of inverse trigonometric functions	54

CHAPTER 6

TRIGONOMETRIC IDENTITIES AND EQUATIONS

6–1	Equations	57
6–2	Fundamental identities	57
6–3	Solution of trigonometric equations	60

CHAPTER 7

ADDITION AND DERIVED FORMULAS

7–1	Addition formulas	63
7–2	Proof of $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$	63
7–3	Proof of $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	64
7–4	Proof of $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$	65
7–5	Double-angle formulas	67
7–6	Half-angle formulas	67
7–7	Product and sum formulas	69

CHAPTER 8

OBLIQUE TRIANGLES

SECTION		PAGE
8–1	Solution of oblique triangles	72
8–2	Law of sines	72
8–3	The law of cosines	73
8–4	Four cases	74
8–5	Case 1. Given one side and two angles	74
8–6	Case 2. The ambiguous case	76
8–7	Cases 3 and 4	78
8–8	First law of tangents	83
8–9	Area of a triangle	83
8–10	Second law of tangents	85

CHAPTER 9

LOGARITHMS

9–1	Logarithms	89
9–2	Laws of logarithms	89
9–3	The two bases	92
9–4	Characteristic and mantissa	92
9–5	Standard position of the decimal point	93
9–6	Table of logarithms	94
9–7	Significant figures	94
9–8	Interpolation	95
9–9	Computation by logarithms	96
9–10	Logarithm to the base e	98
	List of formulas	100
	Index	103
	Answers to problems	105
	TABLES	109

SYMBOLS

$=$	equals
$\doteq$	equals approximately
$\equiv$	is identical with
$\approx$	is equivalent to
$\neq$	is not equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\ll$	is very small, is negligible compared with
$\gg$	is very large compared with
α	varies as
$\rightarrow$	approaches or tends to
∞	is greater than any value, infin te
$\sum_{i=1}^{n} u_n$	the sum of u_i from i = 1 to i = n, that is, $u_1 + u_2 + \cdots + u_n$.

GREEK ALPHABET

Letters	Names	Letters	Names	Letters	Names
α	Alpha	ι	Iota	ρ	Rho
β	Beta	κ	Kappa	σ, Σ	Sigma
γ	Gamma	λ	Lambda	τ	Tau
δ, Δ	Delta	μ	Mu	υ	Upsilon
ϵ	Epsilon	ν	Nu	ϕ	Phi
ζ	Zeta	ξ	Xi	χ	Chi
η	Eta	o	Omicron	ψ	Psi
θ	Theta	π	Pi	ω	Omega

CHAPTER 1

TRIGONOMETRIC FUNCTIONS

1-1 Trigonometry. The word trigonometry is derived from the Greek words *trigon* (triangle) and *metria* (measurement). Plane trigonometry may be divided into geometrical and analytical sections. Geometrical trigonometry mainly treats problems connected with measurements of triangles, such problems as are met with in surveying and navigation. Analytical trigonometry is concerned with the study and application of certain characteristics of angles, called *trigonometric functions*, which are of great importance to pure as well as applied mathematics.

1-2 Angles. When a line OP rotates about the point O, Fig. 1-1, it is said to describe an angle. The initial position of the line,

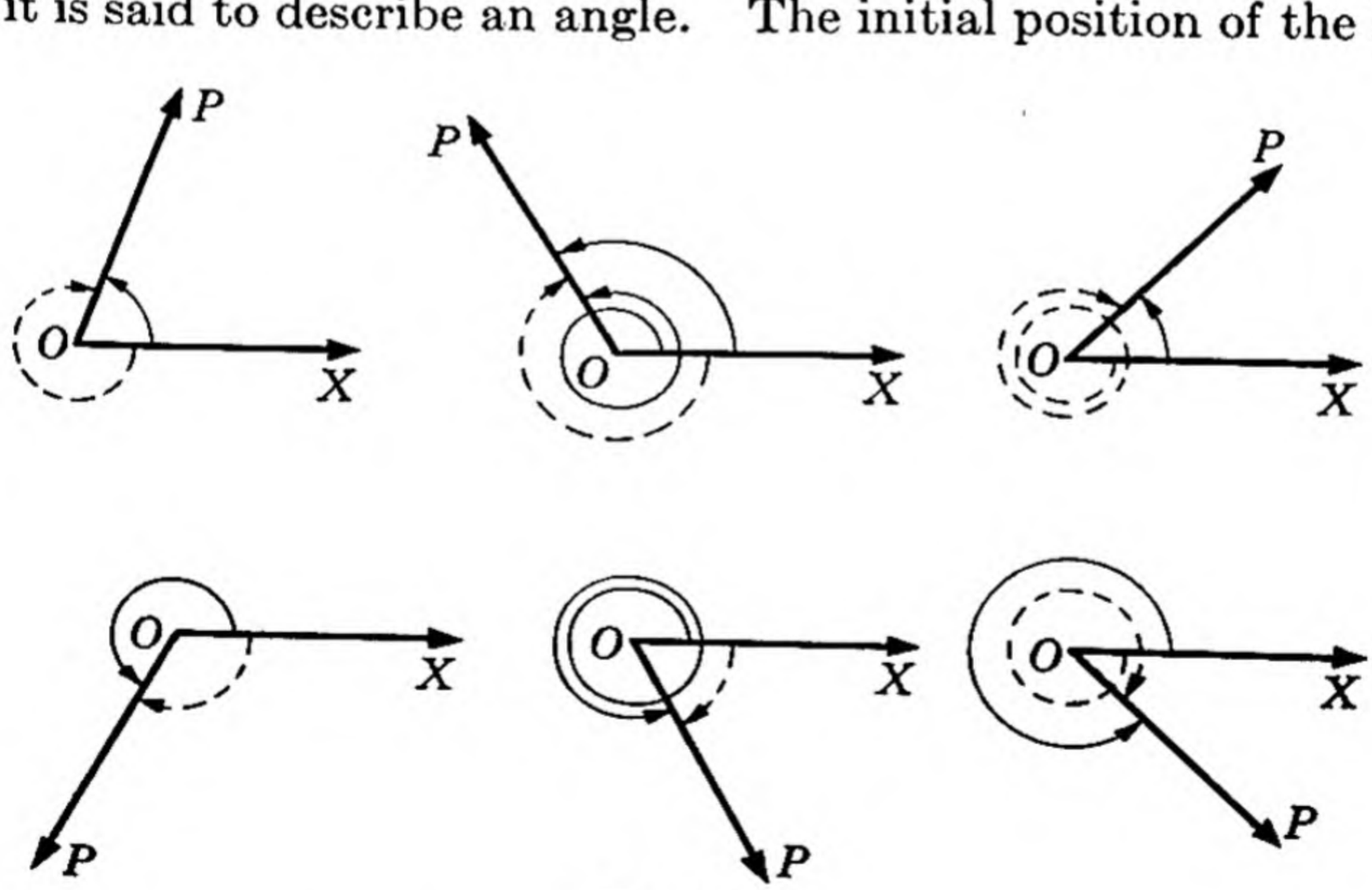


FIG. 1-1

OX, is called the *initial side;* the final position, OP, the *terminal side;* and the point of rotation, the *vertex* of the angle.

An angle may represent a partial rotation, one complete rotation, or more than one complete rotation. An angle is considered *positive* if the rotation is *counterclockwise*, and *negative* if the rotation is *clockwise*. The magnitude as well as the sign of an angle is indicated by a curved arrow. In Fig. 1-1, the continuous curved arrows indicate positive angles and the broken arrows indicate negative angles.

Angles which have the same initial and terminal sides are called *coterminal angles*. The positive and negative angles indicated in each part of Fig. 1-1 are coterminal.

1-3 Units of angle. The angle described in one complete rotation is divided into 360 parts, called *degrees* (Fig. 1-2). The degree

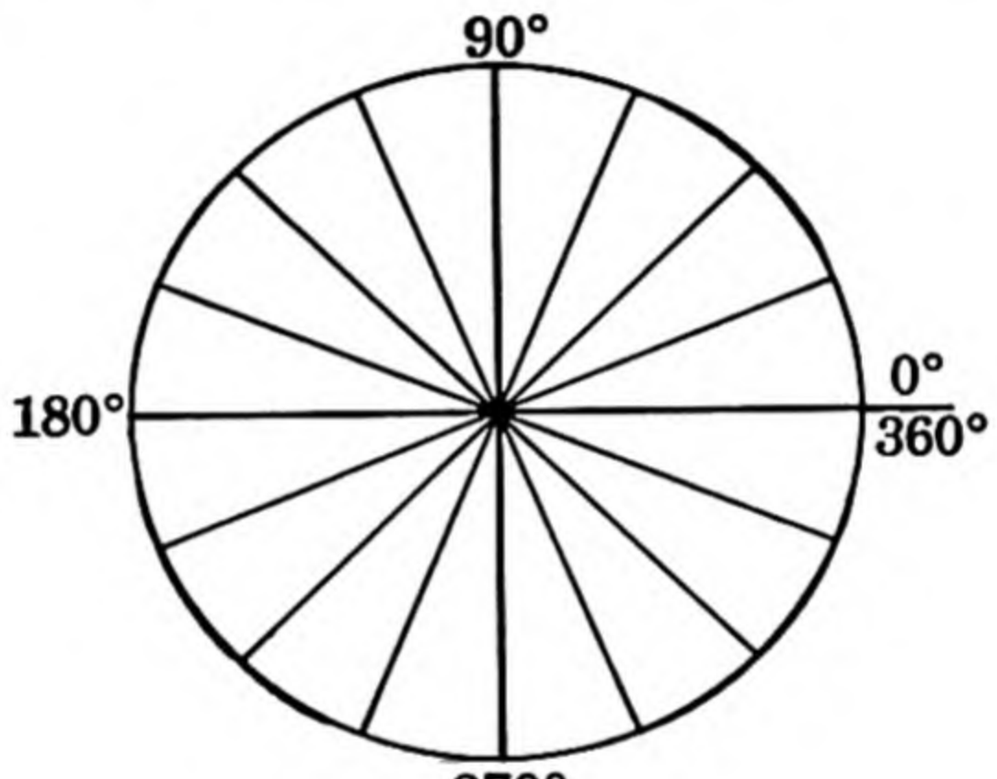


FIG. 1-2

is divided into 60 equal parts, called *minutes*. The minute is divided into 60 equal parts, called *seconds*. The symbols 1°, 1′, and 1″ are used to denote one degree, one minute, and one second, respectively. A second unit of angle, called the *radian*, is introduced in Chapter 4.*

1-4 Rectangular coordinate system. Trigonometric functions, mentioned in Section 1-1, are most conveniently defined by referring angles to a *system of rectangular coordinates*.

* A third unit of angle, called the *mil*, is used by the United States Army. It is defined by the equation 1600 *mils* = 90°, and approximately equals the angle subtended by one yard at a distance of one thousand yards.

The two mutually perpendicular lines of indefinite length through the point O, Fig. 1-3, are called *coordinate axes;* the horizontal line is called the *x-axis*, the vertical line the *y-axis*, the point O the *origin*, and the plane of the axes the *xy-plane*. The four regions into which the coordinate axes divide the plane are called *quadrants* and are numbered by Roman numerals, as in Fig. 1-3.

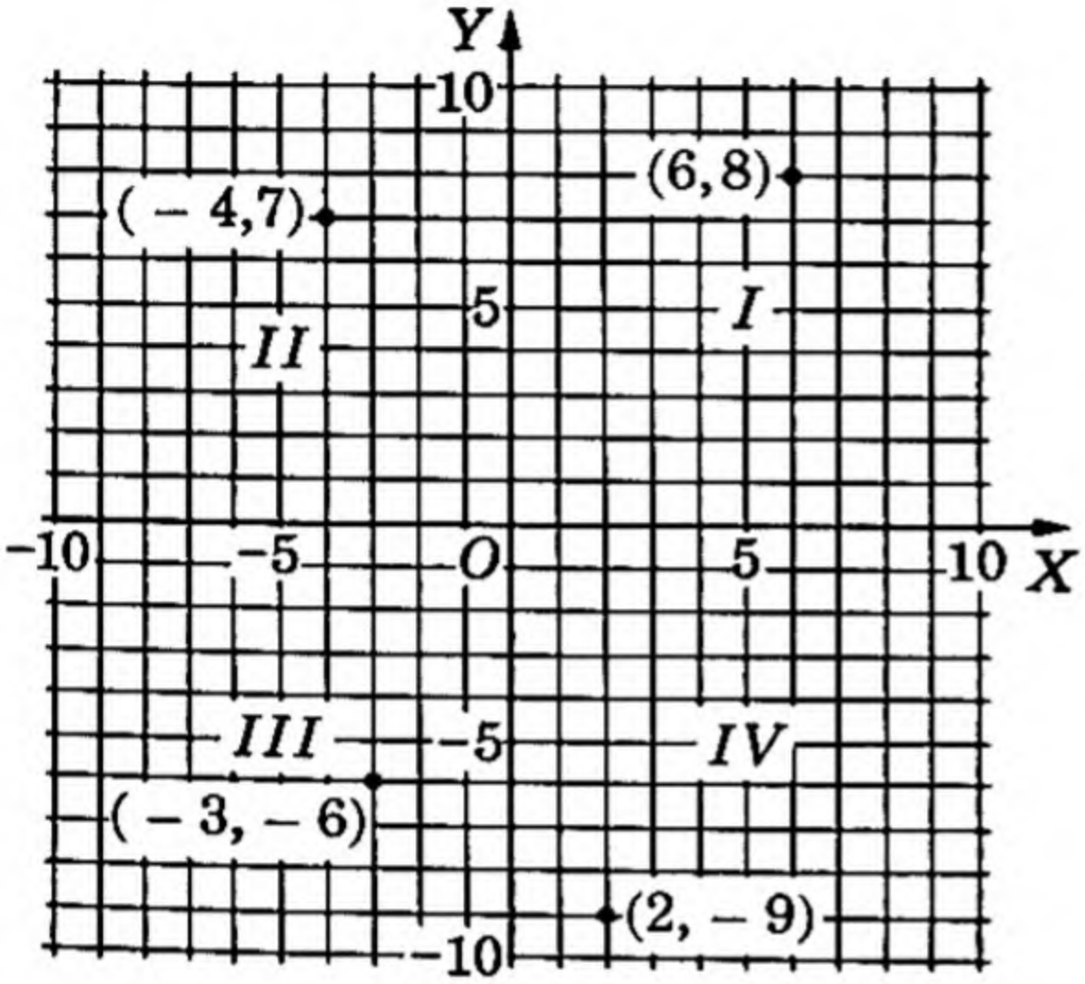


FIG. 1-3

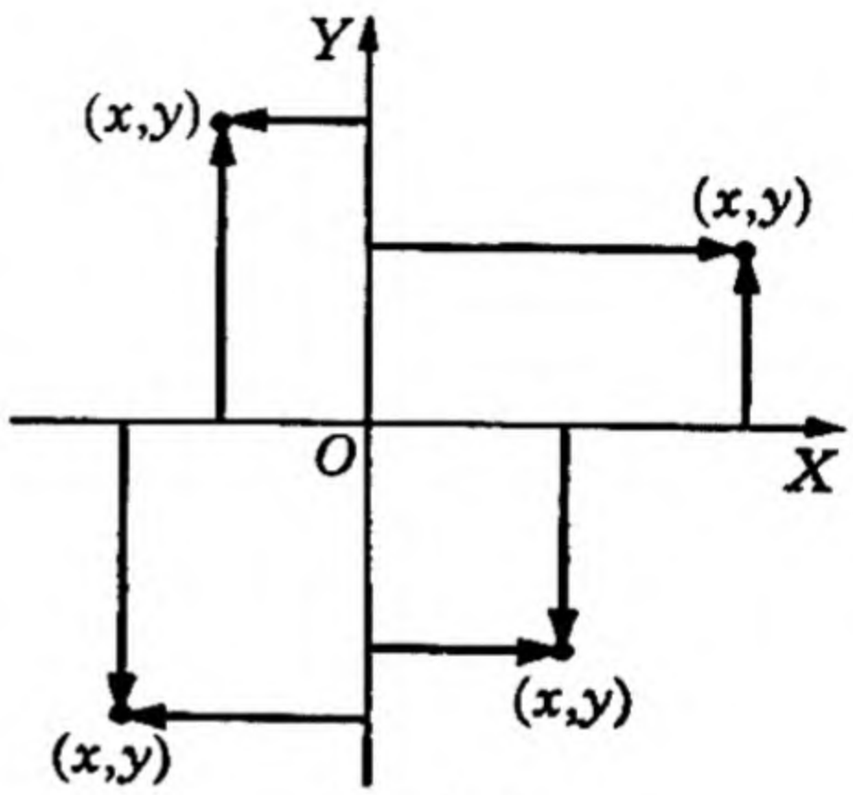


FIG. 1-4

Each coordinate axis is assigned positive and negative directions. The positive direction is left to right in the case of the x-axis and upward in the case of the y-axis, as indicated by the arrowheads. The negative direction is the opposite in each case. This convention gives a directional quality to a distance measured parallel to one of the axes; therefore such a distance is called a *directed distance.*

The position of a point on the xy-plane is specified by giving its directed distances from the coordinate axes. The directed distance of a point from the y-axis is called the *x-coordinate* or the *abscissa;* the directed distance from the x-axis is called the *y-coordinate*, or the *ordinate;* and the two distances together are called the *coordinates* of the point.

The coordinates of any point on the xy-plane are denoted by the symbol (x,y), where x is the abscissa and y the ordinate of the point. The directed lines (straight arrows), Fig. 1-4, indicate the

magnitudes as well as the directions of x and y. In the case of a particular point whose coordinates are given in numbers, the signs as well as the magnitudes of the coordinates are indicated, as shown in Fig. 1-3.

1-5 Standard position of an angle. An angle is said to be in the *standard position* if its vertex is at the origin of a rectangular system of axes and its initial side is OX. The positive and negative angles labeled θ* in Fig. 1-5 are in the standard position.

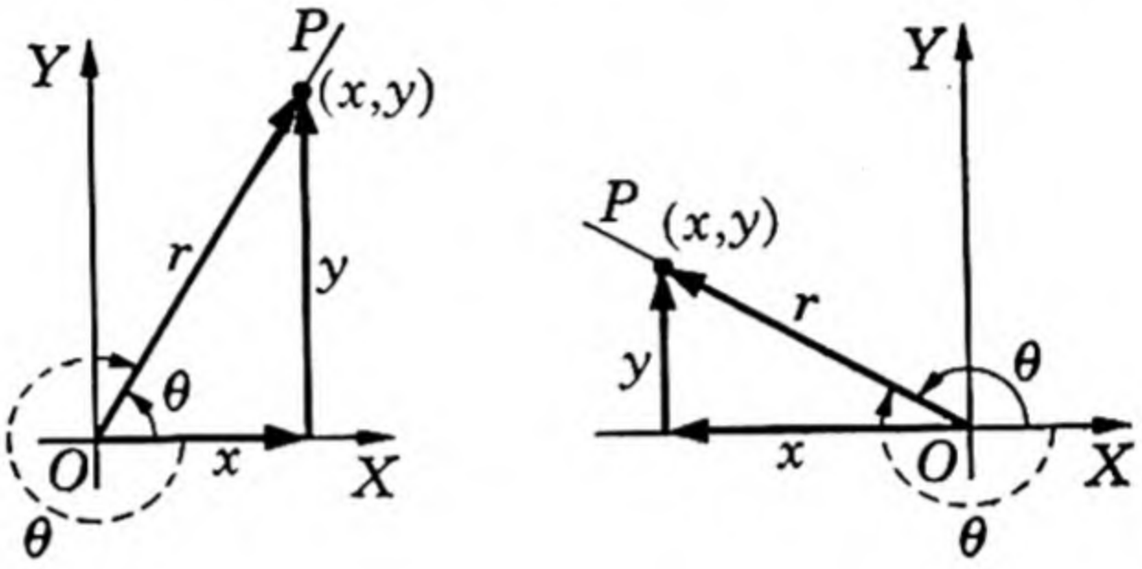


FIG. 1-5

Note: Although the positive and negative angles in Fig. 1-5 are not equal, they are labeled by the same symbol because this makes the presentation of the next section simpler.

1-6 Trigonometric functions. Let θ, Fig. 1-5, be any positive or negative angle in the standard position. From a point (x,y) on the terminal side of θ, drop a perpendicular on the x-axis to form a

* For the names of Greek letters and for meanings of unfamiliar symbols, see p. x.

right triangle. Place an arrowhead at the *terminus* of each side of the triangle—at the end away from the y-axis, the x-axis, or the origin in the case of x, y, or r, respectively. The directions of the arrows which form the legs of the triangle indicate the signs of x and y. The hypotenuse r, called the *radius vector*, is positive because it always points away from the origin.*

The six ratios, obtained by taking two of the sides of the right triangle at a time, are called *trigonometric functions*. The following table gives the names, the abbreviations, and the definitions of the functions.

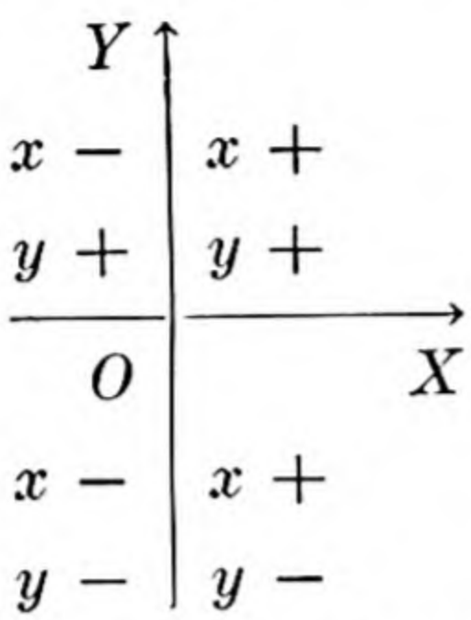
$$\begin{aligned}
\textbf{sine } \theta &= \textbf{sin } \theta = \frac{y}{r}, \\
\textbf{cosine } \theta &= \textbf{cos } \theta = \frac{x}{r}, \\
\textbf{tangent } \theta &= \textbf{tan } \theta = \frac{y}{x}, \\
\textbf{cotangent } \theta &= \textbf{cot } \theta = \frac{x}{y}, \\
\textbf{secant } \theta &= \textbf{sec } \theta = \frac{r}{x}, \\
\textbf{cosecant } \theta &= \textbf{csc } \theta = \frac{r}{y}.
\end{aligned} \tag{A}$$

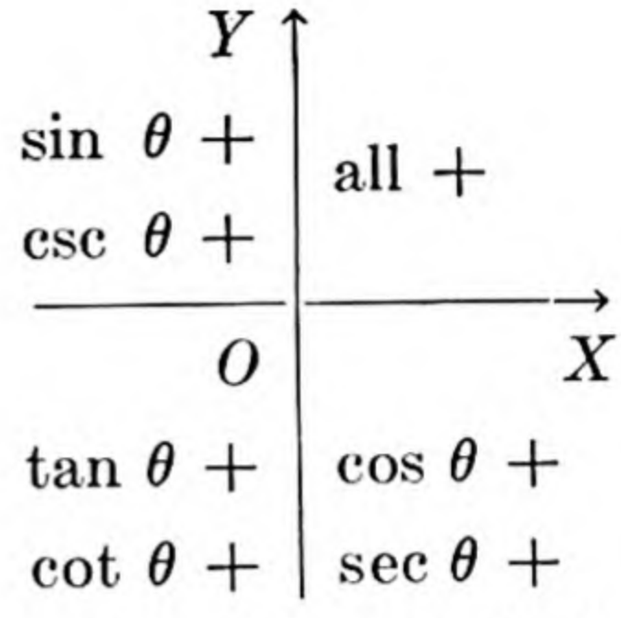
It is important to note the following points:

(a) The value of each of the ratios is not changed when the position of the point (x,y) is changed on the terminal side of the angle because the sides of the triangle change in the same proportion as the point (x,y) moves along the line OP. In other words, the value of each function depends on the angle θ, not on the size of the triangle. On the other hand, the value of each of the ratios changes as the angle changes. This is the reason for the name "trigonometric functions." (*Note:* If the value of a variable depends on the value of another variable, the former is said to be a *function* of the latter.)

* In polar coordinates of analytical geometry, the radius vector may be negative but in that case it does not form the terminal side of the angle. Instead, it represents a distance in the opposite direction from that of the terminal side.

(b) The signs of the functions are determined by the signs of x and y, since r is always positive. The signs of x and y, and of the functions which are positive in the four quadrants, may be indicated as follows:





(c) The value of a function is the same for all coterminal angles.

(d) A comparison of the definitions of the functions yields the following relations:

$$\csc\theta = \frac{1}{\sin\theta}, \qquad (1\text{-}1)$$

$$\sec\theta = \frac{1}{\cos\theta}, \qquad (1\text{-}2)$$

$$\cot\theta = \frac{1}{\tan\theta}, \qquad (1\text{-}3)$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad (1\text{-}4)$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}. \qquad (1\text{-}5)$$

EXAMPLES

1. Find the trigonometric functions of all the angles which are in the standard position and whose common terminal side passes through the point $(-3,4)$.

Solution: since $x = -3$ and $y = 4$, $r = \sqrt{x^2 + y^2} = 5$. Applying the definitions of the functions to Fig. 1-6, we get

$$\sin\theta = \frac{y}{r} = \frac{4}{5}, \qquad \cot\theta = \frac{x}{y} = -\frac{3}{4},$$

$$\cos\theta = \frac{x}{r} = -\frac{3}{5}, \qquad \sec\theta = \frac{r}{x} = -\frac{5}{3},$$

$$\tan\theta = \frac{y}{x} = -\frac{4}{3}, \qquad \csc\theta = \frac{r}{y} = \frac{5}{4}.$$

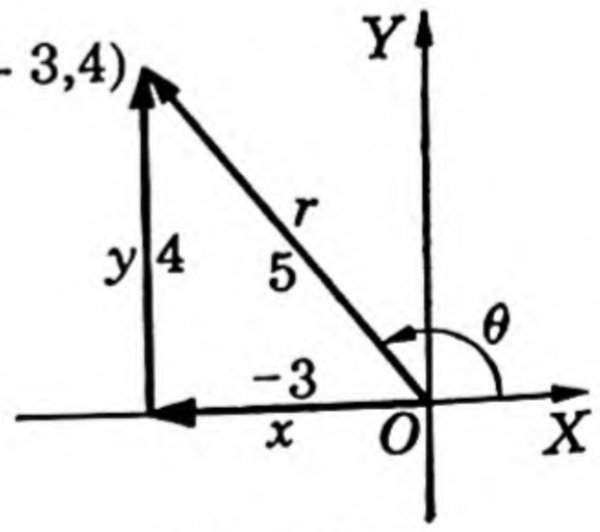


FIG 1-6.

Note: In Fig. 1-6 the positive obtuse angle is labeled θ but these results are valid for any coterminal angle.

2. Find the trigonometric functions of all angles for which $r = 10$ and $y = -5$.

Solution: In this case x has two possible values, namely,

$$x = \pm\sqrt{r^2 - y^2} = \pm\sqrt{75} = \pm 5\sqrt{3}.$$

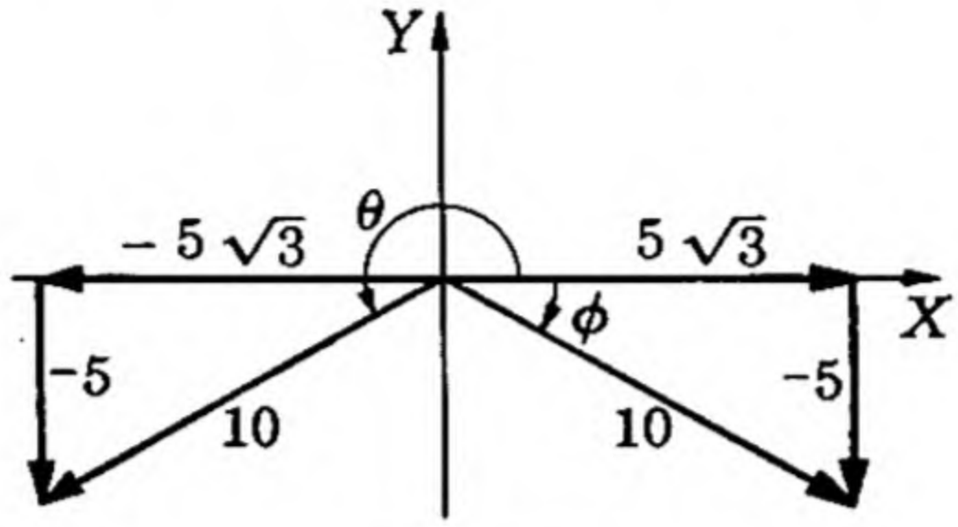


FIG. 1-7

Therefore, from Fig. 1-7, we have

$$\sin\theta = \frac{-5}{10} = -\frac{1}{2}, \qquad \sin\varphi = \frac{-5}{10} = -\frac{1}{2},$$

$$\cos\theta = \frac{-5\sqrt{3}}{10} = -\frac{\sqrt{3}}{2}, \qquad \cos\varphi = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2},$$

$$\tan\theta = \frac{-5}{-5\sqrt{3}} = \frac{1}{\sqrt{3}}, \qquad \tan\varphi = \frac{-5}{5\sqrt{3}} = -\frac{1}{\sqrt{3}},$$

$$\cot\theta = \sqrt{3}, \qquad \cot\varphi = -\sqrt{3},$$

$$\sec\theta = -\frac{2}{\sqrt{3}}, \qquad \sec\varphi = \frac{2}{\sqrt{3}},$$

$$\csc\theta = -2, \qquad \csc\varphi = -2.$$

3. Given $\cos\theta = \frac{2}{3}$, find the other functions of θ.

Solution: It is evident from Fig. 1-8 that for both angles labeled θ, $\cos\theta = \frac{2}{3}$.

Therefore

$$\sin\theta = \pm\frac{\sqrt{5}}{3}, \qquad \cot\theta = \pm\frac{2}{\sqrt{5}},$$

$$\cos\theta = \frac{2}{3}, \qquad \sec\theta = \frac{3}{2},$$

$$\tan\theta = \pm\frac{\sqrt{5}}{2}, \qquad \csc\theta = \pm\frac{3}{\sqrt{5}}.$$

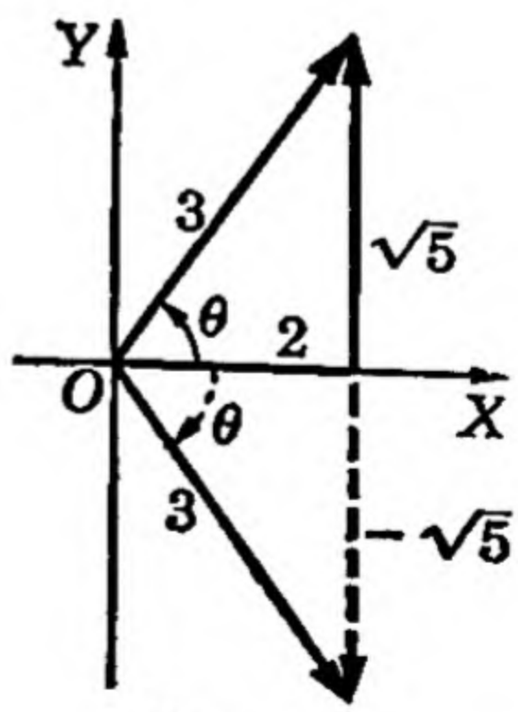


FIG. 1-8

PROBLEMS

Each of the following points is on the terminal side of an angle θ in the standard position. Find the values of the trigonometric functions of θ.

1. (1,2).
2. (3,−4).
3. (−2,5).
4. (7,−6).
5. (6,−4).
6. (8,2).
7. (−9,−3).
8. $(\sqrt{2},\sqrt{6})$.
9. $(-\sqrt{3},\sqrt{6})$.
10. $(-\sqrt{2},1)$.
11. (−2,−4).
12. (−6,1).
13. $(-2,\sqrt{2})$.
14. $(4,2\sqrt{2})$.
15. $(-\sqrt{2},-4)$.

If θ is in the standard position, in what quadrants does the terminal side of θ lie under the following conditions? (*Note:* "> 0" means "is positive," "< 0" means "is negative.")

16. $\sin\theta > 0$.
17. $\cos\theta > 0$.
18. $\tan\theta > 0$.
19. $\cot\theta > 0$.
20. $\sec\theta > 0$.
21. $\csc\theta > 0$.
22. $\sin\theta < 0$.
23. $\cos\theta < 0$.
24. $\tan\theta < 0$.
25. $\cot\theta < 0$.
26. $\sec\theta < 0$.
27. $\csc\theta < 0$.
28. $\sin\theta > 0$, $\cos\theta > 0$.
29. $\cos\theta > 0$, $\sin\theta < 0$.
30. $\sin\theta > 0$, $\tan\theta > 0$.
31. $\cos\theta > 0$, $\tan\theta < 0$.
32. $\sin\theta > 0$, $\cos\theta < 0$.
33. $\cos\theta < 0$, $\tan\theta < 0$.
34. $\sin\theta < 0$, $\tan\theta < 0$.
35. $\sin\theta < 0$, $\cos\theta < 0$.

Under each of the following conditions, locate the position of the terminal side of θ and find the values of the other functions.

36. $\sin\theta = \frac{1}{2}$, $\cos\theta > 0$.
37. $\cos\theta = \frac{3}{4}$, $\sin\theta < 0$.
38. $\sin\theta = -\frac{2}{3}$, $\cos\theta > 0$.
39. $\tan\theta = \frac{4}{3}$, $\cos\theta < 0$.
40. $\cot\theta = \frac{1}{2}$, $\sin\theta > 0$.
41. $\cos\theta = -\frac{4}{5}$, $\sin\theta < 0$.
42. $\sin\theta = -\frac{1}{2}$, $\cos\theta < 0$.
43. $\tan\theta = -3$, $\sin\theta < 0$.
44. $\tan\theta = -1$, $\cos\theta > 0$.
45. $\cot\theta = 2$, $\cos\theta < 0$.

1-7 Functions of a positive acute angle. In defining functions of any angle, the angle is placed at the standard position because the signs of x and y must be taken into account. An inspection of Fig. 1-9 shows that both x and y are positive when the angle is a positive acute angle. Therefore functions of a positive acute angle may be defined in terms of the sides of a right triangle, with-

out reference to coordinate axes (Fig. 1-10). Thus if α is a positive acute angle, we may make α an acute angle of a right triangle,

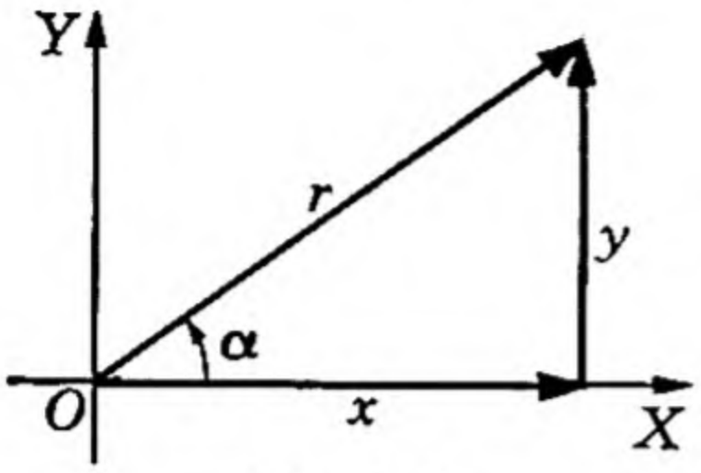


FIG. 1-9

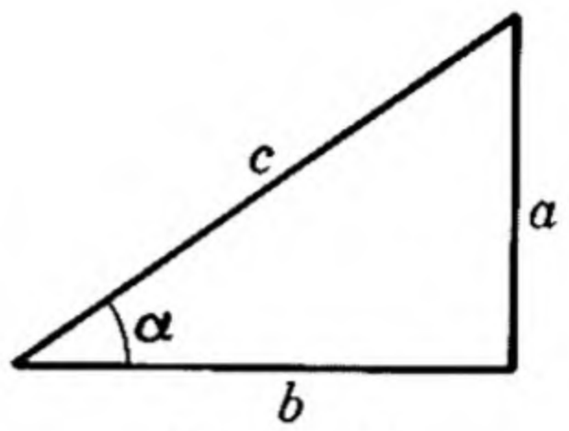


FIG. 1-10

label the sides of the triangle as in Fig. 1-10, and then define the functions of α as follows:

$$\sin \alpha = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}},$$

$$\cos \alpha = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}},$$

$$\tan \alpha = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}},$$

$$\cot \alpha = \frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}},$$

$$\sec \alpha = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}},$$

$$\csc \alpha = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}}. \qquad (A')$$

1-8 Functions of complementary angles. The two acute angles of a right triangle are called *complementary angles*. Applying the verbal definitions of the functions given in (A′) to the complementary angles α and β of Fig. 1-11 and comparing the two sets of functions, we find that

$$\sin \beta = \cos \alpha,$$
$$\cos \beta = \sin \alpha,$$
$$\tan \beta = \cot \alpha,$$
$$\cot \beta = \tan \alpha,$$
$$\sec \beta = \csc \alpha,$$
$$\csc \beta = \sec \alpha.$$

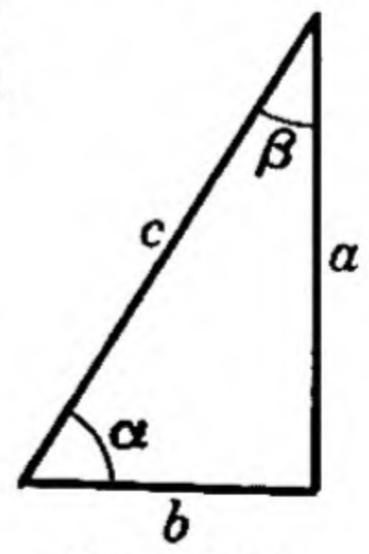


FIG. 1-11

The pairs of functions (sine and cosine, secant and cosecant, and tangent and cotangent) are called *cofunctions*. Thus, each function of an angle equals the corresponding cofunction of the complementary angle, i.e.:

$$\textbf{function of } (90^\circ - \alpha) = \textbf{cofunction of } \alpha. \tag{B}$$

EXAMPLES

1. Given $b = 4$ and $c = 5$, find the functions of the complementary angles α and β.

Solution: $a = \sqrt{5^2 - 4^2} = 3$. Therefore from Fig. 1-12 we have

$$\sin\ \alpha = \frac{3}{5}, \qquad \sin\ \beta = \frac{4}{5},$$

$$\cos\ \alpha = \frac{4}{5}, \qquad \cos\ \beta = \frac{3}{5},$$

$$\tan\ \alpha = \frac{3}{4}, \qquad \tan\ \beta = \frac{4}{3},$$

$$\cot\ \alpha = \frac{4}{3}, \qquad \cot\ \beta = \frac{3}{4},$$

$$\sec\ \alpha = \frac{5}{4}, \qquad \sec\ \beta = \frac{5}{3},$$

$$\csc\ \alpha = \frac{5}{3}, \qquad \csc\ \beta = \frac{5}{4}.$$

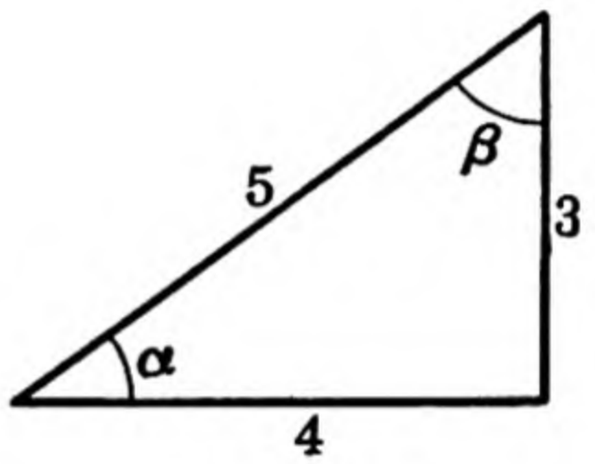


FIG. 1-12

2. If α is a positive acute angle and $\tan\ \alpha = \frac{1}{2}$, find the other functions of α, and the functions of β, the complement of α.

Solution: We construct a right triangle, Fig. 1-13, label the acute angles, and assign 1 to the side opposite α and 2 to the adjacent side, thus making $\tan\ \alpha = \frac{1}{2}$. Then we find the hypotenuse and obtain

$$\sin\ \alpha = \frac{1}{\sqrt{5}}, \qquad \sin\ \beta = \frac{2}{\sqrt{5}},$$

$$\cos\ \alpha = \frac{2}{\sqrt{5}}, \qquad \cos\ \beta = \frac{1}{\sqrt{5}},$$

$$\tan\ \beta = 2,$$

$$\cot\ \alpha = 2, \qquad \cot\ \beta = \frac{1}{2},$$

$$\sec\ \alpha = \frac{\sqrt{5}}{2}, \qquad \sec\ \beta = \sqrt{5},$$

$$\csc\ \alpha = \sqrt{5}. \qquad \csc\ \beta = \frac{\sqrt{5}}{2}.$$

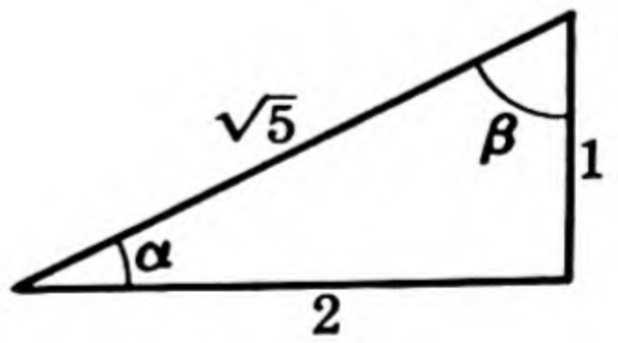


FIG. 1-13

PROBLEMS

From each of the following sets of data, find the values of the functions of the acute angles of the corresponding right triangle.

1. $a = 8, b = 6.$
2. $a = 8, c = 10.$
3. $b = 3, c = 5.$
4. $a = 1, b = 3.$
5. $a = 3, b = 4.$
6. $a = 4, b = 3.$
7. $b = 1, c = \sqrt{2}.$
8. $a = 5, b = 10.$
9. $b = 4, c = 5.$
10. $b = 5, c = 10.$
11. $a = 3, c = 5.$
12. $a = 2, b = 4.$
13. $b = 3, c = 7.$
14. $a = 2, c = 3.$
15. $a = 6, b = 8.$

In each of the following problems α and β denote the acute angles of a right triangle; find the other functions of both angles.

16. $\sin \alpha = \frac{1}{2}.$
17. $\cos \alpha = \frac{3}{4}.$
18. $\tan \alpha = \frac{2}{3}.$
19. $\cot \alpha = \frac{2}{5}.$
20. $\sec \alpha = \frac{4}{3}.$
21. $\csc \alpha = \frac{3}{2}.$
22. $\sin \alpha = \frac{5}{6}.$
23. $\tan \alpha = \frac{3}{2}.$
24. $\csc \beta = \frac{4}{3}.$
25. $\cot \beta = \frac{5}{2}.$
26. $\cos \beta = \frac{2}{5}.$
27. $\tan \beta = 2.$
28. $\cot \beta = 1.$
29. $\csc \beta = 3.$
30. $\sec \beta = 2.$

Verify the following equations.

31. $\sin \alpha \cos (90° - \alpha) \sec (90° - \alpha) = \sin \alpha.$
32. $\cos \alpha \cos (90° - \alpha) \sin (90° - \alpha) \csc \alpha = \cos^2 \alpha.$
33. $\tan (90° - \alpha) \csc (90° - \alpha) \cos \alpha = \cot \alpha.$
34. $\cot (90° - \alpha) \sec (90° - \alpha) \sin \alpha = \tan \alpha.$

1-9 Functions of 45°. If one angle of a right triangle is 45°, the other acute angle also is 45° and the triangle is isosceles. Letting each leg of the triangle equal 1, Fig. 1-14, we have

$$\sin 45° = \frac{1}{\sqrt{2}}, \qquad \csc 45° = \sqrt{2},$$

$$\cos 45° = \frac{1}{\sqrt{2}}, \qquad \sec 45° = \sqrt{2},$$

$$\tan 45° = 1, \qquad \cot 45° = 1.$$

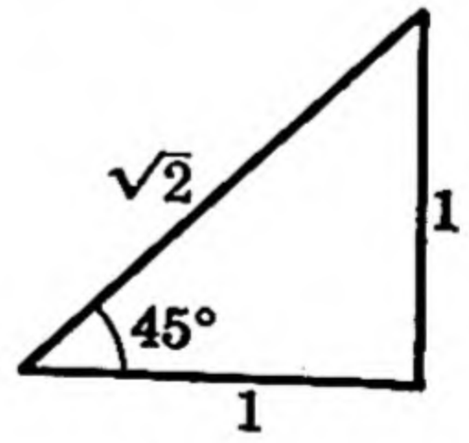


FIG. 1-14

1-10 Functions of 30° and 60°. Draw an equilateral triangle and drop a perpendicular from one vertex to the opposite side,

Fig. 1-15. The acute angles of the right triangles thus obtained are 60° and 30°. Letting one side of the equilateral triangle equal 2, we find the other sides of one of the right triangles and obtain the following values for the functions of 60° and 30°:

$$\sin 60° = \frac{\sqrt{3}}{2}, \qquad \sin 30° = \frac{1}{2},$$

$$\cos 60° = \frac{1}{2}, \qquad \cos 30° = \frac{\sqrt{3}}{2},$$

$$\tan 60° = \sqrt{3}, \qquad \tan 30° = \frac{1}{\sqrt{3}},$$

$$\cot 60° = \frac{1}{\sqrt{3}}, \qquad \cot 30° = \sqrt{3},$$

$$\sec 60° = 2, \qquad \sec 30° = \frac{2}{\sqrt{3}},$$

$$\csc 60° = \frac{2}{\sqrt{3}}. \qquad \csc 30° = 2.$$

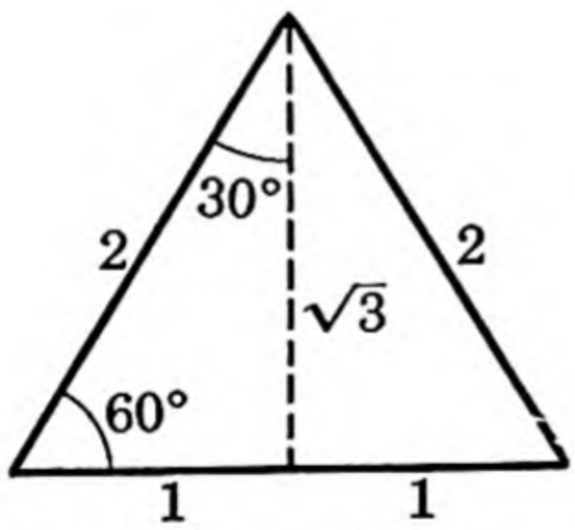


Fig. 1-15

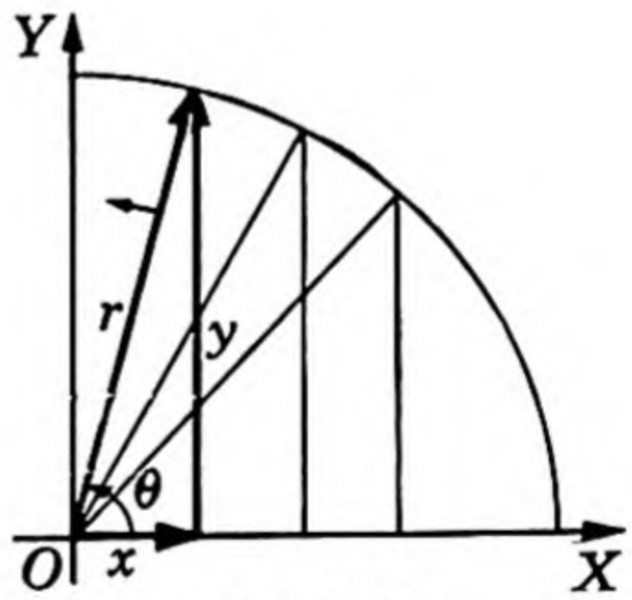


Fig. 1-16

1-11 Functions of 90° and 0°. Let the curve in Fig. 1-16 be the quadrant of a circle of radius r. Then as θ increases and approaches 90°, r remains constant, y increases and approaches r in length, and x decreases and approaches zero; that is,

$$\text{as } \theta \to 90°, \; y \to r, \text{ and } x \to 0.$$

Therefore

$$\sin 90° = \frac{r}{r} = 1,$$

$$\cos 90° = \frac{0}{r} = 0,$$

$$\cot 90° = \frac{0}{r} = 0,$$

$$\csc 90° = \frac{r}{r} = 1.$$

On the other hand, as θ decreases and approaches 0°, Fig. 1-17, y approaches zero, and x approaches r; that is,

$$\text{as } \theta \to 0°,\ y \to 0,\ x \to r.$$

Therefore

$$\sin 0° = \frac{0}{r} = 0,$$

$$\cos 0° = \frac{r}{r} = 1,$$

$$\tan 0° = \frac{0}{r} = 0,$$

$$\sec 0° = \frac{r}{r} = 1.$$

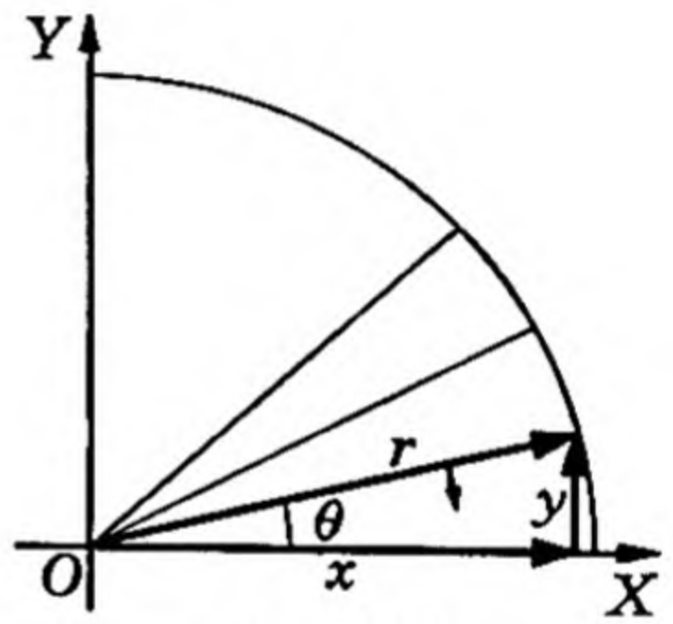


FIG. 1-17

Tan 90°, sec 90°, cot 0°, and csc 0° require special consideration. As θ approaches 90°, y approaches r, x approaches zero, and $\tan\theta$ $\left(=\frac{y}{x}\right)$ increases indefinitely.* This is clearly brought out in Fig. 1-18, where P is the point of intersection of the extension of r with the vertical line through A, $y' = AP$, and $\tan\theta = \frac{y}{x} = \frac{y'}{r}$. As θ increases and approaches 90°, P moves up and y' increases indefinitely, that is, becomes infinite.

---

* If the denominator of a fraction approaches zero while the numerator remains constant (other than zero) or increases, the value of the fraction increases indefinitely if it is positive and decreases indefinitely if it is negative.

It is convenient to use the symbol ∞ to denote infinity and to write

$$\tan 90^\circ = \infty.$$

The student is warned against taking this to be an equation in the strict sense of the term. Infinity is not a definite number; consequently it is not proper to say "tan 90° equals infinity." It will be shown in Chapter 5 that tan θ decreases indefinitely, that is, becomes $-\infty$, as θ approaches 90° from the side of the second quadrant. For this reason, one should write $\tan 90^\circ = \pm\infty$.

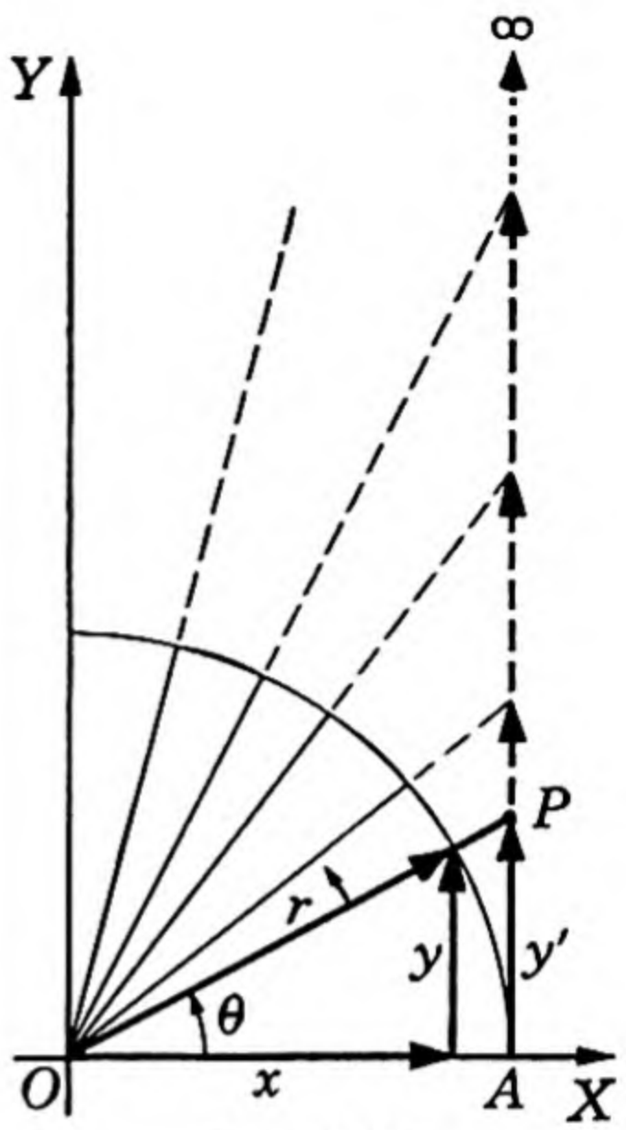


FIG. 1-18

By similar considerations and with similar reservations, we have

$$\sec 90^\circ = \pm\infty, \cot 0^\circ = \pm\infty, \text{ and } \csc 0^\circ = \pm\infty.$$

PROBLEMS

Find the exact value of each of the following expressions.

1. $\sin^2 60^\circ + \cos^2 30^\circ$.
2. $\sin^2 30^\circ + \cos^2 60^\circ$.
3. $\sin^2 60^\circ + \cos^2 45^\circ$.
4. $\sin^2 60^\circ - \sin^2 30^\circ$.
5. $\cos^2 45^\circ - \sin^2 30^\circ$.
6. $\cos^2 60^\circ - \cos^2 30^\circ$.
7. $(\sin 60^\circ + \sin 30^\circ)(\sin 60^\circ - \cos 30^\circ)$.
8. $(\cos 45^\circ - \sin 30^\circ)(\cos 45^\circ + \sin 30^\circ)$.
9. $(\cos 60^\circ + \cos 30^\circ)(\cos 60^\circ - \cos 30^\circ)$.
10. $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$.
11. $\tan 45^\circ \cot 45^\circ - \sin 30^\circ \csc 30^\circ$.
12. $\tan 60^\circ \cot 30^\circ + \tan 30^\circ \cot 60^\circ$.

Verify the following equations by introducing the exact values of the functions.

13. $\sin^2 30^\circ + \cos^2 30^\circ = 1$.
14. $\sin^2 45^\circ + \cos^2 45^\circ = 1$.
15. $\sin^2 60^\circ + \cos^2 60^\circ = 1$.
16. $1 + \tan^2 60^\circ = \sec^2 60^\circ$.
17. $1 + \tan^2 30^\circ = \sec^2 30^\circ$.
18. $1 + \cot^2 60^\circ = \csc^2 60^\circ$.
19. $2 \sin 30^\circ \cos 30^\circ = \sin 60^\circ$.
20. $\cos^2 30^\circ - \sin^2 30^\circ = \cos 60^\circ$.
21. $(1 + \tan 45^\circ)(1 - \tan 45^\circ) = 0$.
22. $2 \sin 45^\circ \cos 45^\circ = 1$.
23. $1 + \tan^2 45^\circ = \sec^2 45^\circ$.
24. $1 + \cot^2 45^\circ = \csc^2 45^\circ$.
25. $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = 1$.
26. $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = 0$.
27. $(1 - \tan^2 30^\circ) \tan 60^\circ = 2 \tan 30^\circ$.
28. $(1 + \tan 30^\circ \tan 60^\circ) \tan 30^\circ = \tan 60^\circ - \cot 60^\circ$.
29. $(\sin 30^\circ + \cos 60^\circ)(\sec 45^\circ + \csc 45^\circ) = 2\sqrt{2}$.
30. $(\tan 45^\circ - \sin 30^\circ)(\cot 60^\circ + \cos 45^\circ) = \dfrac{2\sqrt{3} + 3\sqrt{2}}{12}$.

1-12 Tables of trigonometric functions.* Functions of most angles can not be found by such elementary methods as those of the preceding three sections. Furthermore, their exact numerical values can not be found by any method. Therefore tables are prepared which give approximate values of the first four functions. Table IV (at the back of the book) is called a table of *natural trigonometric functions.* It gives the values of the functions to five decimals. Table III gives the logarithms of the functions to the same number of decimals.

Neither table gives secants or cosecants because their inclusion would greatly increase the size of the tables. Since these functions are reciprocals of sines and cosines, however, the tables can be used to find their values.

The tables contain only positive acute angles because functions of all other angles can be reduced to functions of positive acute angles. The use of the tables is similar to the use of a table of logarithms of numbers, with the following exceptions:

Trigonometric tables are read from the top down for angles between 0° and 45° and from the bottom up for angles between 45° and 90°.

In Table III the logarithms are 10 units greater than the logarithms of the functions; therefore -10 should be appended to the logarithm of a function obtained from this table. For example, log sin 34° should be written as $9.74756 - 10$, although it is given as 9.74756 in the table.

EXAMPLES

1. Find cos 25° 48.4′ and log cos 25° 48.4′.

Solution: (a) From Table IV, we have

$$\left.\begin{array}{l}\cos 25^\circ\, 48' = 0.90032\\ \cos 25^\circ\, 49' = 0.90019\end{array}\right| -13,$$

where -13 is the tabular difference, that is, the change in the value of the cosine due to an increase of the angle by 1′. But since 25° 48.4′ is 0.4′ greater than 25° 48′, its cosine is found by adding to the cos 25° 48′ the whole number nearest to $0.4 \times (-13)$, the proportional part. Hence

$$\cos 25^\circ\, 48.4' = 0.90027.$$

* If the class has not studied logarithms, Chapter 9 should be studied before proceeding further.

This method of finding intermediate values is called *interpolation.**

(b) From Table III we get

$$\left.\begin{array}{l}\log \cos 25^\circ 48' = 9.95440 - 10 \\ \log \cos 25^\circ 49' = 9.95434 - 10\end{array}\right| -6.$$

In this case interpolation gives

$$\log \cos 25^\circ 48.4' = 9.95438 - 10.$$

2. Given $\log \tan \alpha = 0.13412$; find α.

Solution: Since this logarithm is positive, $\tan \alpha$ must be greater than unity; therefore α must be greater than 45°. Looking over the columns marked *log tan* at the bottom of Table III, we find that α falls between 53° 42′ and 53° 43′.

$$\left.\begin{array}{l}\log \tan 53^\circ 42' = 10.13397 - 10 \\ \log \tan \alpha \quad\quad = 10.13412 - 10\end{array}\right] 15 \Big] 26.$$
$$\log \tan 53^\circ 43' = 10.13423 - 10 \quad\quad \Big]$$

Therefore α is greater than 53° 42′ by an amount which equals about $\frac{15}{26}$ of one minute, that is,

$$\alpha = 53^\circ 42.6'.$$

PROBLEMS

Use Tables III and IV to find the values and the logarithms of the following:

1. sin 19° 26′.
2. cos 31° 39′.
3. tan 58° 41′.
4. cot 35° 17′.
5. sin 65° 14′.
6. cos 52° 48′.
7. tan 77° 22′.
8. cot 46° 59′.
9. sin 38° 26.4′.
10. cos 42° 17.8′.
11. tan 34° 54.1′.
12. cot 11° 49.5′.
13. sin 74° 23.1′.
14. cos 47° 19.9′.
15. tan 82° 25.6′.
16. cot 50° 51.3′.

Use Table IV to find the positive acute angle α from the following data:

17. $\sin \alpha = 0.40756$.
18. $\cos \alpha = 0.51419$.
19. $\tan \alpha = 0.63402$.
20. $\cot \alpha = 4.2327$.
21. $\sin \alpha = 0.83012$.
22. $\cos \alpha = 0.10324$.
23. $\tan \alpha = 12.194$.
24. $\cot \alpha = 4.7572$.

Use Table III to find the positive acute angle α from the following data:

25. $\log \sin \alpha = 9.66132 - 10$.
26. $\log \cos \alpha = 9.52445 - 10$.
27. $\log \tan \alpha = 9.87485 - 10$.
28. $\log \cot \alpha = 10.37972 - 10$.
29. $\log \sin \alpha = 9.90635 - 10$.
30. $\log \cos \alpha = 9.48120 - 10$.
31. $\log \tan \alpha = 10.15142 - 10$.
32. $\log \cot \alpha = 9.51612 - 10$.

---

* A fuller explanation of interpolation is given in Sec. 9-8.

CHAPTER 2

RIGHT TRIANGLES

2-1 Solution of right triangles. The three sides and the three angles of a triangle are called *parts* of the triangle. If three of the parts are given, at least one of which is a side, the triangle can be solved; that is, the other parts can be determined.

In the case of a right triangle one of the parts, the right angle, is known; therefore it can be solved when any two sides or when one side and one acute angle are given.

EXAMPLES

1. Given $a = 2$ and $b = 4$; solve the right triangle.

Solution: Using the notation of Fig. 2-1, we have

$$c = \sqrt{a^2 + b^2}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}.$$

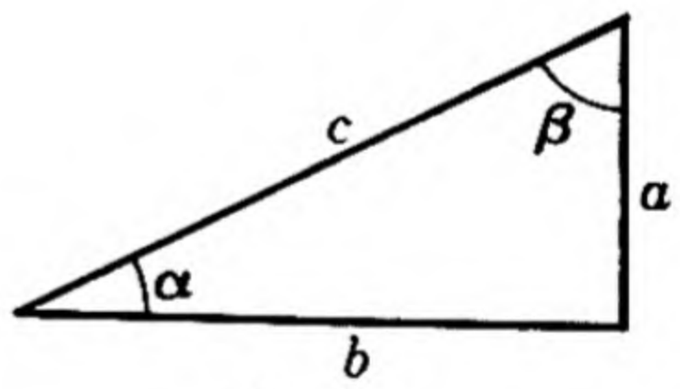


FIG. 2-1

$$\tan \alpha = \frac{a}{b} = 0.50000,$$
$$\alpha = 26^\circ\, 33.9',$$
$$\beta = 90^\circ - \alpha = 63^\circ\, 26.1'.$$

2. Given $c = 12.435$ and $\beta = 38^\circ\, 20.1'$; using logarithms, solve the triangle and find its area.

Solution: Using the notation of Fig. 2-1, we have

$$\alpha = 90^\circ - \beta = 51^\circ\, 39.9'.$$

$$a = c \cos \beta.$$

$$\begin{aligned} \log c &= 1.09464 \\ \log \cos \beta &= \underline{9.89454 - 10} \\ \log a &= 0.98918 \end{aligned}$$

$$a = 9.7540.$$

$$b = c \sin \beta.$$

$$\begin{aligned} \log c &= 1.09464 \\ \log \sin \beta &= \underline{9.79258 - 10} \\ \log b &= 0.88722 \end{aligned}$$

$$b = 7.713.$$

$$A = \frac{1}{2}ab$$
$$= \frac{1}{2}c\cos\beta \cdot c\sin\beta$$
$$= \frac{1}{2}c^2\sin\beta\cos\beta.$$

$$\log \tfrac{1}{2} = 9.69897 - 10$$
$$\log c = 1.09464$$
$$\log c = 1.09464$$
$$\log \sin\beta = 9.79258 - 10$$
$$\log \cos\beta = \underline{9.89454 - 10}$$
$$\log A = 1.57537$$
$$A = 37.616.$$

Note: In computing the area, the logarithms of a and b can be used. But this is not desirable because errors made in the computation of a and b would be carried into the value of A. *It is almost always desirable to express each required magnitude in terms of the symbols of the given magnitudes before computing its value.* (For suggestions on how to solve problems efficiently, consult the author's booklet, *How to Study—How to Solve*, Addison-Wesley Publishing Company.)

2-2 Application to surveying and navigation. Before applying the solution of the right triangle to problems of surveying and navigation, it is necessary to understand the meanings of the following technical terms which may appear in such problems.

(*a*) *Angle of inclination.* The angle which a line or a plane makes with the horizontal is called the *angle of inclination*.

(*b*) *Angles of elevation and depression.* The angle which the line of sight to an object makes with the horizontal is called the *angle of elevation* if the object viewed is at a higher level than the eye of the observer, and the *angle of depression* if the object viewed is at a lower level, Fig. 2-2.

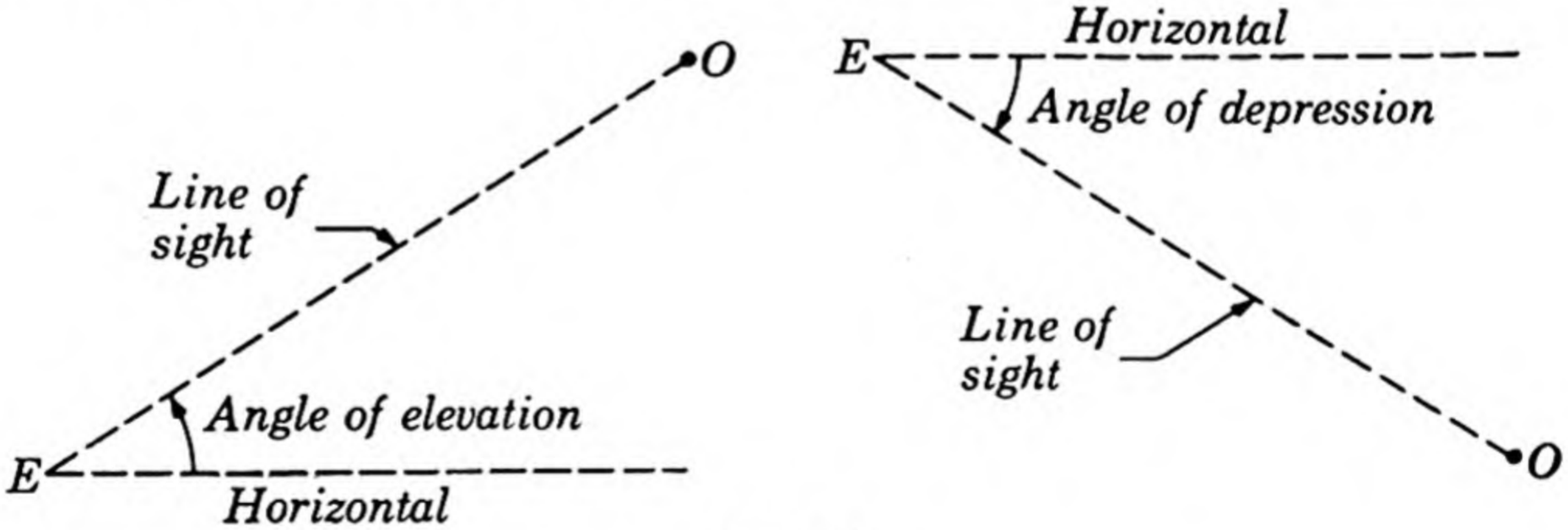


FIG. 2-2

(*c*) *Bearing.* The acute angle which a line OP makes with the north-south line is called the *bearing* of P from O. In Fig. 2-3, the bearings of P_1, P_2, P_3 and P_4 from O are respectively:

North 55° East, or N 55° E, South 20° West, or S 20° W,
North 30° West, or N 30° W, South 70° East, or S 70° E.

In surveying and navigation, the term "bearing" is defined as the angle OP makes with ON (the true North), measured in the clockwise direction. However, here we shall use the first definition because in this chapter we are concerned with acute angles.

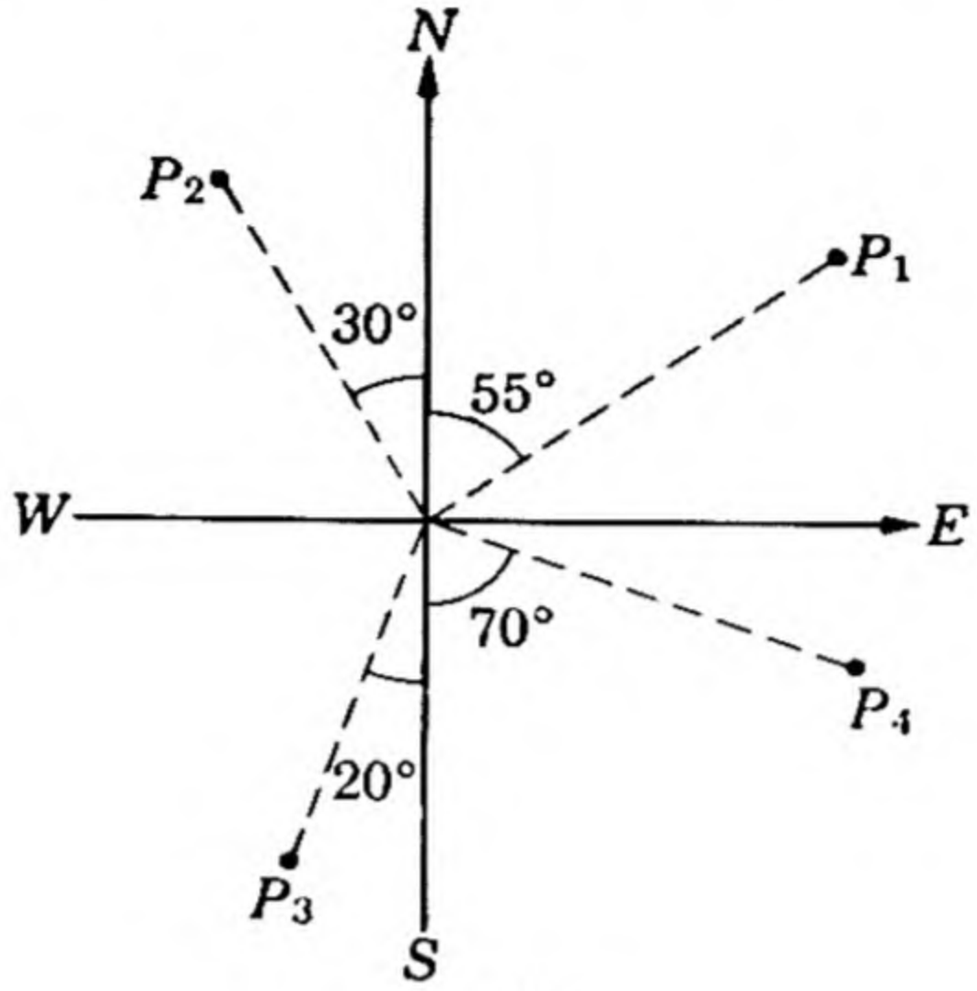


FIG. 2-3

(*d*) *Knot.* In nautical circles, the speed of a ship is expressed in *knots*. One knot equals one nautical mile per hour; one nautical mile equals 6080.27 feet. Thus one knot = 6080.27 ft/hr.

EXAMPLES

1. A flagpole stands on top of a tower. From a point 500 feet from the base of the tower, the angles of elevation to the top and the base of the pole are 34° 35.7′ and 29° 14.1′, respectively; find the height of the pole.

Given: $a = 500$ ft $\quad \beta = 29°\ 14.1'$
$\alpha = 34°\ 35.7' \quad h = ?$

Solution: Using the notation of Fig. 2-4, we get

$$H + h = a \tan \alpha$$

from the larger right triangle, and

$$H = a \tan \beta$$

from the smaller right triangle. Solving these equations for h we have

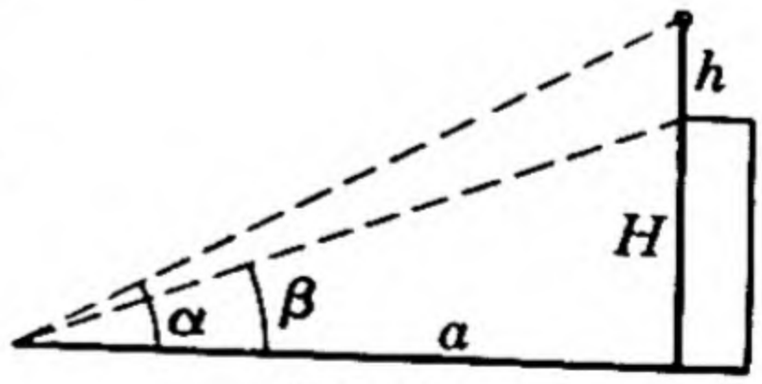


FIG. 2-4

$$\begin{aligned} h &= a \tan \alpha - a \tan \beta \\ &= (\tan \alpha - \tan \beta)\, a \\ &= (0.68972 - 0.55968)\ 500 \text{ ft} \\ &= 65.02 \text{ ft.} \end{aligned}$$

Note: Since the value of H is not required, H was eliminated instead of finding its value and using it to find the value of h. This procedure not only saves time but also reduces the chance of error.

2. The bearing of a lighthouse from a ship is N 25° 54′ E; 50 minutes 28 seconds later it is due East. Find the distance of the lighthouse from the second position of the ship if it is heading North at the rate of 21.6 knots.

Given:

$$\alpha = 25° 54',$$

$$t = 50 \text{ min } 28 \text{ sec}$$

$$= \frac{757}{900} \text{ hr},$$

$$v = 21.6 \text{ knots}$$

$$= \frac{21.6 \times 6080}{5280} \text{ m/hr}.$$

$$a = ?$$

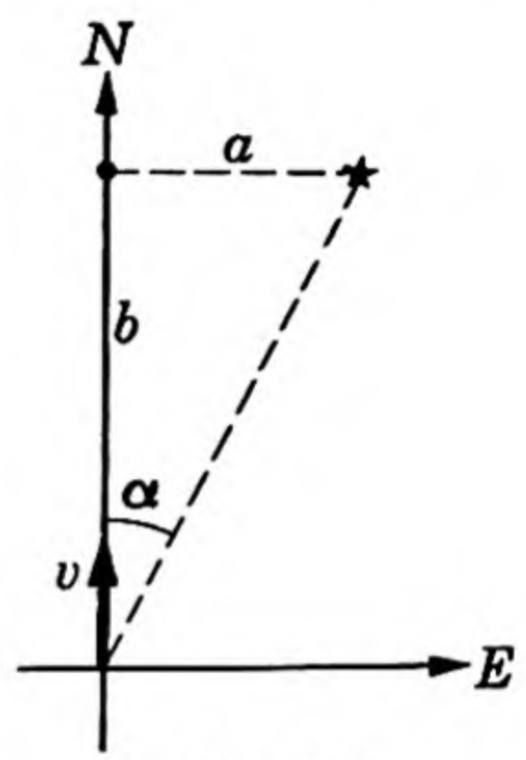


FIG. 2-5

Solution: Using the notation of Fig. 2-5, we have

$$a = b \tan \alpha$$
$$= vt \tan \alpha$$
$$= \frac{21.6 \times 6080 \times 757 \times \tan \alpha}{5280 \times 900} m$$
$$= 10.16\, m.$$

$$\log 21.6 = 1.33445$$
$$\log 6080 = 3.78390$$
$$\log 757 = 2.87910$$
$$\log \tan \alpha = \underline{9.68626 - 10}$$
$$7.68361$$
$$\underline{6.67687}$$
$$\log a = 1.00674$$

$$\log 5280 = 3.72263$$
$$\log 900 = \underline{2.95424}$$
$$6.67687$$

Note: The number 6080 is used here, instead of 6080.27, because the rest of the data do not warrant the use of six significant figures.

PROBLEMS

In each of the following problems, one side and one angle of a right triangle are given. Without using tables, find the exact values of the other two sides and the area of the triangle.

1. $a = 5$, $\alpha = 45°$.
2. $c = 2$, $\alpha = 60°$.
3. $c = 10$, $\alpha = 45°$.
4. $c = 8$, $\alpha = 30°$.
5. $b = 3$, $\beta = 60°$.
6. $b = 4$, $\beta = 30°$.
7. $a = 15$, $\beta = 60°$.
8. $a = 10$, $\beta = 30°$.
9. $b = 6$, $\beta = 60°$.
10. $c = 20$, $\alpha = 30°$.

In each of the following problems, use tables to find the area of the right triangle.

11. $c = 62.9$, $\alpha = 13° 58'$.
12. $c = 31.69$, $\alpha = 52° 30'$.
13. $c = 481.6$, $\beta = 49° 28'$.
14. $a = 242.5$, $\alpha = 76° 38'$.
15. $a = 62.08$, $\beta = 81° 30'$.
16. $b = 19.26$, $\alpha = 23° 48.3'$.

In each of the following problems, use tables to find the other sides of the right triangle.

17. $c = 2.137$, $\alpha = 48° 13'$.
18. $c = 53.16$, $\alpha = 76° 24'$.
19. $a = 36.51$, $\alpha = 23° 56'$.
20. $b = 393.7$, $\alpha = 51° 33'$.
21. $b = 11.82$, $\alpha = 43° 2'$.
22. $a = 18.94$, $\beta = 21° 11.2'$.

23. When the angle of elevation of the sun is 32° 40′, a tree casts a shadow 65.4 feet long. Find the height of the tree.

24. Find the height of a kite, if 200 yards of string is out and the angle of inclination of the string is 34° 25′.

25. A tower 100 feet high stands on one bank of a river. At the point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 4° 20′. Find the width of the river.

26. From a point 400 feet from the Washington Monument the angle of elevation of the top is 54° 14.6′. Find its height.

27. From a "blimp" 1000 feet directly above one end of an island, the angle of depression to the other end is 13° 24′. Find the length of the island.

28. From a lighthouse 200 feet above the surface of the sea the angle of depression to a ship is 9° 37′. Find the distance of the ship.

29. A lighthouse is directly west of a ship at 10:10 a.m.; at 11:25 a.m. its bearing is S 31° 16′ W. Find the distance of the lighthouse at the later time if the ship is proceeding North at 10 knots.

30. Find the area of a hexagon inscribed in a circle of 10-inch radius.

31. Find an expression for the area of a regular polygon of n sides inscribed in a circle.

32. Find an expression for the perimeter of a regular polygon of n sides inscribed in a circle.

33. From a point on a plain the angle of elevation of the summit of a mountain is 32° 47′. From a point 1000 feet nearer, the angle of elevation is 35° 16′. Find the height of the mountain above the plain.

34. From the summit of a mountain 5000 feet above a lake the angles of depression to the ends of the lake are 13° 27′ and 11° 38′. Find the length of the lake if the two ends and the summit are in the same vertical plane.

CHAPTER 3

APPLICATION TO PHYSICS

3-1 Vectors. One of the most important applications of trigonometry is to problems of physics and mechanics in which directed quantities are involved.

A quantity which has magnitude but no direction is called a *scalar quantity*. Positive and negative numbers, volume, mass, and energy are examples of scalar quantities.

A quantity which has direction as well as magnitude is called a *vector quantity*. Velocity, acceleration, force, and momentum are examples of vector quantities.

A vector quantity is represented by a straight arrow, called a *vector*. The length of the arrow represents the magnitude and the direction in which the arrow points indicates the direction of the vector.

Two vectors are equal if they are of the same length, are parallel, and point in the same direction. In other words, a vector is not changed by being moved parallel to itself.

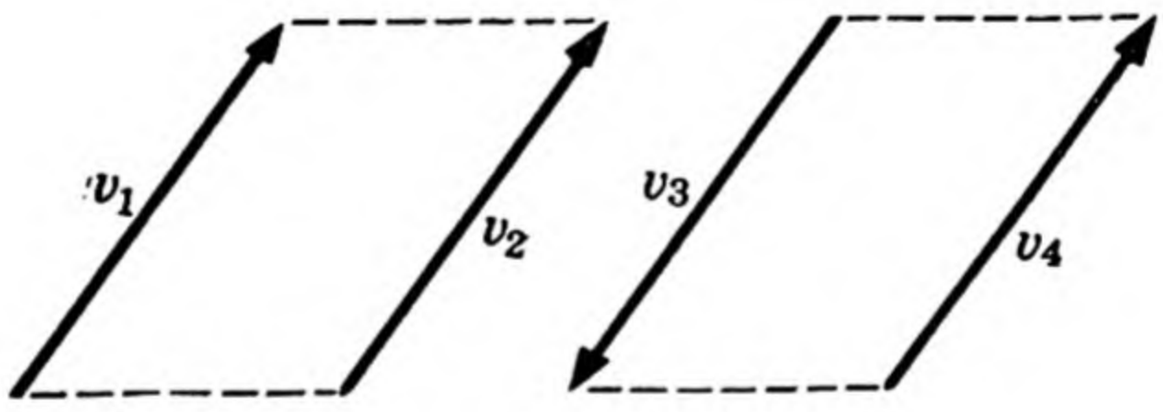


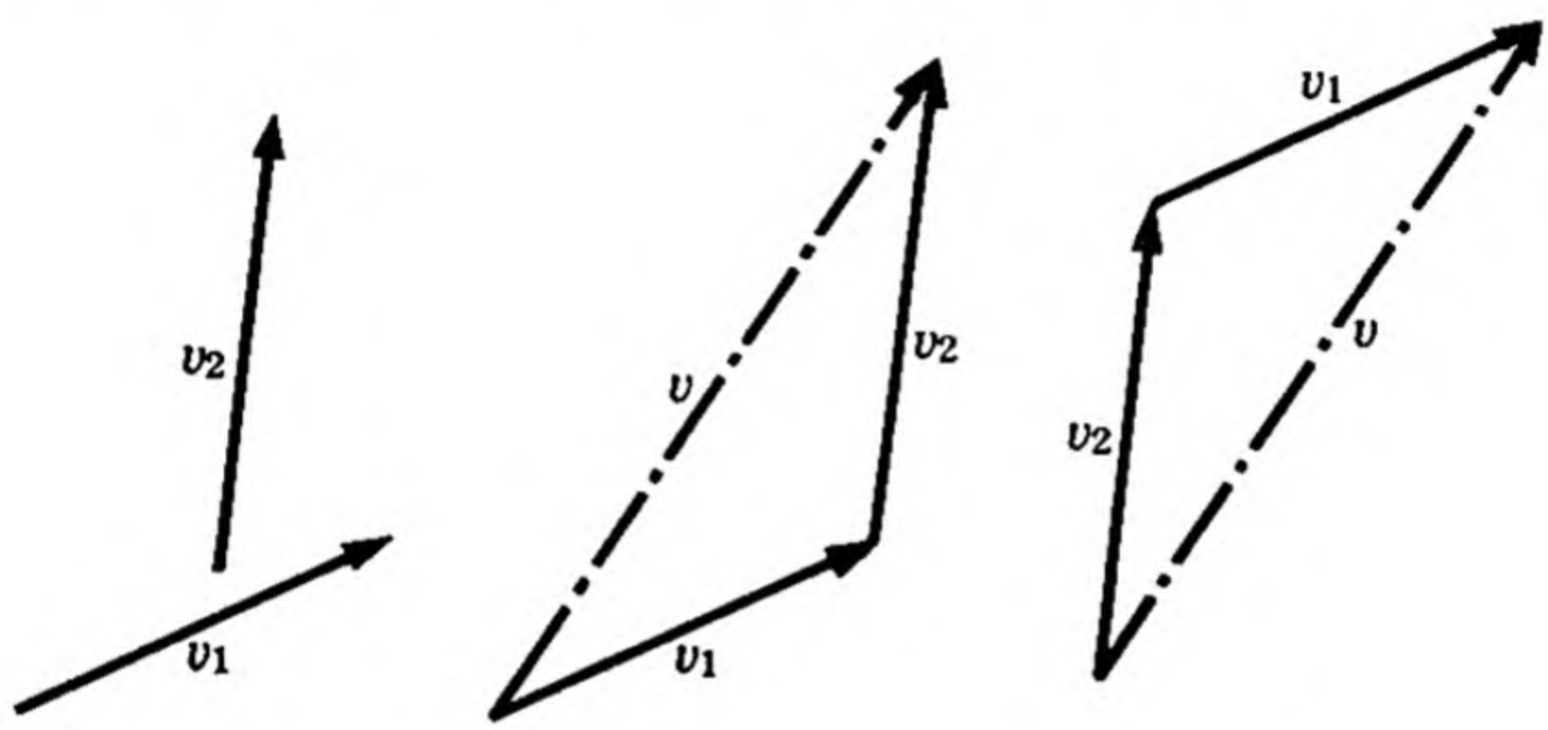
FIG. 3-1

A vector is the negative of another vector if the two are parallel and equal in length but point in opposite directions. In Fig. 3-1, $v_1 = v_2$, and $v_3 = -v_4$.

3-2 Geometric addition of vectors. In arithmetic, addition means the addition of numbers of the same sign. In school algebra, the meaning of the word is extended to include the addition

of numbers of opposite signs. In vector algebra, the meaning is further extended to take into account the direction of vectors. Thus we have arithmetical, algebraic, and vector (or geometric) additions.

To add two vectors geometrically we move one of them, keeping it parallel to its original position, until its *origin* (initial end) touches the *terminus* (arrow end) of the other; then we complete the triangle by drawing a vector whose terminus is at the terminus of the displaced vector. Thus in Fig. 3-2b and Fig. 3-2c, v is the geometric, or vector, sum of v_1 and v_2 of Fig. 3-2a.



(*a*) (*b*) (*c*)

FIG. 3-2

The sum of two or more vectors is called the *resultant* of the vectors. To add three or more vectors geometrically, we add two of the vectors, then a third to the resultant of the two, then a fourth to the resultant of the three, and so on. The order in which vectors are added does not affect the resultant.

3-3 Projection. Components of a vector. If the points Q_1 and Q_2, Fig. 3-3, are the feet of the perpendiculars dropped on the line AB from the points P_1 and P_2, then Q_1, Q_2, and Q_1Q_2 are called, respectively, the *projections* of P_1, P_2, and P_1P_2 on AB. If P_1P_2 is a vector, then Q_1Q_2 is called the *component* of the vector along the line AB. Since two parallel vectors which have the same length and direction are equal, $P_1P'_2$ also is a component of the vector P_1P_2 along the line AB.

3-4 Resolution of vectors. A vector may be replaced by two or more component vectors which, when added geometrically, give the original vector. When this is done, the vector is said to be *resolved* into components. The most convenient components into which a vector may be resolved are the components parallel to the axes of a system of rectangular coordinates.

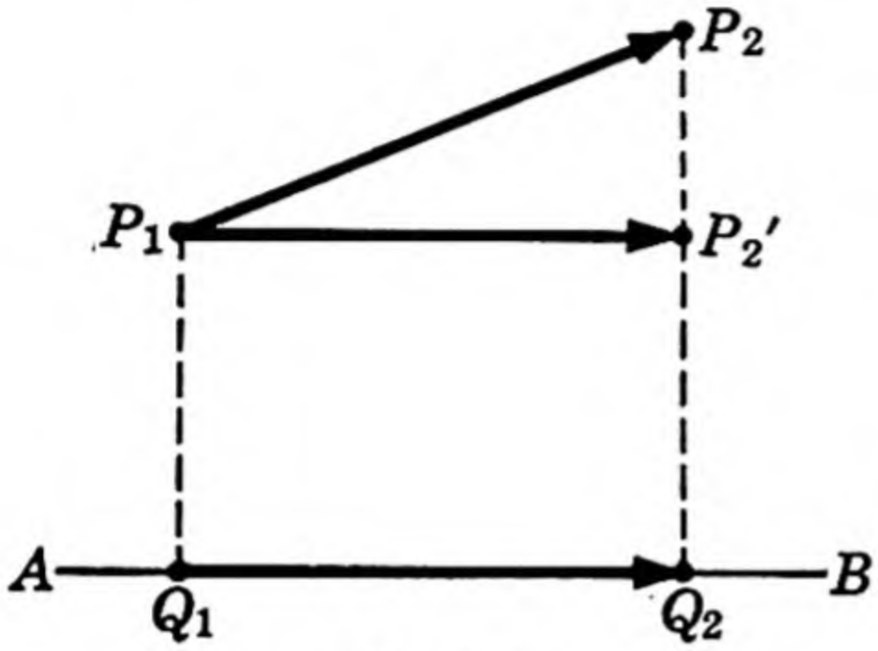


FIG. 3-3

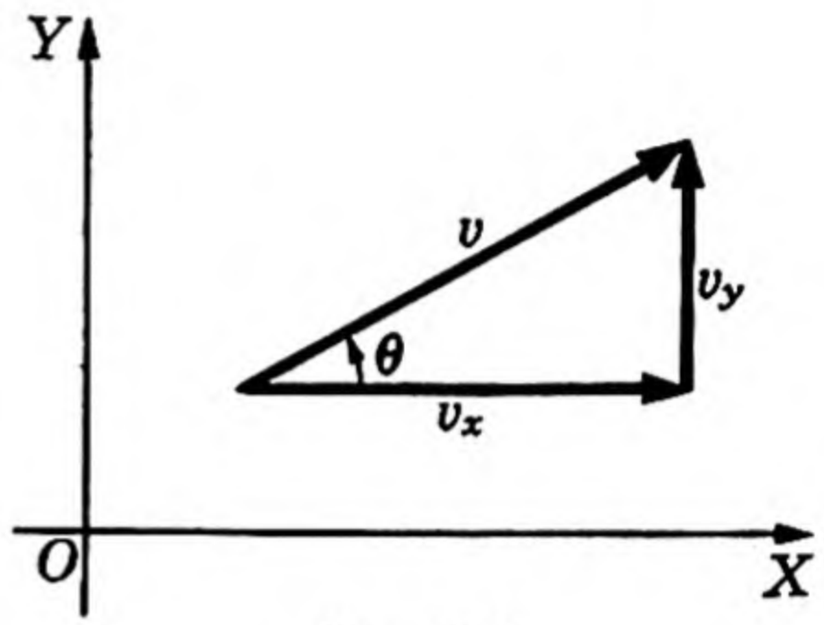


FIG. 3-4

In Fig. 3-4, vector v is resolved into v_x and v_y parallel to OX and OY, respectively. Evidently

$$v_x = v \cos \theta,$$

$$v_y = v \sin \theta,$$

$$v = \sqrt{v_x^2 + v_y^2},$$

$$\tan \theta = \frac{v_y}{v_x}.$$

Since a vector is the resultant of its components, v is the resultant, or vector sum, of v_x and v_y.

3-5 Analytical addition of vectors. The geometric addition of vectors, described in Section 3-2, is convenient when only two vectors are to be added, especially when the two vectors are at right angles. When more than two vectors are to be added, it is more convenient to add them *analytically*. Vectors are added analytically as follows:

1. Components of the vectors parallel to each axis of a rectangular system are added algebraically in order to obtain the corresponding components of the resultant.

2. The magnitude and the direction of the resultant are then found by the use of the equations

$$v = \sqrt{v_x^2 + v_y^2}, \qquad \tan \theta = \frac{v_y}{v_x},$$

where v is the resultant, v_x and v_y are the algebraic sums of the components of the vectors parallel to the x-axis and the y-axis, respectively, and θ is the angle the resultant makes with the x-axis.

EXAMPLES

1. A man who can row at the rate of 5 miles per hour wants to cross a river and to land at the point directly opposite the starting point. Find the direction in which he should keep his boat headed and his actual velocity, if the river flows at the rate of 4 miles per hour.

Given: $v_1 = 5 \frac{\text{m}}{\text{hr}}$ $\qquad v = ?$

$v_2 = 4 \frac{\text{m}}{\text{hr}}$ $\qquad \theta = ?$

Solution: Since only two vectors (v_1 and v_2) are to be added and v is to be perpendicular to v_2, we use the geometric method and draw Fig. 3-5 so that v_1 is about $\frac{5}{4}$ v_2 in length. Then

$$v = \sqrt{v_1^2 - v_2^2}$$

$$= \sqrt{25 - 16} \text{ m/hr}$$

$$= 3 \text{ m/hr.}$$

$$\sin \theta = \frac{v_2}{v_1}$$

$$= \frac{4}{5}$$

$$= 0.8.$$

$$\theta = 53° 8'.$$

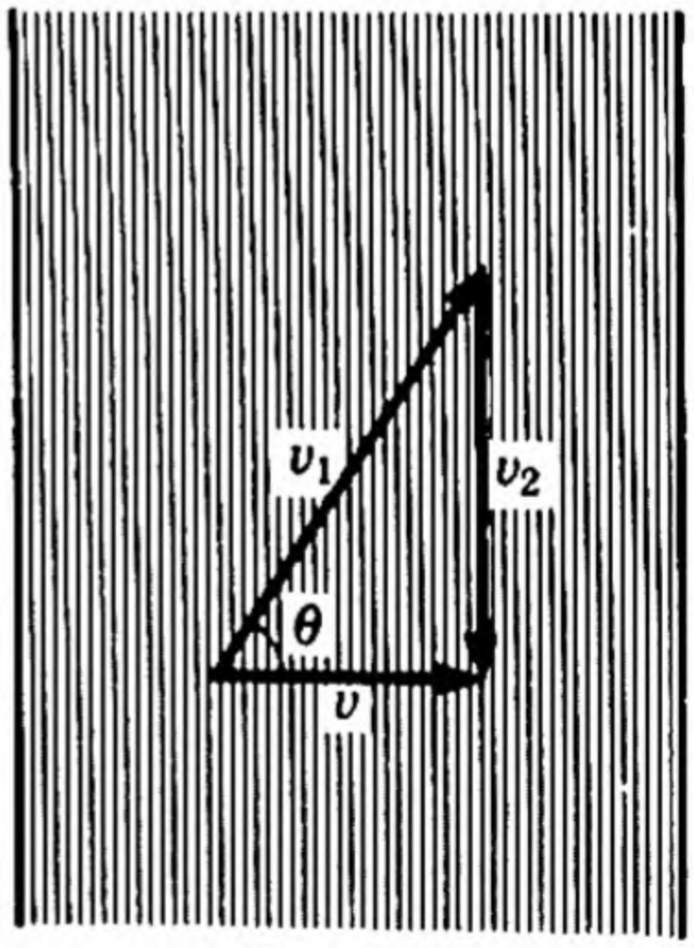


FIG. 3-5

Note: Sin θ, instead of tan θ, is used here in order to obtain θ directly from the given data.

2. Find the magnitude and direction of the resultant of the four forces indicated in Fig. 3-6, where $F_1 = 120$ lb, $F_2 = 80$ lb, $F_3 = 100$ lb, $F_4 = 90$ lb, $\alpha = 60°$, $\beta = 45°$, and $\gamma = 30°$.

Solution: Let F denote the magnitude of the resultant, F_x and F_y its components, and θ the angle it makes with the x-axis. Then

$$F_x = F_1 \cos \alpha + 0 - F_3 \cos \beta + F_4 \cos \gamma$$
$$= (120 \times 0.5 - 100 \times 0.707 + 90 \times 0.866) \text{ lb}$$
$$= 67.24 \text{ lb.}$$
$$F_y = F_1 \sin \alpha + F_2 + F_3 \sin \beta - F_4 \sin \gamma$$
$$= (120 \times 0.866 + 80 + 100 \times 0.707 - 90 \times 0.5) \text{ lb}$$
$$= 209.6 \text{ lb.}$$
$$\tan \theta = \frac{F_y}{F_x}.$$
$$\log \tan \theta = 0.49383.$$
$$\theta = 72^\circ\ 13'.$$

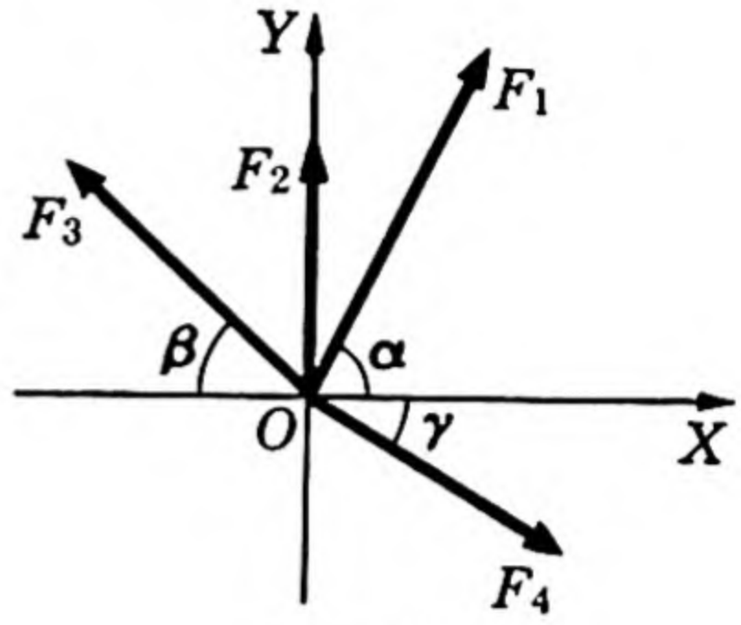


FIG. 3-6

Note: Observe (a) that components which point to the left or downward have negative signs and (b) that the numerical values obtained are kept within a reasonable number of significant figures.

PROBLEMS

1. If an airplane takes off at an angle of 15° with a velocity of 100 miles per hour, at what rates does it (a) rise vertically and (b) move horizontally?

2. A man pushes a crate with a force of 150 pounds. If the crate is on an inclined plane which makes an angle of 20° and if the force is exerted horizontally, how much of the force is effective in pushing the crate up the plane?

3. One man is pulling at an object with a force of 150 pounds toward the East and another man with a force of 120 pounds toward the Northeast. Find the magnitude and direction of the resultant force.

4. A man keeps his motorboat pointed at right angles to the stream while crossing a river. If the power supplied by the engine can give the boat a velocity of 10 miles per hour in still water and if the velocity of the river is 6 miles per hour, find the magnitude and direction of the actual velocity of the boat.

5. If the river of the preceding problem is 500 feet wide, how far downstream will the man land?

6. In what direction should the man of Problem 4 keep the boat headed so as to land directly opposite the starting point?

7. A ship is moving East at the rate of 10 miles per hour; a man walks across the deck at the rate of 4 miles per hour; find the magnitude and direction of his velocity relative to the ocean.

8. A man in a motorboat wants to intercept a ship in the shortest time. The ship is 10 miles East of him and is moving North at the rate of 15 miles per hour. Find the direction in which he should move if the motorboat can make 20 miles per hour.

9. In the preceding problem, find the time.

10. An automobile is traveling East at the rate of 40 miles per hour; the wind is blowing from the Northeast at the rate of 30 miles per hour. Find the magnitude and direction of the velocity of the wind relative to the automobile.*

11. Three equal forces act on a body in the directions N 20° E, N 30° W, and S 60° E. Find the magnitude and direction of their resultant.

12. Four forces F, 2F, 3F, and 4F act, respectively, in the directions N 15° E, N 25° W, S 10° W, and S 20° E. Find the magnitude and direction of their resultant.

3-6 Friction. The force of one body on another due to action through contact may be divided into *normal* and *tangential* components, that is, components perpendicular and parallel to the surfaces in contact. The normal component is due to the rigidity of the bodies; the tangential component is due to the roughness of the surfaces in contact. The normal component is called the *normal reaction* and the tangential component the *frictional force.*

If a body is on the point of motion or in motion, the frictional force is proportional to the normal reaction, that is,

$$F = \mu N,$$

where F denotes the frictional force, N the normal reaction, and μ the constant of proportionality, called the *coefficient of friction.*

The frictional force acting on a body is always opposed to the direction in which the body is urged to move by the forces which act on it.

If the surfaces in contact are perfectly smooth, μ and the frictional force are zero.

If a body is not on the point of motion or in motion, there may be a frictional force but the relation $F = \mu N$ does not hold. In such a case the frictional force is less than μN, and equals the tangential component of the resultant of all the forces, except the frictional force, which act on the body. Thus μN is the greatest value the frictional force can have.

3-7 Condition of the equilibrium of a particle. In a problem of mechanics, a body of any size or shape may be considered as a *particle* if the lines of action of the forces which act on it pass through the same point.

* The velocity of one body relative to another body in motion is obtained by finding the resultant of the velocities of the two bodies after the velocity of the second (reference) body is reversed.

A particle is said to be in *equilibrium* if the resultant of the forces which act on it equals zero. But if the resultant is zero, its component along any direction also is zero. Furthermore, a component of the resultant equals the algebraic sum of the corresponding components of the forces. Therefore the condition of the equilibrium of a particle states:

The algebraic sum of the components of the forces along any direction equals zero.

To apply this condition to a problem of the equilibrium of a particle,* proceed as follows:

(1) Draw a figure in which the particle and the forces (pulls and pushes by other bodies) which act *on* it are indicated, and label the forces, the lengths, and the angles involved.

(2) Draw a rectangular system of axes, and set the algebraic sum of the components along each axis equal to zero.

The axes may be drawn in any way, but if they are so chosen that the number of forces parallel or perpendicular to one of the axes is greatest, the number of sines and cosines becomes smallest and, consequently, the equations become simplest.

(3) Solve the resulting equations simultaneously to find expressions for the required forces in terms of the given quantities.

(4) In case of a numerical problem, introduce the values of the given quantities in the final expressions and carry out the indicated computations.

EXAMPLES

1. A weight of 50 pounds, which is suspended by a rope, is pulled to one side by a horizontal force so that the rope makes an angle of 30° with the vertical. Find the horizontal force and the pull of the rope.

Given:

$W = 50$ lb
$\theta = 30°$
$F = ?$
$P = ?$

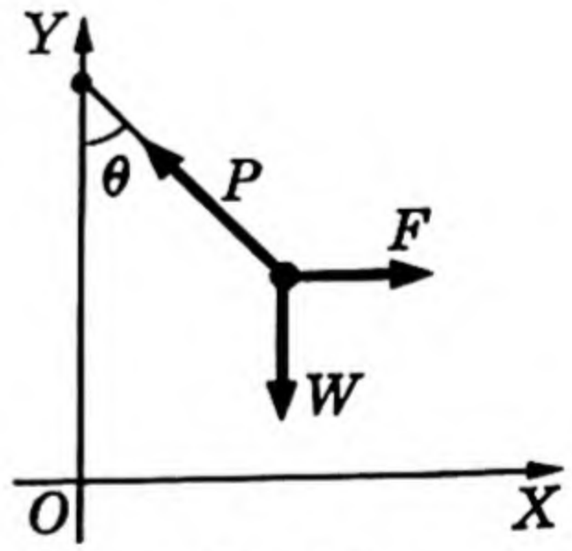


FIG. 3-7

* The particle whose equilibrium is under consideration may be a point-particle, as in Example 1 or a sizable body, as in Example 2. If several bodies are involved in an equilibrium problem, it is desirable to consider first the equilibrium of the body on which the given force or forces act.

Solution: Using the notation of Fig. 3-7 and setting the algebraic sums of the components of the forces in the horizontal and vertical directions equal to zero, we have

$$F - P \sin \theta = 0,$$
$$P \cos \theta - W = 0.$$

$$P = \frac{W}{\cos \theta}$$
$$= \frac{50}{\frac{1}{2}\sqrt{3}} \text{ lb}$$
$$= \frac{100}{\sqrt{3}} \text{ lb}$$
$$= 57.7 \text{ lb.}$$

$$F = P \sin \theta$$
$$= \frac{W}{\cos \theta} \cdot \sin \theta$$
$$= W \tan \theta$$
$$= \frac{50}{\sqrt{3}} \text{ lb}$$
$$= 28.9 \text{ lb.}$$

Discussion: If $\theta = 0°$, $\cos \theta = 1$ and $\tan \theta = 0$, then $P = W$ and $F = O$, as they should. If $\theta = 90°$, $\cos \theta = 0$ and $\tan \theta = \infty$; then both P and F become infinite. This means that no horizontal force, however great, can maintain the weight at a position which makes the rope horizontal.

2. A uniform bar, which is 6 feet long and weighs 70 pounds, is suspended by

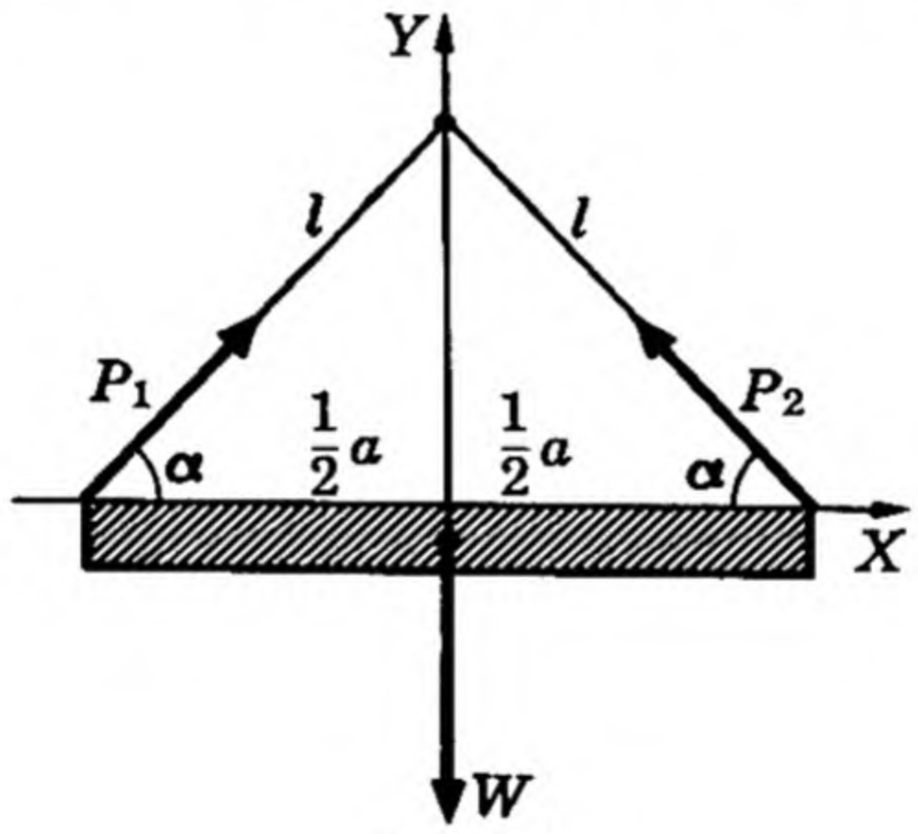


FIG. 3-8

two ropes, each of which is 5 feet long. The upper ends of the ropes are fastened to a peg and the lower ends to the ends of the bar. Find the pull of each rope on the bar.

Given:
$a = 6$ ft
$l = 5$ ft
$W = 70$ lb
$P_1 = ?$
$P_2 = ?$

Solution: Using the notation of Fig. 3-8, we have

$$P_1 \cos \alpha - P_2 \cos \alpha = 0, \quad \text{(a)}$$
$$P_1 \sin \alpha + P_2 \sin \alpha - W = 0. \quad \text{(b)}$$

From Eq. (a) we find that $P_1 = P_2$. Making use of this fact in (b), we get

$$P_1 = P_2 = \frac{W}{2 \sin \alpha}$$
$$= \frac{W}{\frac{2\sqrt{l^2 - (\frac{1}{2}a)^2}}{l}}$$
$$= \frac{l}{\sqrt{4l^2 - a^2}} W$$
$$= \frac{5}{8} \times 70 \text{ lb}$$
$$= 43.75 \text{ lb.}$$

Discussion: The following conclusions may be drawn from the last literal expression for P_1 and P_2:

(a) As l is increased indefinitely, a^2 becomes negligible compared with $4l^2$, the strings become parallel, and P_1 and P_2 approach $\frac{1}{2} W$, as they should.

(b) As l is decreased so as to make $2l$ approach a, the denominator approaches zero and, consequently, P_1 and P_2 increase indefinitely.

Note: It is important to distinguish, as in Fig. 3-7 and Fig. 3-8 between the length, say, of a rope and the length of the vector which represents the pull of the rope.

3. A weight of 200 pounds is being moved up a rough inclined plane by two men who exert equal forces. If one of the men pushes it horizontally, and the other pulls it up parallel to the plane, find the force exerted by each man, taking the coefficient of friction to be 0.5 and the angle of inclination of the plane to be 30°.

Given:

$$\alpha = 30°$$
$$W = 200 \text{ lb}$$
$$\mu = 0.5$$
$$P = ?$$

Solution: Since most of the forces are parallel or perpendicular to the inclined

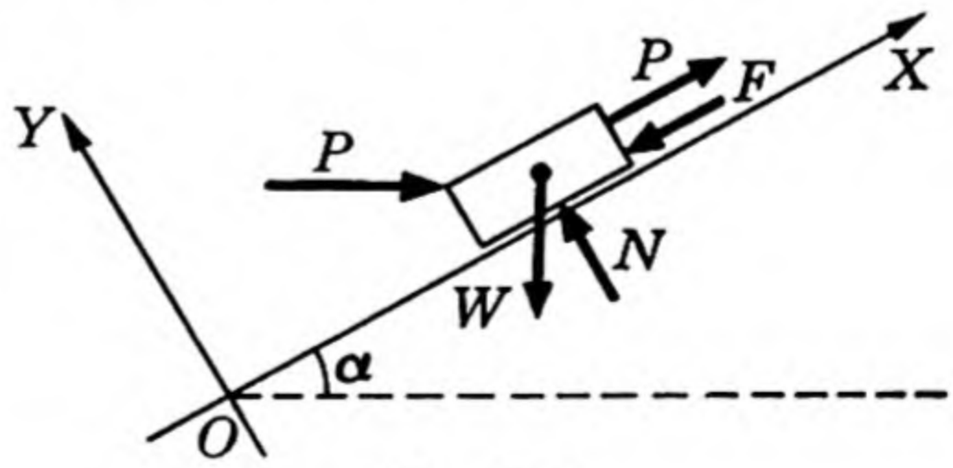


FIG. 3-9

plane, it is desirable to choose the axes as in Fig. 3-9. Using the notation of the figure and applying the condition of equilibrium, we have

$$P \cos \alpha + P - F - W \sin \alpha = 0, \quad \text{(a)}$$
$$N - P \sin \alpha - W \cos \alpha = 0, \quad \text{(b)}$$
$$F = \mu N. \quad \text{(c)}$$

Note: Observe that the action of the plane is represented in Fig. 3-9 by its normal and frictional components.

Replacing F in (a) by μN, multiplying (b) by μ, and adding the equations, we obtain

$$P \cos \alpha + P - \mu P \sin \alpha - \mu W \cos \alpha - W \sin \alpha = 0.$$

Hence:

$$\begin{aligned} P &= \frac{\mu \cos \alpha + \sin \alpha}{\cos \alpha + 1 - \mu \sin \alpha} W \\ &= \frac{0.5 \times 0.866 + 0.5}{0.866 + 1 - 0.5 \times 0.5} \times 200 \text{ lb} \\ &= \frac{0.933 \times 200 \text{ lb}}{1.616} \\ &= 115.4 \text{ lb}. \end{aligned}$$

Discussion: If $\alpha = \mu = 0$, then $P = 0$, as it should. If $\mu = 0$, $P = \dfrac{\sin \alpha}{1 + \cos \alpha} W$.

PROBLEMS

(*Note:* Solve numerical problems as literal problems first. Discuss the expressions for the required forces.)

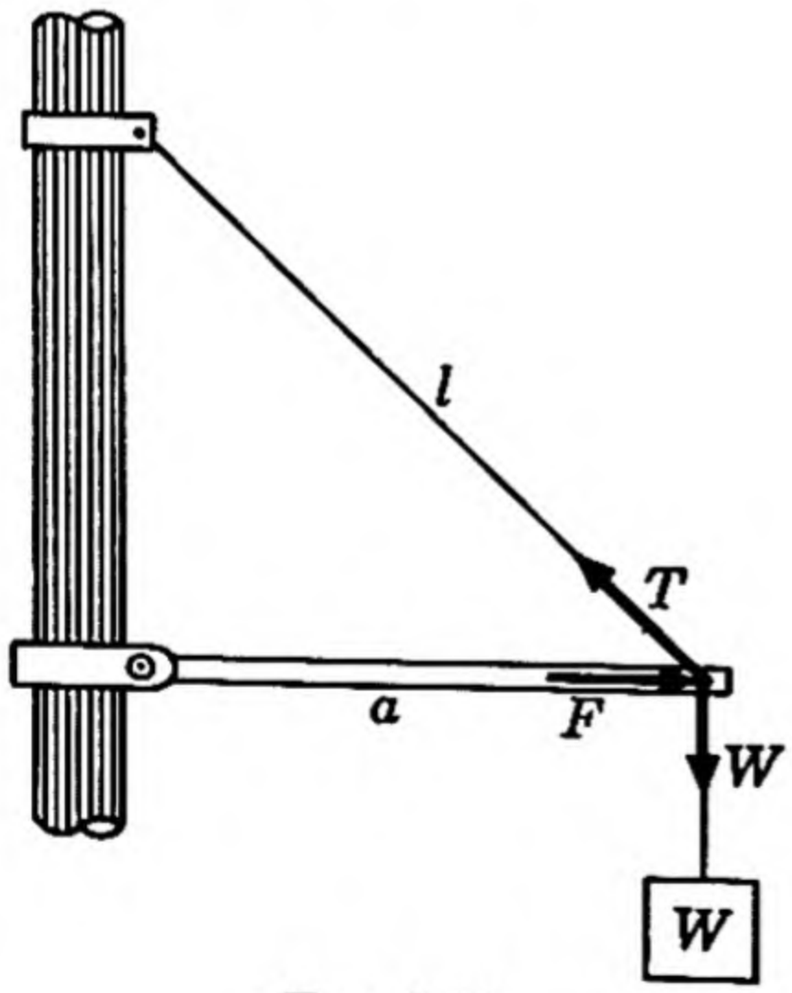


FIG. 3-10

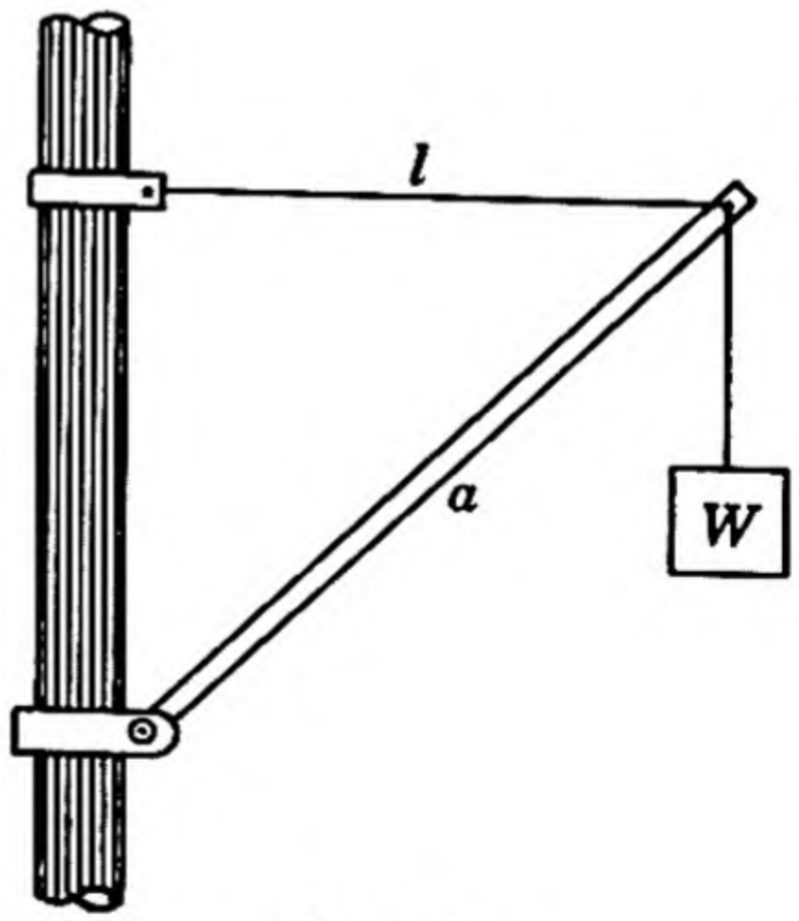


FIG. 3-11

1. The ends of a rope 24 feet long are fastened to two hooks which are at the same level but 10 feet apart. If a weight of 100 pounds is suspended from the middle of the rope, find the pull of the rope on each side of the point of suspension.

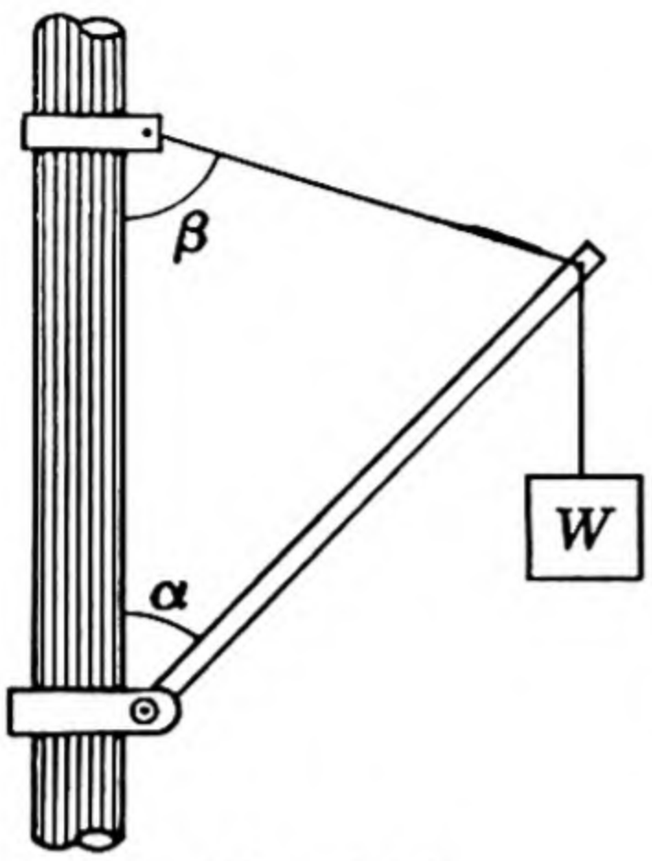


FIG. 3-12

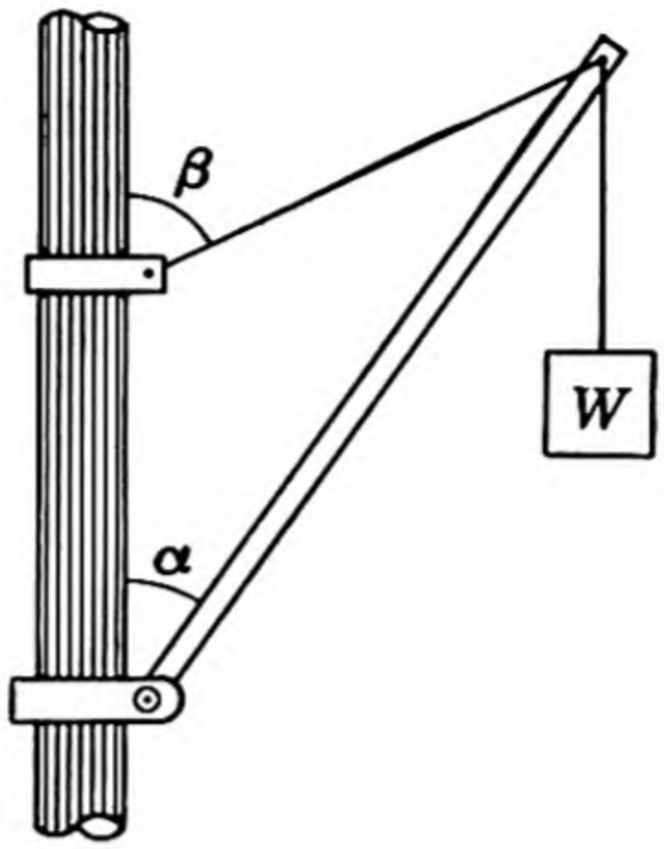


FIG. 3-13

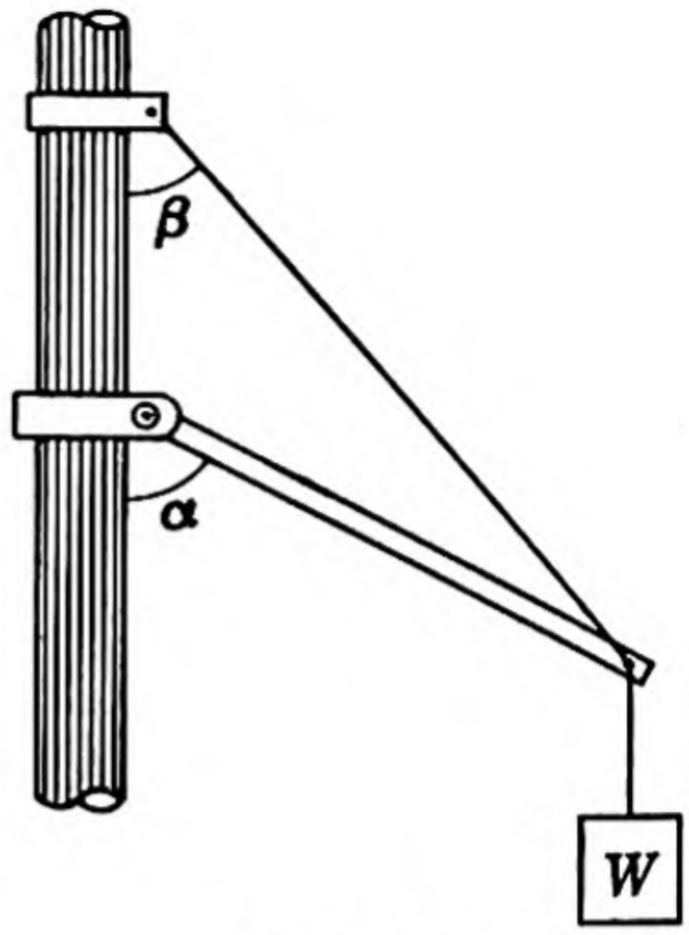


FIG. 3-14

2. Same as Problem 1, except that the points of suspension are 8 feet apart.

3. A man weighing 150 pounds is resting in a hammock. If the supporting ropes make angles of 20° and 25° with the horizontal, find the pull of each rope.

4. An automobile weighing 3500 pounds is prevented from rolling down an incline of 15° by pushing it parallel to the ground; find the pushing force.

5. Same as Problem 4, but the force is horizontal.

6. In Fig. 3-10, a denotes the length of the horizontal rod which is hinged at the left end, l the length of the supporting rope, and W the weight of the suspended body. Find expressions for the force of compression (push) of the rod and the force of tension (pull) of the rope.

7. Same as Problem 6, but Fig. 3-11 is to be used.

8. Find expressions for the force of compression of the rod and the force of tension of the supporting rope in Fig. 3-12.

9. Same as Problem 8, but Fig. 3-13 is to be used.

10. Same as Problem 8, but Fig. 3-14 is to be used.

11. Same as Problem 8, but Fig. 3-15 is to be used.

12. In Example 3 (Fig. 3-9), suppose the body to be on the point of sliding down.

13. In Fig. 3-16, W is the weight of a sphere which rests between two smooth inclined planes. Find the force which each plane exerts on the sphere.

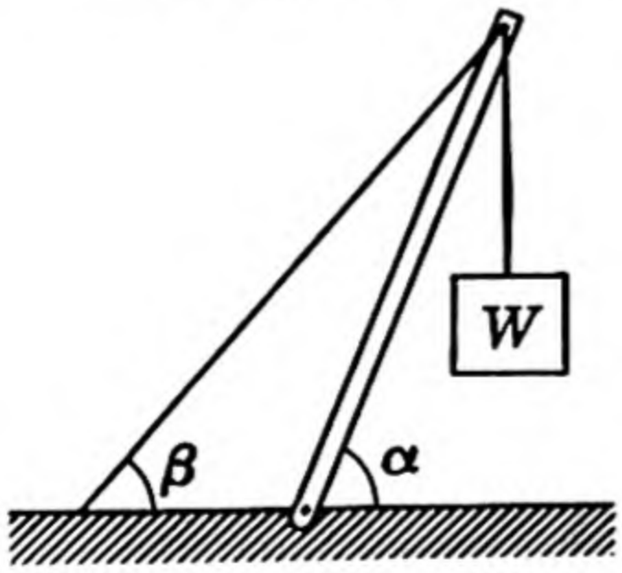


FIG. 3-15

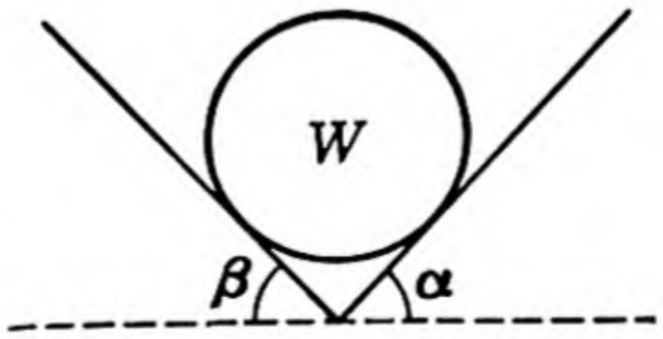


FIG. 3-16

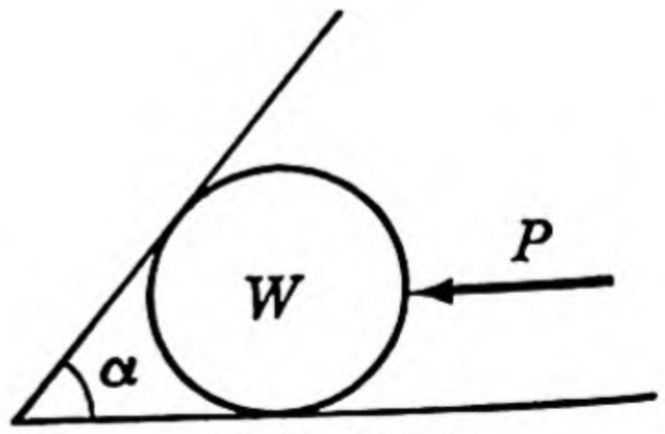


FIG. 3-17

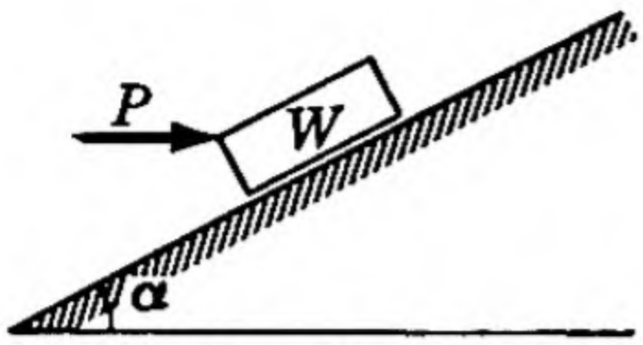


FIG. 3-18

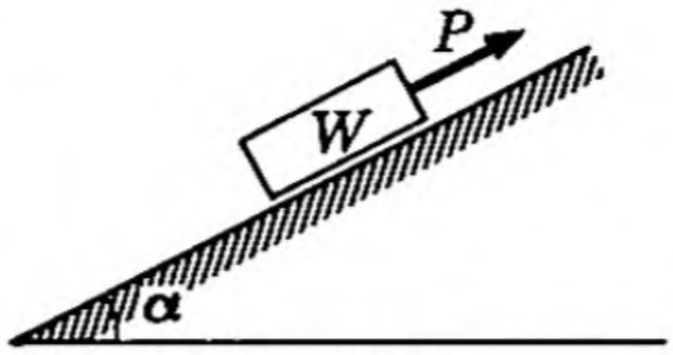


FIG. 3-19

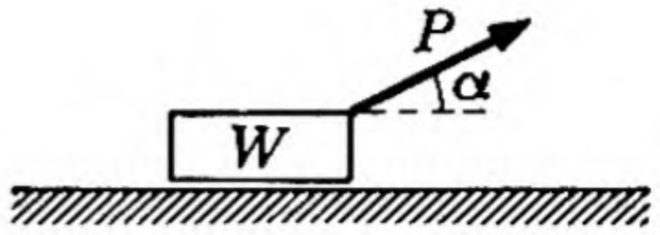


FIG. 3-20

14. Same as Problem 13, but Fig. 3-17 is to be used.

15. The contact between the body and the inclined plane in Fig. 3-18 is smooth. Find an expression for the horizontal force P which prevents the weight from sliding down the plane.

16. Same as Problem 15, but Fig. 3-19 is to be used.

17. The contact between the plane and the body in Fig. 3-20 is rough. Find an expression for the force P necessary for moving the body.

18. Same as Problem 17, but Fig. 3-21 is to be used.

FIG. 3-21

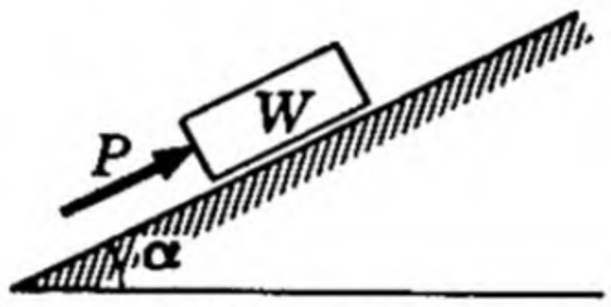


FIG. 3-22

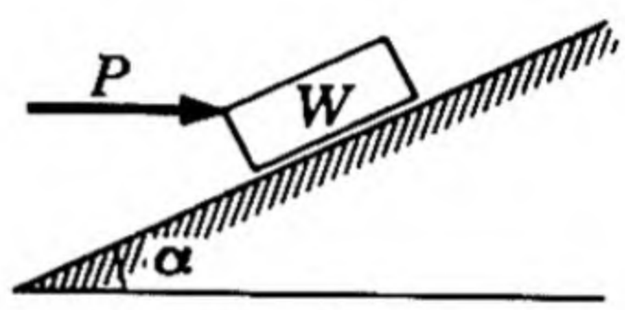


FIG. 3-23

19. The contact in Fig. 3-22 is rough. Find the force P necessary to prevent the body from sliding down the inclined plane.

20. Same as Problem 19, but the body is to be moved up the inclined plane.

21. The contact in Fig. 3-23 is rough. Find the horizontal force P necessary to move the body up the plane.

22. Same as Problem 21, but the body is to be prevented from sliding down.

23. The contact in Fig. 3-24 is rough. Find the force P necessary to prevent the body from sliding down.

24. Same as Problem 23, but the body is to be moved up the plane.

25. The contact in Fig. 3-25 is rough. Find P if the body is to be pushed up the plane.

26. Same as Problem 25, but the body is to be prevented from sliding down the plane.

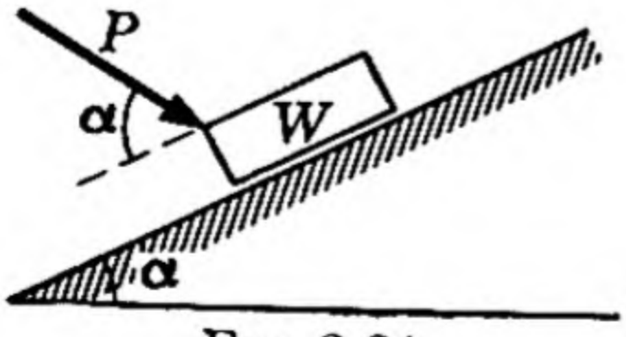


FIG. 3-24

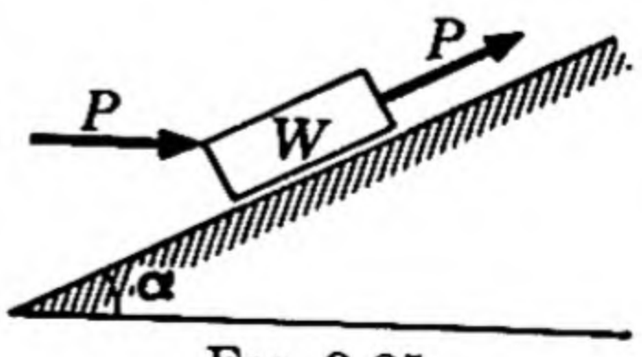


FIG. 3-25

CHAPTER 4

REDUCTION TO ACUTE ANGLES— CIRCULAR MEASURE

4-1 Absolute and numerical values. When the sign of a number is disregarded, the result is called the *absolute value* of the number. The absolute value of a number is indicated by placing the number between two vertical lines, thus:

$$| -2 | = 2.$$

Evidently the absolute value of a positive number is the same as the number; therefore this notation is not necessary in the case of a positive number.

Some authors use the term *numerical value* in the same sense as the absolute value. In this book we distinguish between the two terms: we use "numerical value" to mean the number, positive or negative, which precedes the unit of the value of a geometrical or physical magnitude. Thus the numerical values of 2 ft, −2 ft/sec, and 3 lb are 2, −2 and 3, respectively.

4-2 Reduction to acute angles. Let θ denote not only the angle so marked in each of the figures of Fig. 4-1, but also any other coterminal angle; then the acute angle φ, considered positive, is called the *reference angle* of θ. By the definitions of trigonometric functions (Sec. 1-6), a function of θ and the same function of φ are given by the ratio of the same pair of sides of the right triangle of which φ is an acute angle, except that a function of θ may be negative, whereas all functions of φ are positive, since φ is a positive acute angle. Therefore, *the absolute value of a function of any angle equals the value of the same function of its reference angle*, that is,

$$\textbf{| function of } \theta \textbf{ | = same function of } \varphi.$$

The following *reduction rules* enable us to reduce a function of any positive or negative angle to the same function of the reference angle.

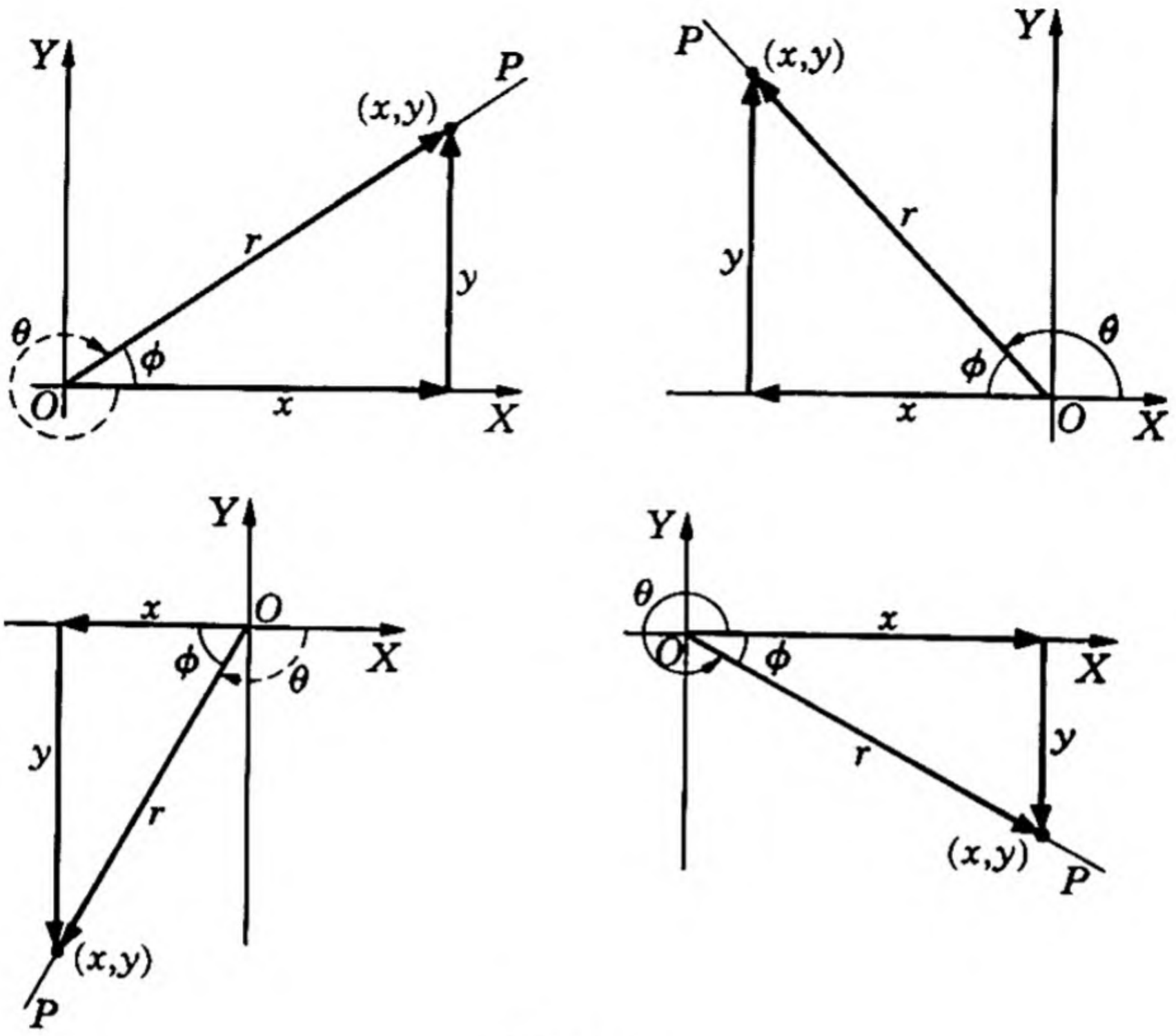


FIG. 4-1

1. Place the given angle in the standard position and form a right triangle by dropping a perpendicular on the x-axis from a point on the terminal side of the angle.

2. Label each side of the triangle as in Fig. 4-1 and place arrowheads at the termini of x, y, and r to indicate the signs of x and y; r is always positive.

3. Equate each function of the angle to the same function of its reference angle and assign a negative sign to the latter if x is negative but not y, or vice versa.

EXAMPLES

1. Reduce functions of 150° to functions of its reference angle.

From Fig. 4-2 we have:

$\sin 150° = \sin 30°$, $\csc 150° = \csc 30°$,
$\cos 150° = -\cos 30°$, $\sec 150° = -\sec 30°$,
$\tan 150° = -\tan 30°$, $\cot 150° = -\cot 30°$.

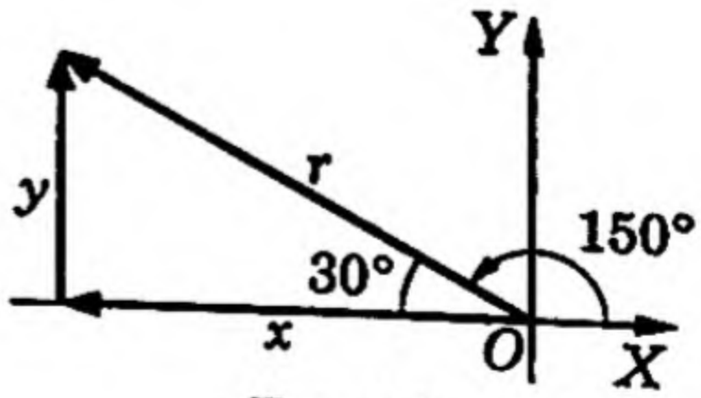


FIG. 4-2

2. Reduce functions of $-137°$ to functions of its reference angle.

From Fig. 4-3 we have:

$$\sin(-137°) = -\sin 43°, \qquad \csc(-137°) = -\csc 43°,$$
$$\cos(-137°) = -\cos 43°, \qquad \sec(-137°) = -\sec 43°,$$
$$\tan(-137°) = \tan 43°, \qquad \cot(-137°) = \cot 43°.$$

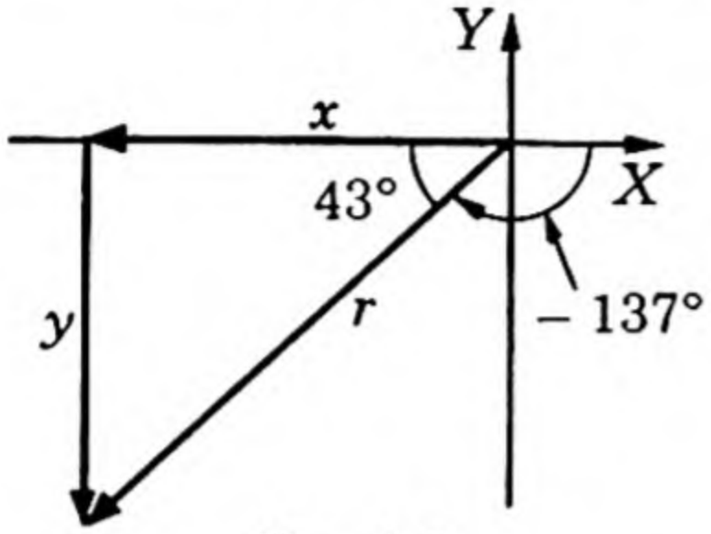


FIG. 4-3

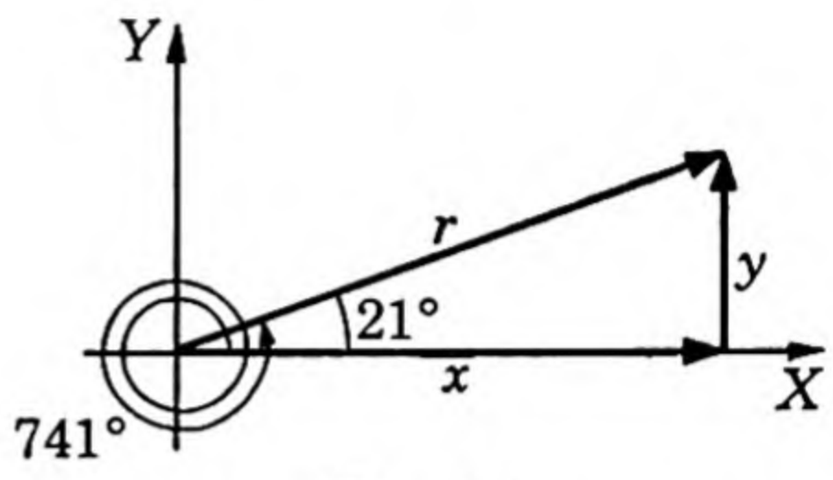


FIG. 4-4

3. Reduce functions of $741°$ to functions of its reference angle.

From Fig. 4-4 we have:

$$\sin 741° = \sin 21°, \qquad \csc 741° = \csc 21°,$$
$$\cos 741° = \cos 21°, \qquad \sec 741° = \sec 21°,$$
$$\tan 741° = \tan 21°, \qquad \cot 741° = \cot 21°.$$

4-3 Functions of $(90° \pm \theta)$, $(180° \pm \theta)$, $(270° \pm \theta)$, and $(360° \pm \theta)$.

When the value of θ is given, functions of these angles can be reduced to functions of a positive acute angle by following the reduction rules of the preceding section. When the value of θ is not given, the reduction can be achieved by treating θ as a positive acute angle and then following the reduction rules. It can be proved that the results thus obtained are valid also when θ is not a positive acute angle.

EXAMPLES

1. Reduce functions of $(180° - \theta)$ to functions of θ.

Solution: Drawing Fig. 4-5 according to the first two of the reduction rules, we observe that θ is the reference angle of $(180° - \theta)$. Therefore:

$$\sin(180° - \theta) = \sin\theta,$$
$$\cos(180° - \theta) = -\cos\theta,$$
$$\tan(180° - \theta) = -\tan\theta,$$
$$\cot(180° - \theta) = -\cot\theta,$$
$$\sec(180° - \theta) = -\sec\theta,$$
$$\csc(180° - \theta) = \csc\theta.$$

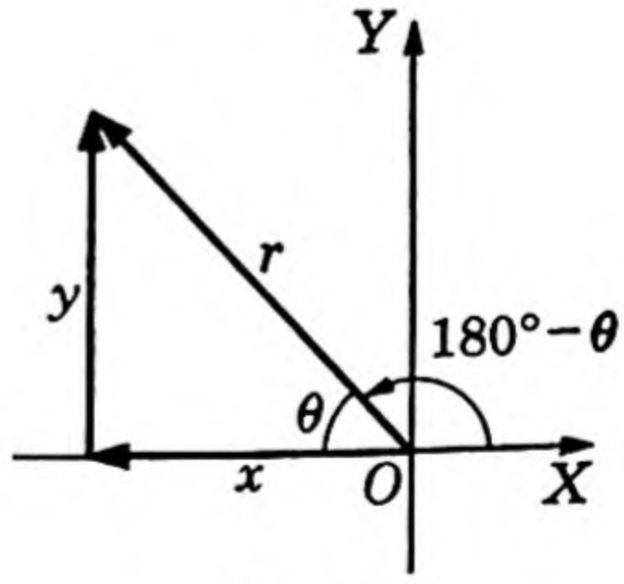


FIG. 4-5

2. Reduce functions of $(270° + \theta)$ to functions of θ.

Solution: Drawing Fig. 4-6 according to the first two reduction rules, we observe that θ is the complement of the reference angle φ. Therefore:

$$\begin{aligned}
\sin\ (270° + \theta) &= -\sin\ \varphi = -\cos\ \theta,\\
\cos\ (270° + \theta) &= \quad \cos\ \varphi = \quad \sin\ \theta,\\
\tan\ (270° + \theta) &= -\tan\ \varphi = -\cot\ \theta,\\
\cot\ (270° + \theta) &= -\cot\ \varphi = -\tan\ \theta,\\
\sec\ (270° + \theta) &= \quad \sec\ \varphi = \quad \csc\ \theta,\\
\csc\ (270° + \theta) &= -\csc\ \varphi = -\sec\ \theta.
\end{aligned}$$

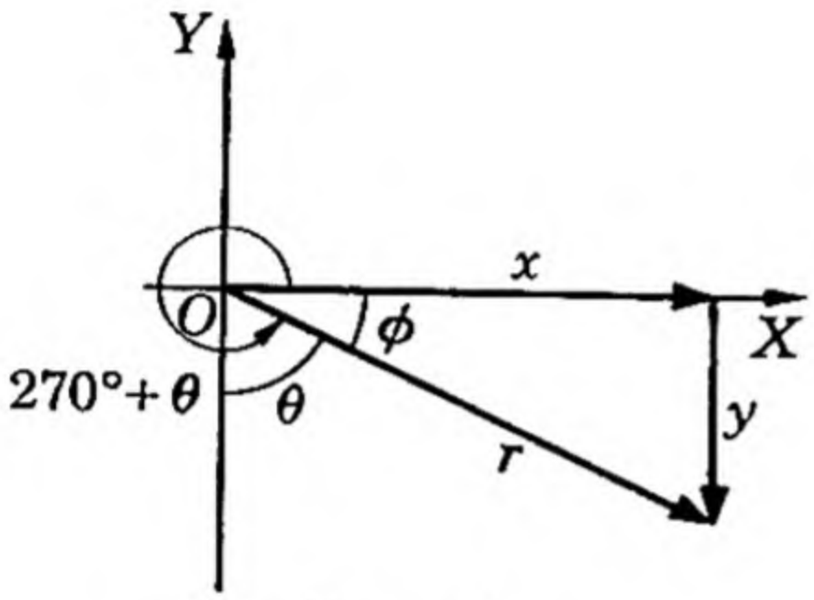


FIG. 4-6

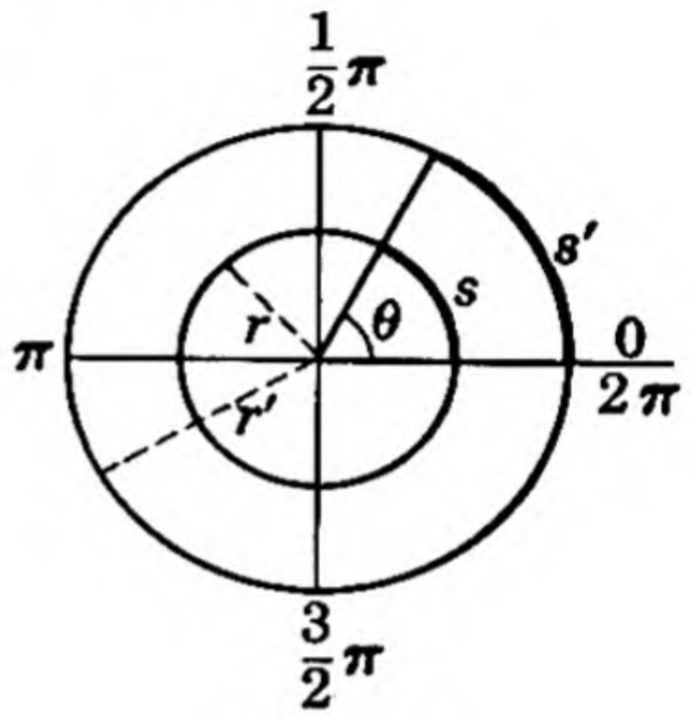


FIG. 4-7

PROBLEMS

Reduce the functions of the following angles to the functions of their reference angles:

1. 120°.
2. 135°.
3. 150°.
4. 210°.
5. 225°.
6. 240°.
7. 300°.
8. 315°.
9. 330°.
10. 375°.
11. 420°.
12. 610°.
13. −15°.
14. −84°.
15. −125°.
16. −164°.
17. −215°.
18. −255°.
19. −275°.
20. −290°.
21. −310°.
22. −375°.
23. −420°.
24. −585°.
25. 740°.
26. 800°.
27. 900°.
28. −760°.
29. −815°.
30. −920°.
31. 1035°.
32. −1123°.

Reduce the functions of the following angles to functions of θ:

33. $90° - \theta$.
34. $90° + \theta$.
35. $180° + \theta$.
36. $270° - \theta$.
37. $360° + \theta$.
38. $360° - \theta$.

4-4 Circular measure of angles. Let θ, Fig. 4-7, be an angle at the center of any two concentric circles of radii r and r' and let s and s' be the subtended arcs. Then θ is to 360° as each subtended arc is to the circumference of the corresponding circle; that is,

$$\frac{\theta}{360^\circ} = \frac{s}{2\pi r} \quad \text{and} \quad \frac{\theta}{360^\circ} = \frac{s'}{2\pi r'}.$$

Hence $$\frac{s}{2\pi r} = \frac{s'}{2\pi r'} \quad \text{or} \quad \frac{s}{r} = \frac{s'}{r'}.$$

Thus we find that the ratio of the arc to the radius is the same for the same central angle irrespective of the size of the circle; in other words, the value of this ratio depends solely on the angle. Therefore we may use this ratio as a measure of angles and write

$$\theta = \frac{s}{r}. \tag{4-1}$$

The right-hand member of this equation is called the *circular measure* of angles.

If $s = r$, then $\theta = 1$. The unit angle thus obtained is called the *radian*. Hence *a radian is the angle which subtends, when placed at the center of a circle, an arc equal to the radius.*

The degree is an arbitrary unit. The radian is a natural unit, not only because of the way it is derived, but also because it comes, unsought, into pure mathematics and into the mathematical formulation of many laws of Nature.

By definition, the number of radians around the center of a circle equals $\frac{2\pi r}{r}$; therefore

$$2\pi \text{ rad.} = 360^\circ,$$

or $$1 \text{ rad.} = \frac{360^\circ}{2\pi} = 57.296^\circ,$$

and $$1^\circ = 0.017453 \text{ rad.}$$

When an angle is expressed in radians the word radian is usually omitted. Thus 360°, 270°, 180°, 90°, 60°, 45°, and 30° are written as 2π, $\frac{3}{2}\pi$, π, $\frac{1}{2}\pi$, $\frac{1}{3}\pi$, $\frac{1}{4}\pi$, and $\frac{1}{6}\pi$, respectively.

4-5 Area of a sector of a circle. Let A denote the area of a sector of a circle, Fig. 4-8. Then

$$\frac{A}{\pi r^2} = \frac{s}{2\pi r}.$$

Hence $$A = \frac{1}{2} rs, \qquad [s = \mathrm{r}\theta]$$

or $$\mathbf{A} = \frac{1}{2} \mathbf{r}^2\theta, \tag{4-2}$$

where θ is measured in radians.

4-6 Relation between linear and angular velocities. Let s be an arc of a circle of radius r, Fig. 4-9, described by a particle in the time t. Then

$$s = r\theta, \tag{a}$$

and

$$\frac{s}{t} = r\,\frac{\theta}{t}, \tag{b}$$

which may be written as $$\mathbf{v} = \mathbf{r}\omega, \tag{4-3}$$

where v denotes the *linear velocity* of the particle and ω the *angular velocity* of the line which joins the particle to the center of the circle. In Eq. (b) s/t and θ/t equal the average velocity and the average angular velocity, respectively; but both Eq. (b) and Formula (4-3) are valid whether the velocities are constant or variable, so long as the motion is in a circle. Since θ is in radians in (a) and (b), ω is in radians per unit time in (4-3).

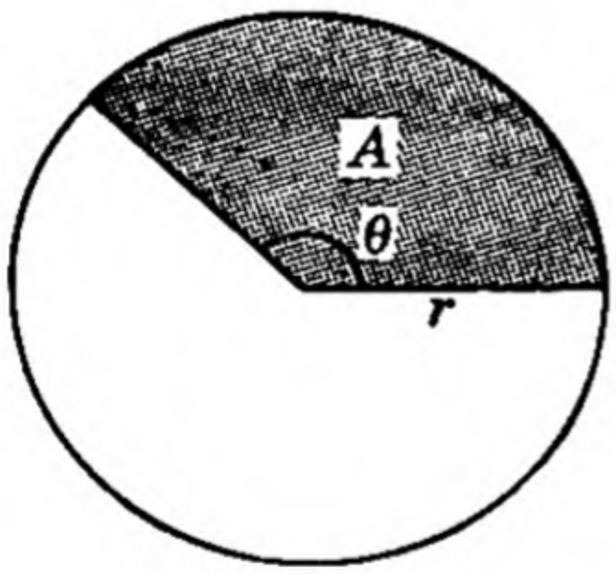


FIG. 4-8

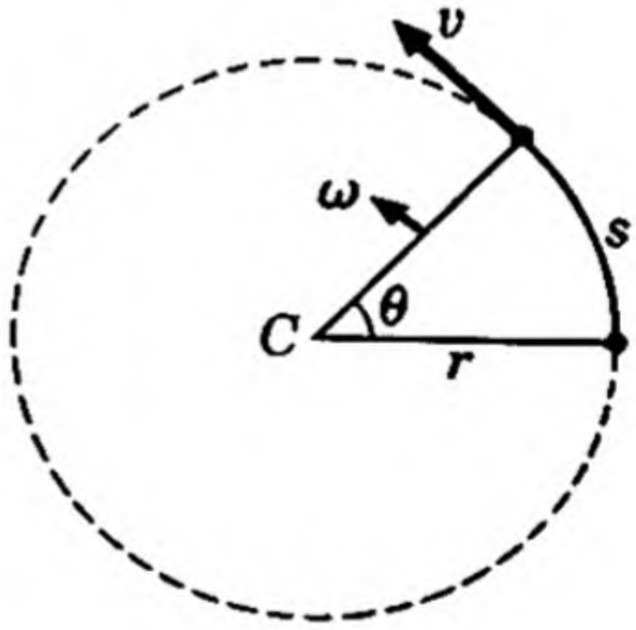


FIG. 4-9

PROBLEMS

The following angles are at the center of a circle 20 inches in diameter; find the arc subtended by each, to one hundredth of one inch.

1. 24° 16′.
2. 43° 15′.
3. 79° 36′.
4. 128° 17′.
5. 143° 24′.
6. 167° 42′.

7. Find the area of a sector of a circle of 10-inch radius if the central angle is 36° 47′.

8. Find an expression for the shaded part of the sector in Fig. 4-10.

9. The *latitude* (angle λ in Fig. 4-11, where the horizontal great circle represents the equator) of New York City is 40° 30.5′. Taking the radius of the Earth to be 3960 miles, find the distance of New York from the equator.

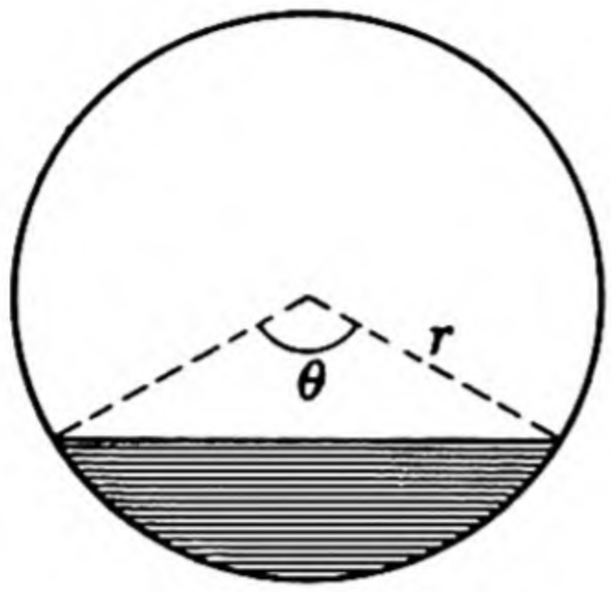


FIG. 4-10

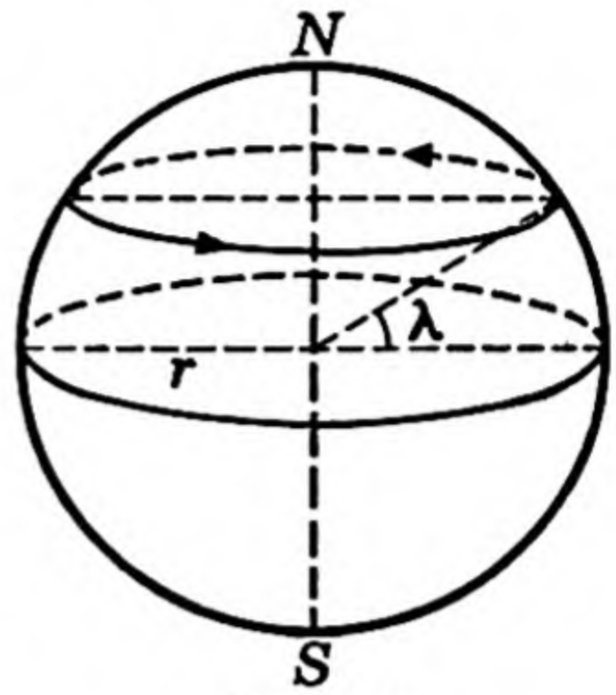


FIG. 4-11

10. Using the data of the preceding problem, find the linear velocity of New York due to the rotation of the Earth about its axis.

11. The latitude of Boston is 42° 20′; find its distance from the North Pole.

12. Find the linear velocity of Boston due to the rotation of the Earth.

13. The latitudes of New Orleans and Chicago are about 30° and 41.8°, respectively. Considering the two cities to be near the same line of *longitude* (North-South line), find the distance between them.

14. If the tires of an automobile are 28 inches in diameter, find their angular velocity when the automobile is moving at the rate of 50 miles per hour.

15. Find the angular velocity of the driving wheels of a locomotive, if the wheels are 6 feet in diameter and the velocity of the locomotive is 70 miles per hour.

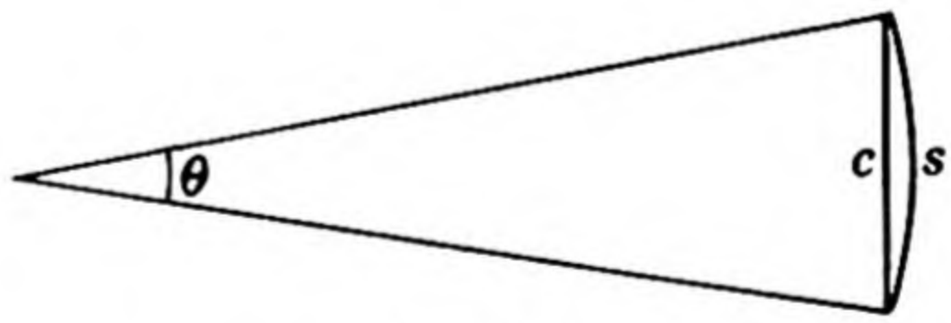


FIG. 4-12

16. A pulley, 4 feet in diameter, has an angular velocity of 200 revolutions per minute. Find the linear velocity of the belt.

17. Find the linear velocity of the Earth in its orbit, considering the orbit to be a circle 92.8×10^6 miles in radius and the year to be 365.25 days.

18. The propellers of a superfortress are 16.5 feet from tip to tip; find the linear velocity of the tips if the propeller makes 1000 revolutions per minute at cruising speed.

19. The diameter of the moon subtends an angle of 31′ at the Earth. Find the distance of the moon if the diameter is 2160 miles.*

20. The Sun subtends an angle of 32′ at the Earth. Taking the distance of the Sun to be 92.8×10^6 miles, find the radius.

21. The *parallax* of *Proxima* (the nearest known star) is 0.76″. Find its distance in *light-years*.†

22. The parallax of Sirius is 0.38″. Find its distance in light-years.

---

* If the central angle θ, Fig. 4-12, is very small, the lengths of the chord c and the arc s are very nearly equal; therefore the equation $s = r\theta$ may be replaced by $c = r\theta$, where θ is in radians.

† The parallax of a star is the angle which the radius of Earth's orbit (92.8×10^6 miles) subtends at the star. A light-year is the distance light travels in one year and equals 5.88×10^{12} miles.

CHAPTER 5

VARIATIONS AND GRAPHS OF TRIGONOMETRIC FUNCTIONS

5-1 Variation, range, and graph of sin x. In the study of the variation of the value of a trigonometric function with the angle, it is convenient to represent the angle by x, the function by y, and the relationship of y to x by a curve on the xy-plane.

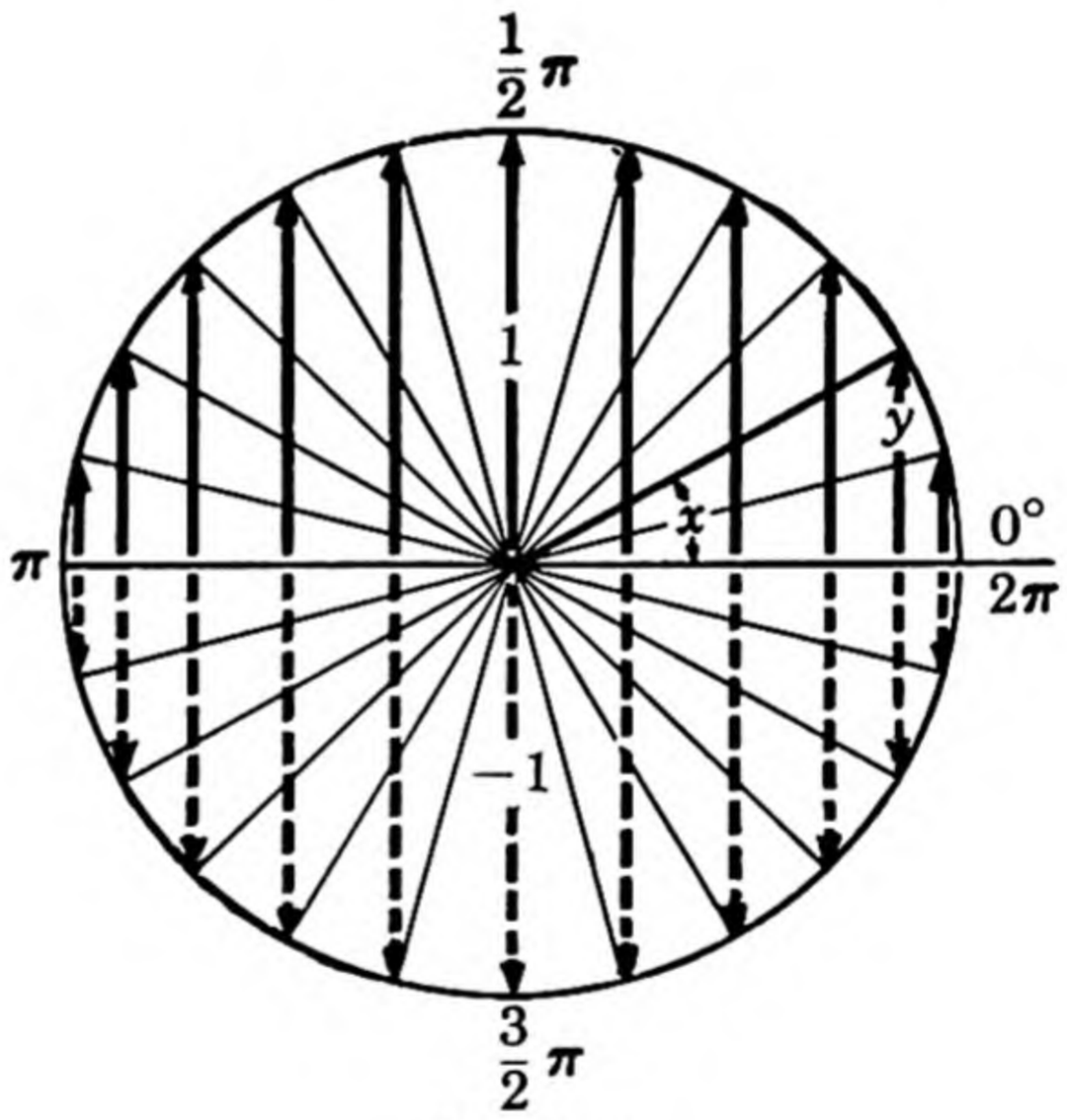


FIG. 5-1

Let x, Fig. 5-1, denote an angle at the center of a circle of unit radius, say one inch, and the length of the line marked y be y inches; then by definition, $\sin x = \dfrac{y \text{ inch}}{1 \text{ inch}} = y$, or $y = \sin x$.

It is important to note that y is the numerical value of the length, not the length, of the line labeled y; in other words, $\sin x$ and, consequently, y is a pure number, not a length.

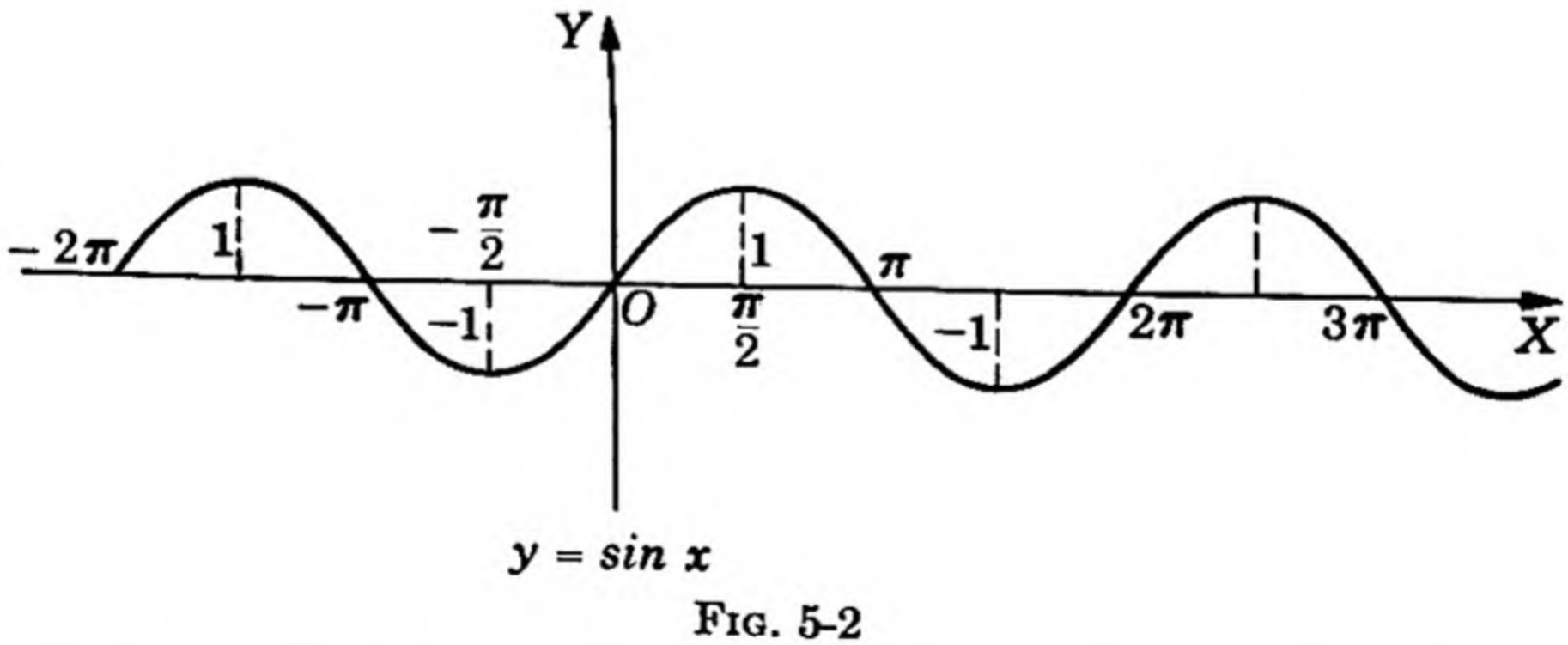
As the radius which forms the terminal side of angle x starts

from the position where $x = 0°$ and rotates in the counterclockwise direction, the value of $\sin x\ (= y)$ starts from zero and changes as indicated by the vertical arrows. The arrows in broken lines indicate negative values of $\sin x$.

If the rotation takes place in the opposite direction, $\sin x$ takes on the same sequence of values but in the reverse order. If the rotation is continued, the same sequence of values is repeated during every complete rotation. The variation of $\sin x$, as x increases from 0° to 360°, is indicated also by the following table:

x	$\sin x\ (=y)$
$0° \rightarrow \frac{\pi}{2}$	$0 \rightarrow 1$
$\frac{\pi}{2} \rightarrow \pi$	$1 \rightarrow 0$
$\pi \rightarrow \frac{3\pi}{2}$	$0 \rightarrow -1$
$\frac{3\pi}{2} \rightarrow 2\pi$	$-1 \rightarrow 0$

These results are graphically portrayed by the *sine curve*, Fig. 5-2, where the x-coordinate of a point on the curve represents an angle,



$y = \sin x$

FIG. 5-2

and the y-coordinate the sine of the angle. In the figure, the curve is drawn in different scales along the two coordinate axes.

The following important conclusions may be drawn from the sine curve:

(a) The values of sin x *range* from -1 to 1, that is,

$$-1 \leq \sin x \leq 1.$$

(b) The sine of a negative angle equals minus the sine of a positive angle of the same size, that is,

$$\sin(-x) = -\sin x.$$

(c) Sin x is a *periodic function* of x, and has a *period* of 2π.

This is a concise way of stating that the value of sin x changes with the value of x in such a way as to make the sine curve a repetition of the same pattern as that between any two points on the curve whose x-coordinates differ by 2π. The following is an even more concise way of stating the same thing.

$$\sin(x \pm 2\pi n) = \sin x,$$

where n is any integer.

5-2 Variation, range, and graph of cos x. Let the circle, Fig. 5-3, have a unit radius and x be an angle at the center; then $y = \cos x$.

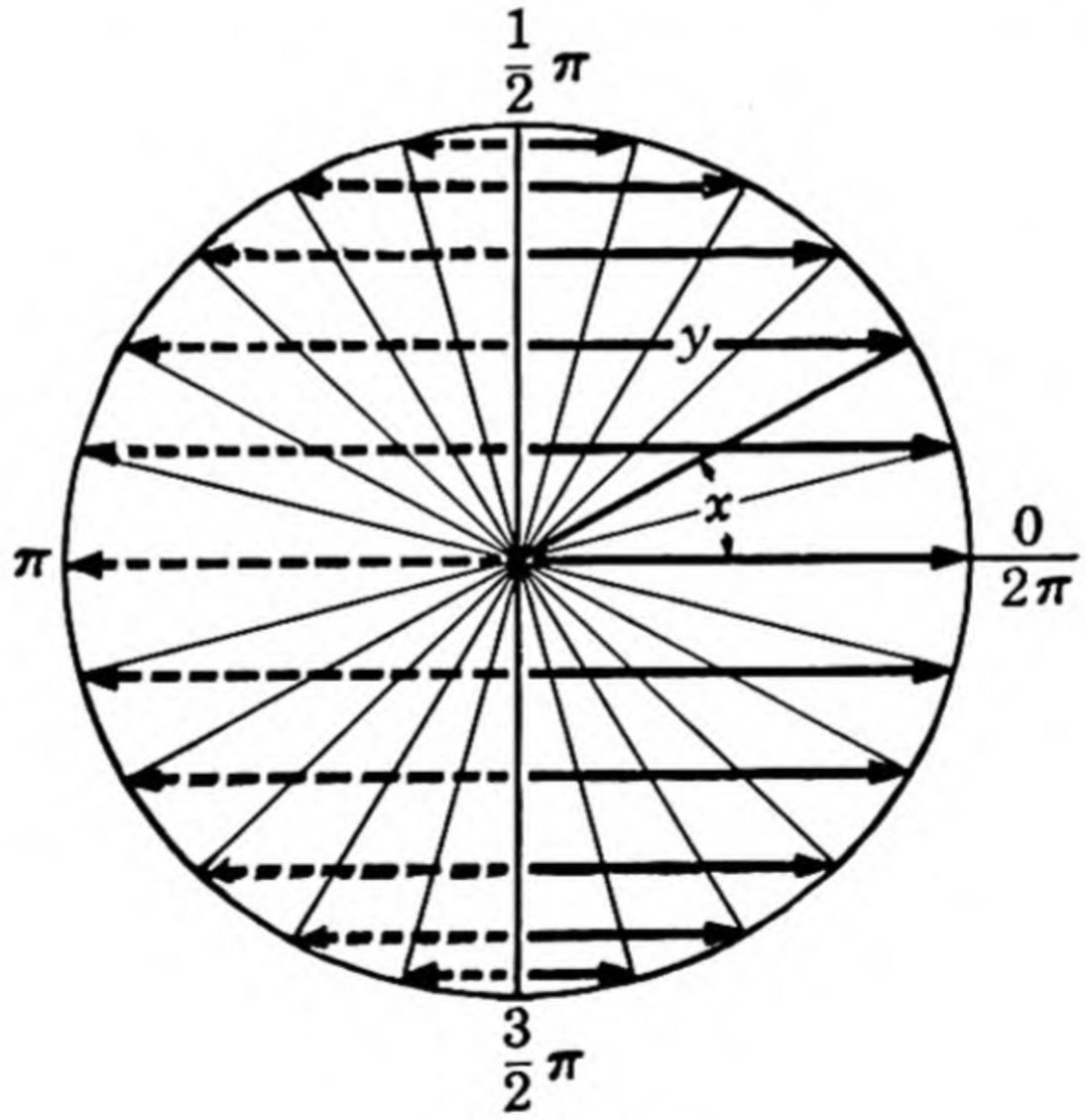


FIG. 5-3

As the radius which forms the terminal side of x starts from the position where $x = 0°$ and rotates counterclockwise, the value of y starts from 1 and changes as indicated by the arrows in the figure.

If the rotation takes place in the opposite direction, cos x takes on the same sequence of values but in the reverse order. If the rotation is continued, the same sequence of values is repeated during every complete rotation.

The variation of cos x with x is also indicated by the following table:

x	$\cos x \ (=y)$
$0° \to \frac{\pi}{2}$	$1 \to 0$
$\frac{\pi}{2} \to \pi$	$0 \to -1$
$\pi \to \frac{3\pi}{2}$	$-1 \to 0$
$\frac{3\pi}{2} \to 2\pi$	$0 \to 1$

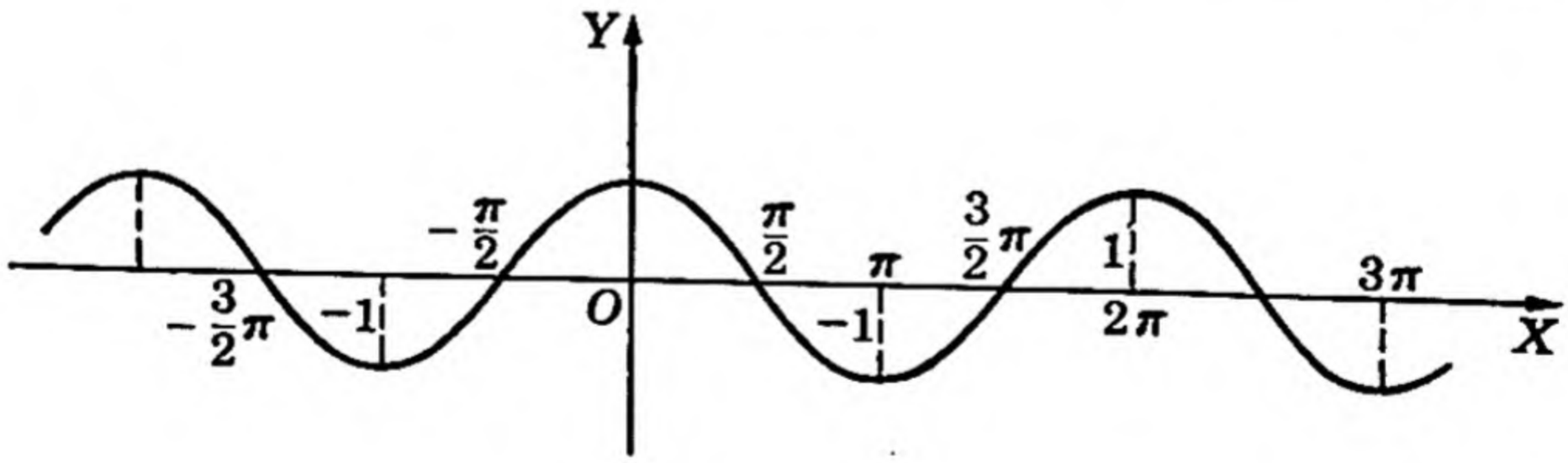
The following conclusions are drawn from the *cosine curve*, Fig. 5-4:

(a) $-1 \leq \cos x \leq 1.$

(b) $\cos(-x) = \cos x.$

(c) $\cos(x \pm 2\pi n) = \cos x, \qquad P = 2\pi.$

The sine and cosine curves are identical but are differently placed



$y = \cos x$

FIG. 5-4

relative to the y-axis. The cosine curve becomes a sine curve if it is shifted toward the right through $\frac{1}{2}\pi$. This is to be expected because the arrows in Fig. 5-3, which represent cos x, assume the

same positions as those in Fig. 5-1, when the former is rotated through 90° in the counterclockwise direction.

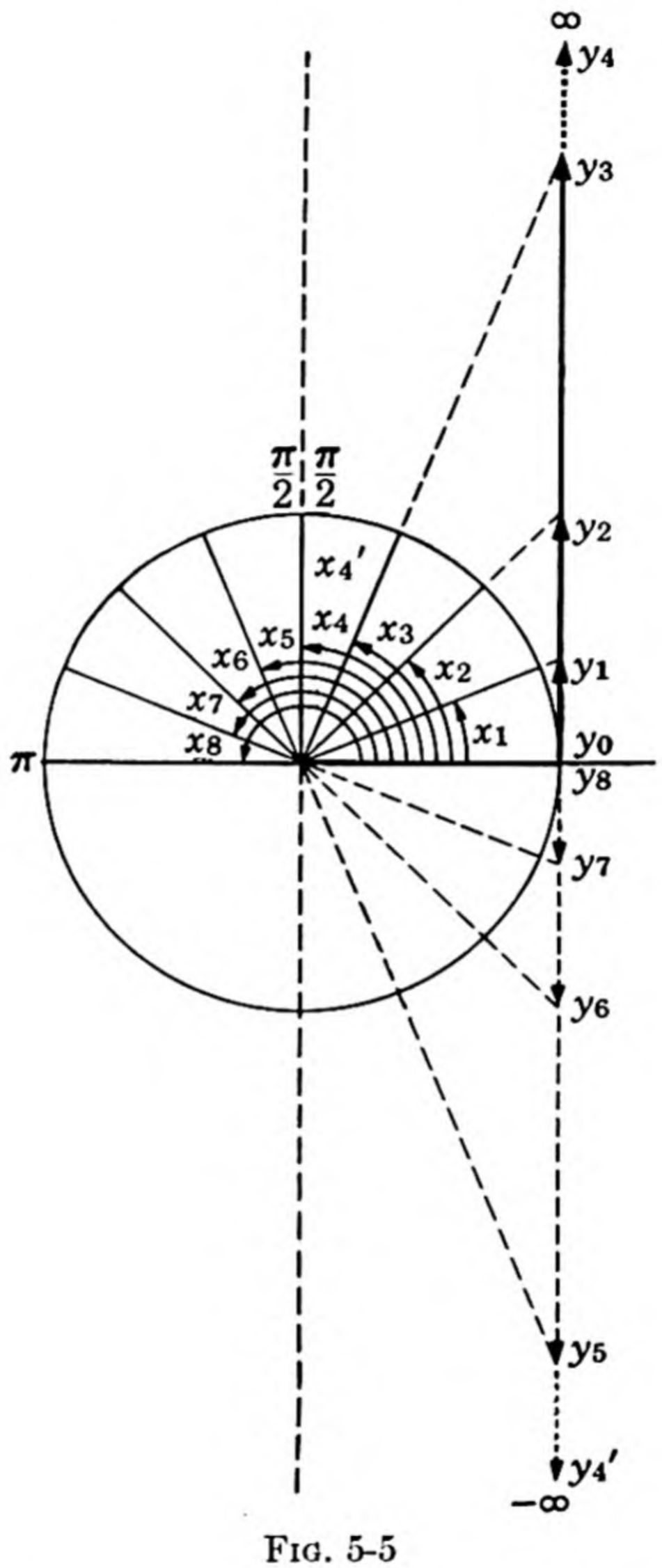


FIG. 5-5

5-3 Variation, range, and graph of tan x. Let the circle, Fig. 5-5, have a unit radius and x denote an angle at the center; then $y = \tan x$, where y is the numerical value of a vertical arrow measured from the point labeled y_0 $(=0)$. As the angle starts from x_0 $(=0°)$ and assumes the values x_1, x_2, x_3 and x_4 $(= 90°)$, tan x

takes on the numerical values of the arrows labeled $y_o\,(=0)$, y_1, y_2, y_3, and becomes $y_4\,(=\infty)$. As the angle passes 90° ($x_4 = x'_4 = 90°$) and assumes the values x_5, x_6, x_7 and $x_8\,(=180°)$, tan x suddenly becomes $y'_4\,(=-\infty)$ and takes on the values y_5, y_6, y_7, $y_8\,(=0)$. As the angle x increases and reaches 360°, the same sequence of values is repeated in the same order. These results are summarized in the following table:

x	$\tan x\ (=y)$
$0° \to \frac{\pi}{2}$	$0 \to \infty$
	$\uparrow$
	$\vdots$
	$\downarrow$
$\frac{\pi}{2} \to \pi$	$-\infty \to 0$
$\pi \to \frac{3\pi}{2}$	$0 \to \infty$
	$\uparrow$
	$\vdots$
	$\downarrow$
$\frac{3\pi}{2} \to 2\pi$	$-\infty \to 0$

The symbol "$\leftarrow\cdot\rightarrow$" is used to indicate that the change from ∞ to $-\infty$ or from $-\infty$ to ∞ takes place suddenly.

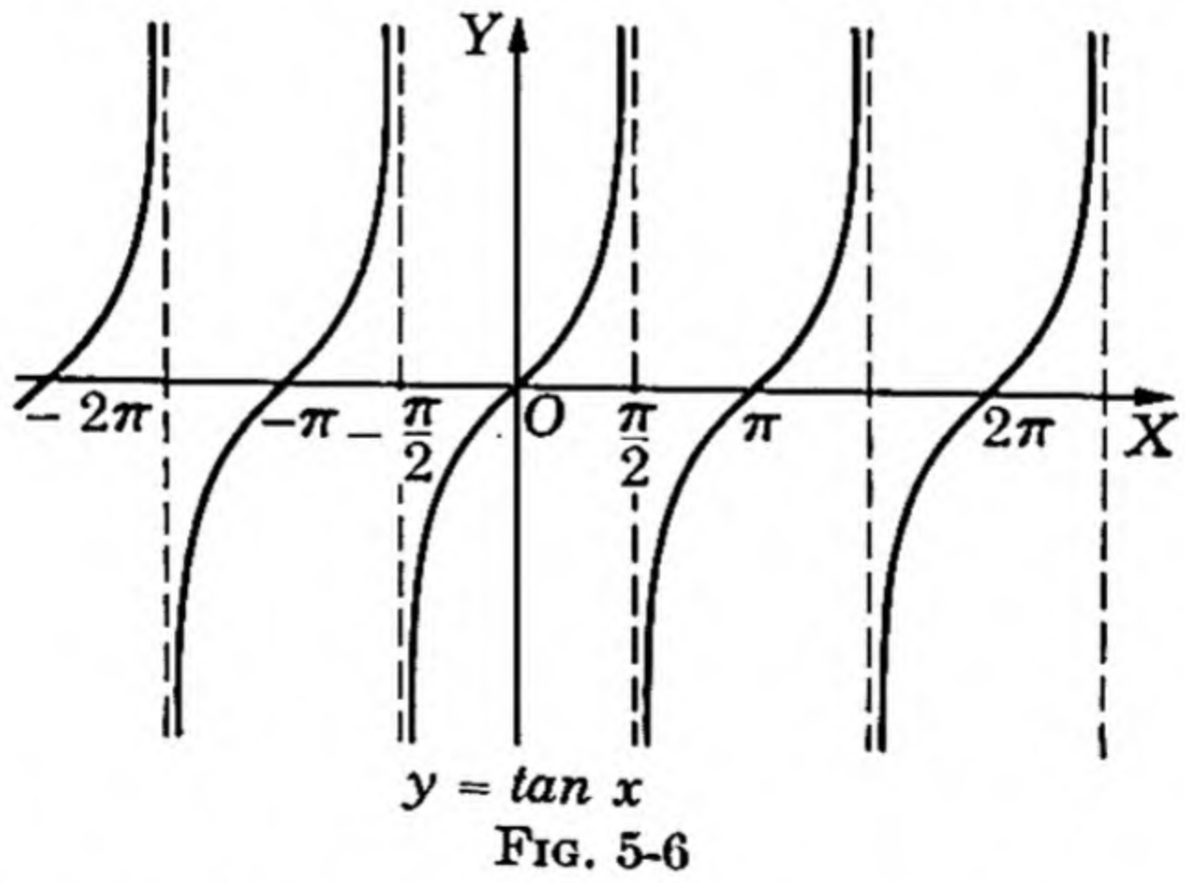


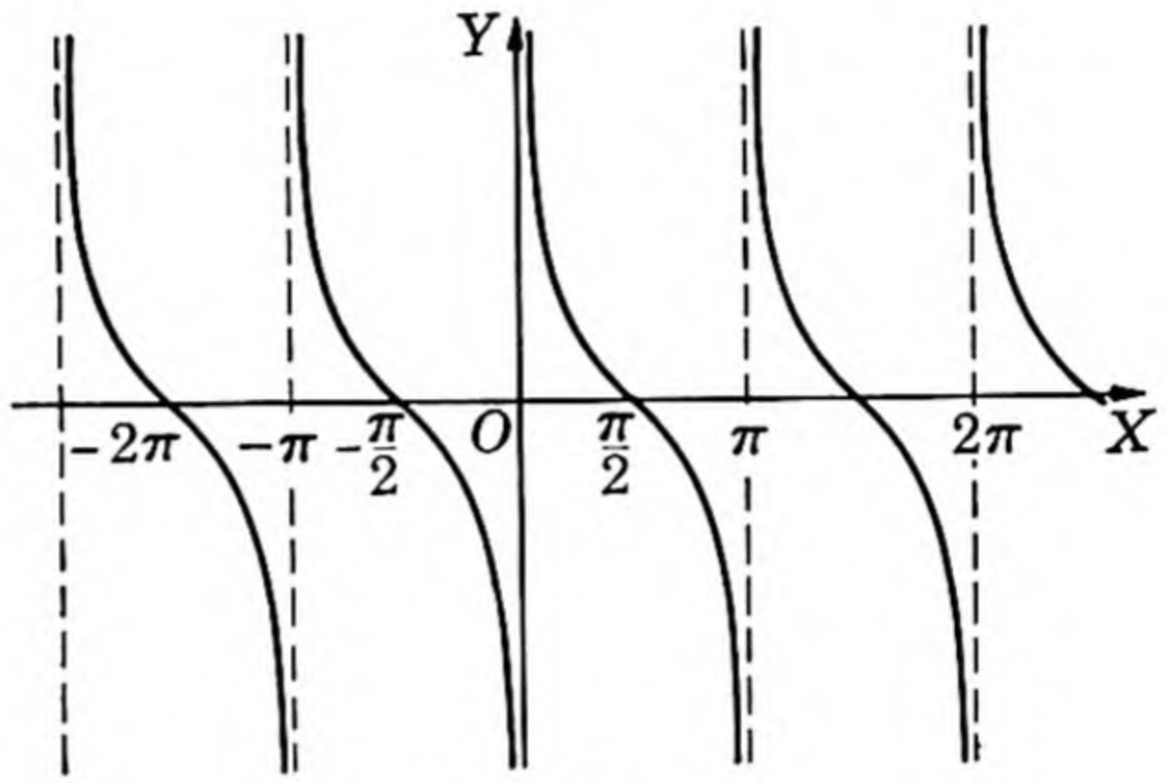
FIG. 5-6

The following conclusions are drawn from the *tangent curve*, Fig. 5-6:

(a) $-\infty < \tan x < \infty$.
(b) $\tan(-x) = -\tan x$.
(c) $\tan(x \pm n\pi) = \tan x, \qquad P = \pi$.

The tangent curve differs from the sine and the cosine curves in three important respects: its ordinate has an unlimited range of values; the curve is *discontinuous* at $x = \pm\frac{1}{2}\pi$, $\pm\frac{3}{2}\pi$, $\pm\frac{5}{2}\pi$, etc.; and the period is π.

5-4 Graphs of cot x, sec x, and csc x. The variation, the range and the period of each of these functions are indicated by the curves in Figs. 5-7, 5-8, and 5-9.

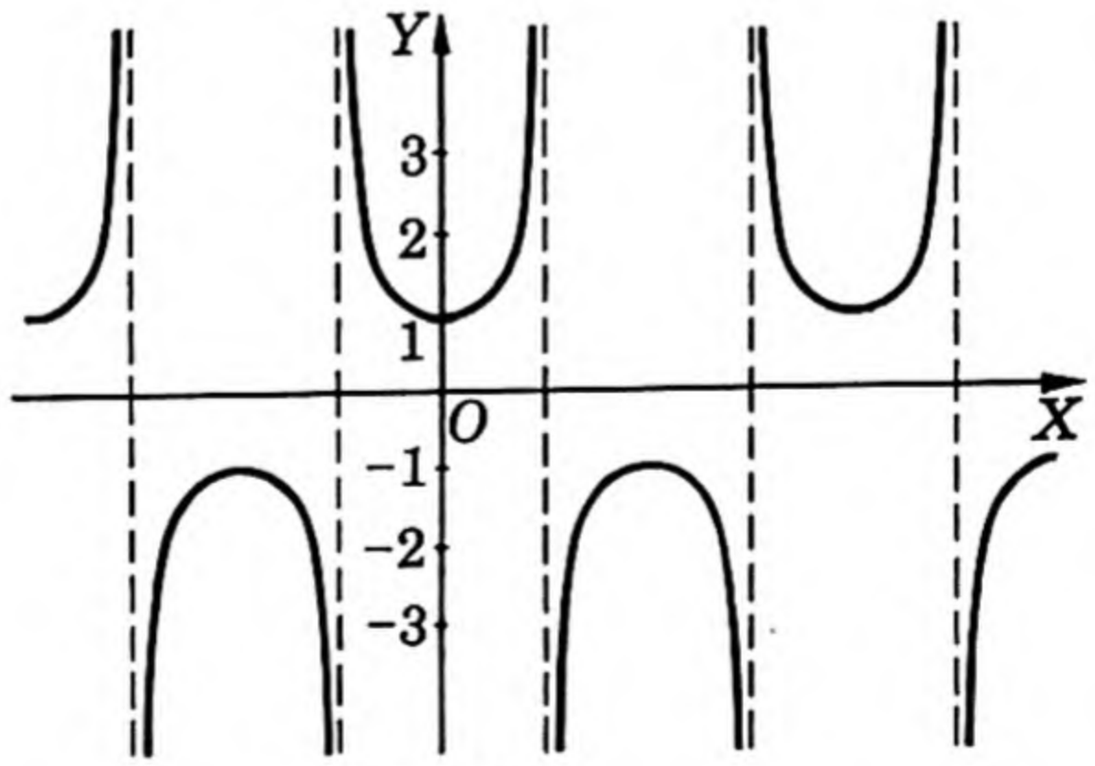


$y = cot\ x$

FIG. 5-7

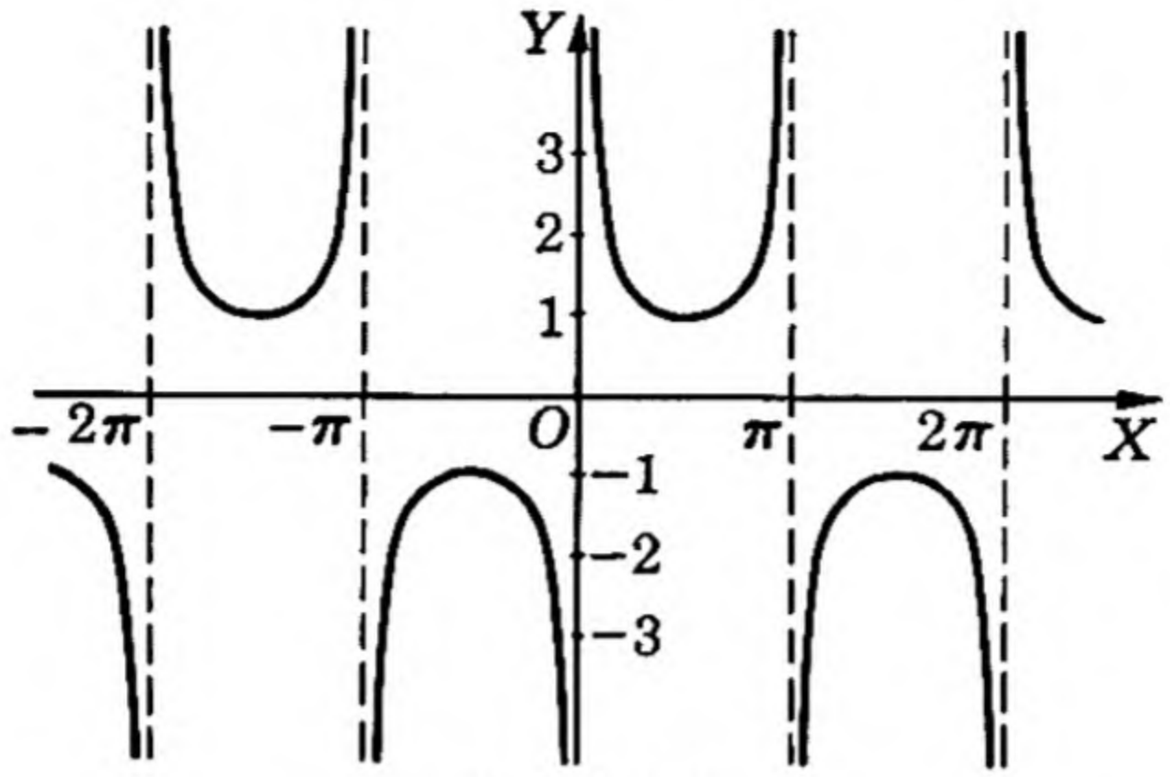
5-5 Geometric addition of curves. The graph of $y = \sin x + \cos x$ may be obtained by plotting points which satisfy this equation but the following method is much more convenient.

Let curves I and II, Fig. 5-10, be the graphs of $y = \sin x$ and



$y = sec\ x$

FIG. 5-8



$y = csc\ x$

FIG. 5-9

$y = \cos x$, respectively. Then if the ordinates of points on these curves having the same abscissa are added, the corresponding point on the graph (curve III) of the original equation is obtained. For instance, the algebraic sum of the ordinates of the points A' and A'', B' and B'', C' and C'' give the ordinates of the points A, B, C, respectively. After finding a few strategically located points in this manner, a smooth curve is drawn through them to obtain the graph of the original equation.

This method can be used to obtain the graph of any composite function—that is, the curve of any equation which, when solved

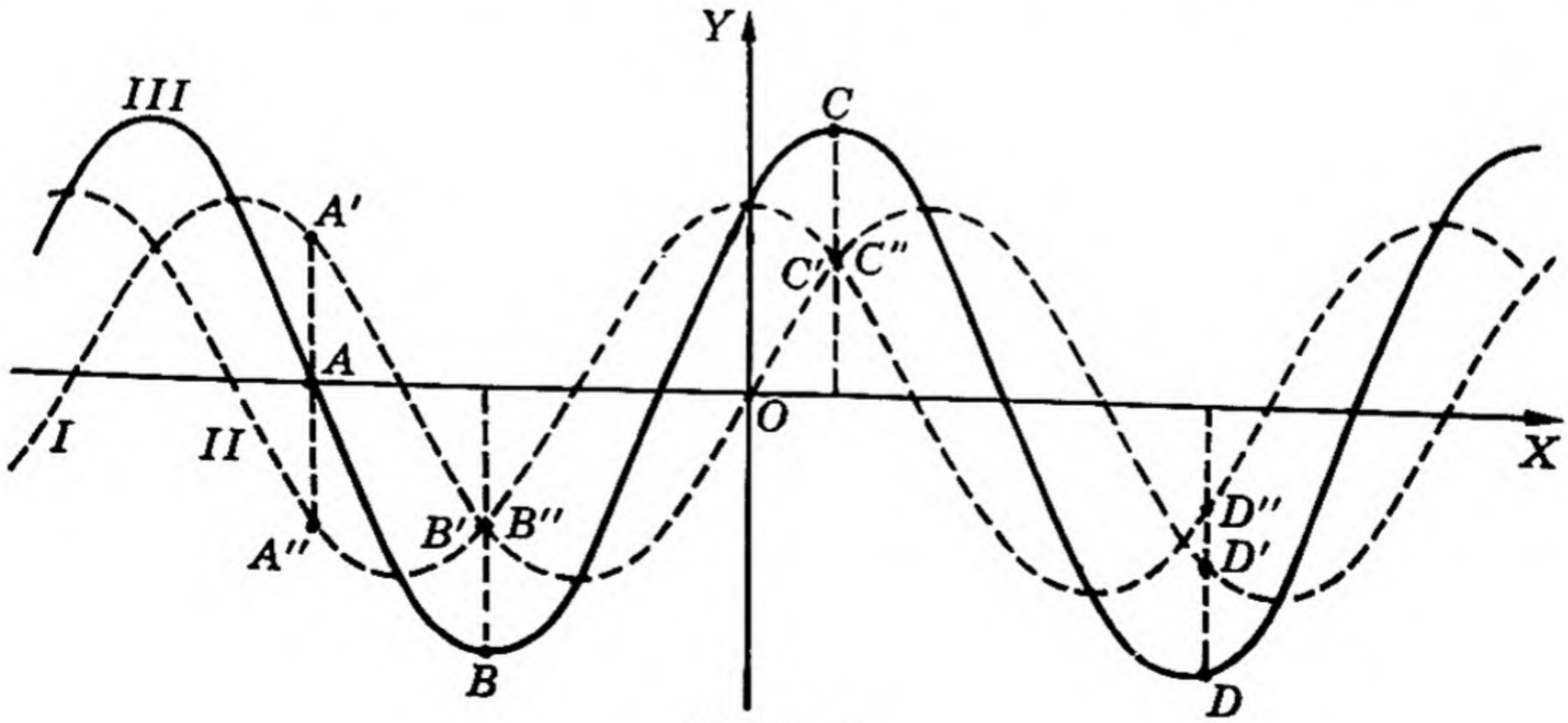


FIG. 5-10

for y, contains more than one term in the right-hand member. But the method is most useful when the graphs of the individual terms can be drawn more easily than the graph of the composite function.

5-6 Application to periodic phenomena. Some of the phenomena of Nature are obviously periodic, such as the successions of day and night, of the four seasons, and of waves. Scientists have shown that the phenomena of sound, light, electromagnetism, and many others, are also periodic.

In the mathematical study of periodic phenomena, trigonometric functions, particularly the sine and the cosine, play an indispensable part. The sine and the cosine are important not only because they are the simplest periodic functions but also because complex periodic functions can be expressed in their terms.

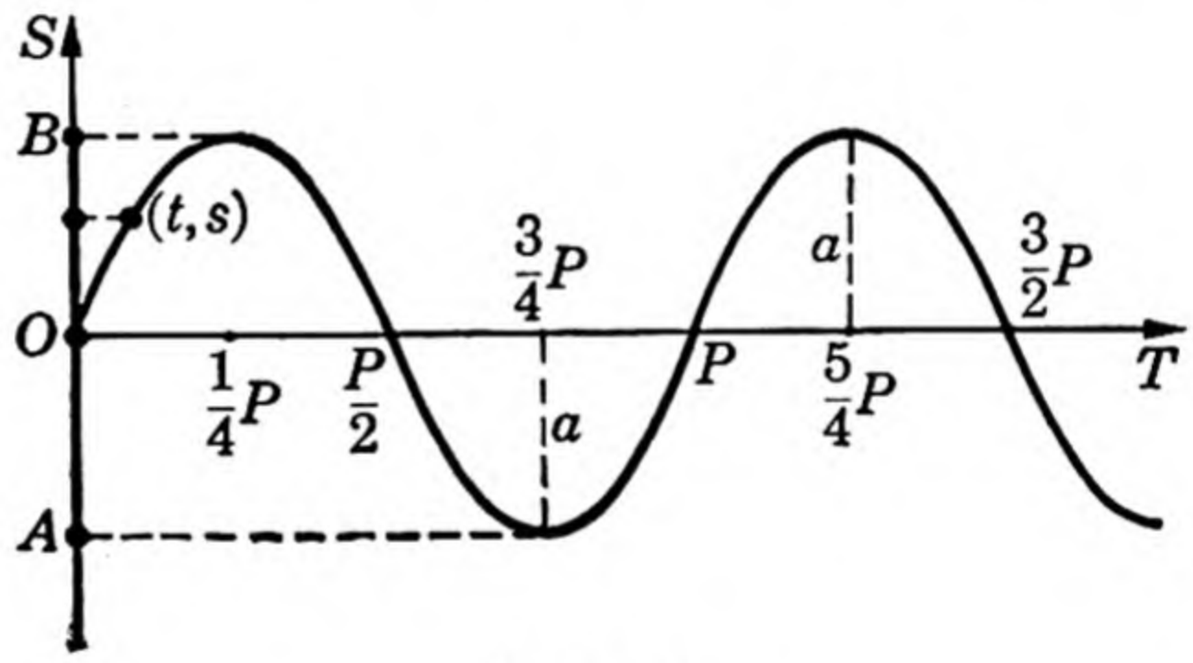


FIG. 5-11

The simplest periodic phenomenon is the motion, called a *simple harmonic motion*, of a particle which vibrates about a fixed point. If a particle vibrates simple harmonically between A and B about the point O, Fig. 5-11, the relationship between the variable distance, s, of the particle from the point O and the time t is indicated by the sine curve, where P denotes the period and a the *amplitude* of the motion.

A more complex periodic motion can be analyzed into simple harmonic motions, and its graph can be shown to be the result of a combination of sine and cosine curves. For example, the curve in Fig. 5-12, which is the photograph of the sound from a violin,* is

* Dayton C. Miller, *The Science of Musical Sounds*, Macmillan Co., New York, 1934. p. 100.

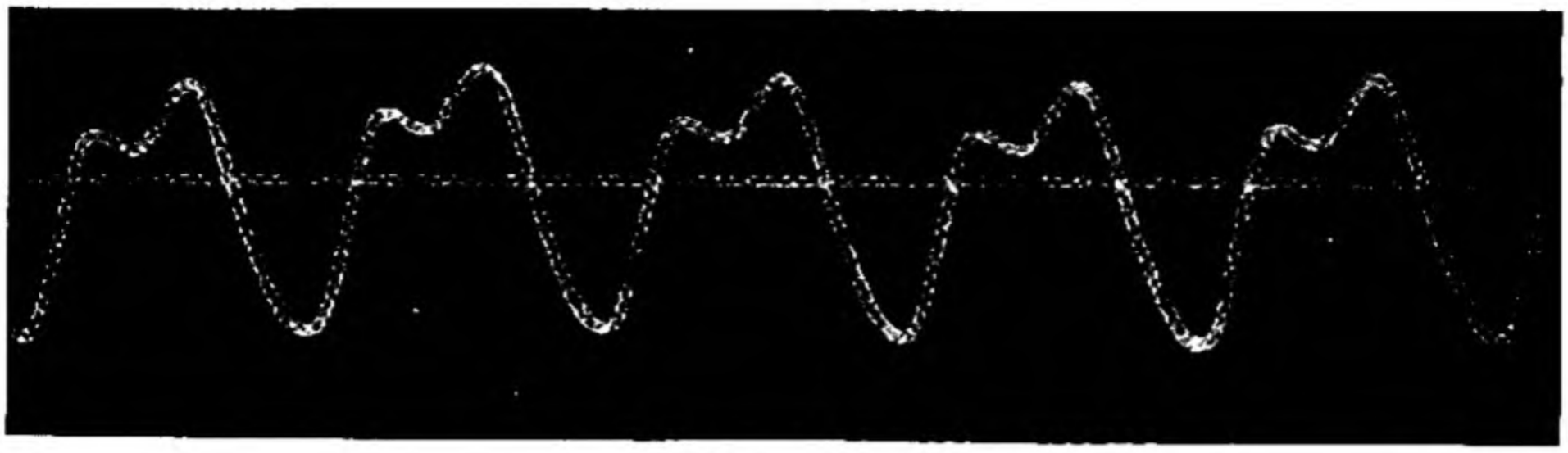


FIG. 5-12

(*From Miller: The Science of Musical Sound, copyright 1916; by permission of Macmillan Publishing Company.*)

analyzed by methods of higher mathematics or by an instrument called a *harmonic analyzer* into the three sine and cosine curves shown in broken lines in Fig. 5-13. When these three curves are added, the continuous curve which represents the sound is obtained. Of the three curves, the one with the greatest period represents the *fundamental tone* of the sound, and the other two the *overtones*. The quality of the sound produced by a musical instrument depends on the number and the amplitudes of the overtones and also on the timing of the overtones relative to the fundamental tone.

Trigonometric functions, particularly the sine and the cosine, are of great importance not only to the solution of problems of surveying, navigation, engineering, and the physical sciences but also to the study of higher mathematics and its application to many fields of knowledge.

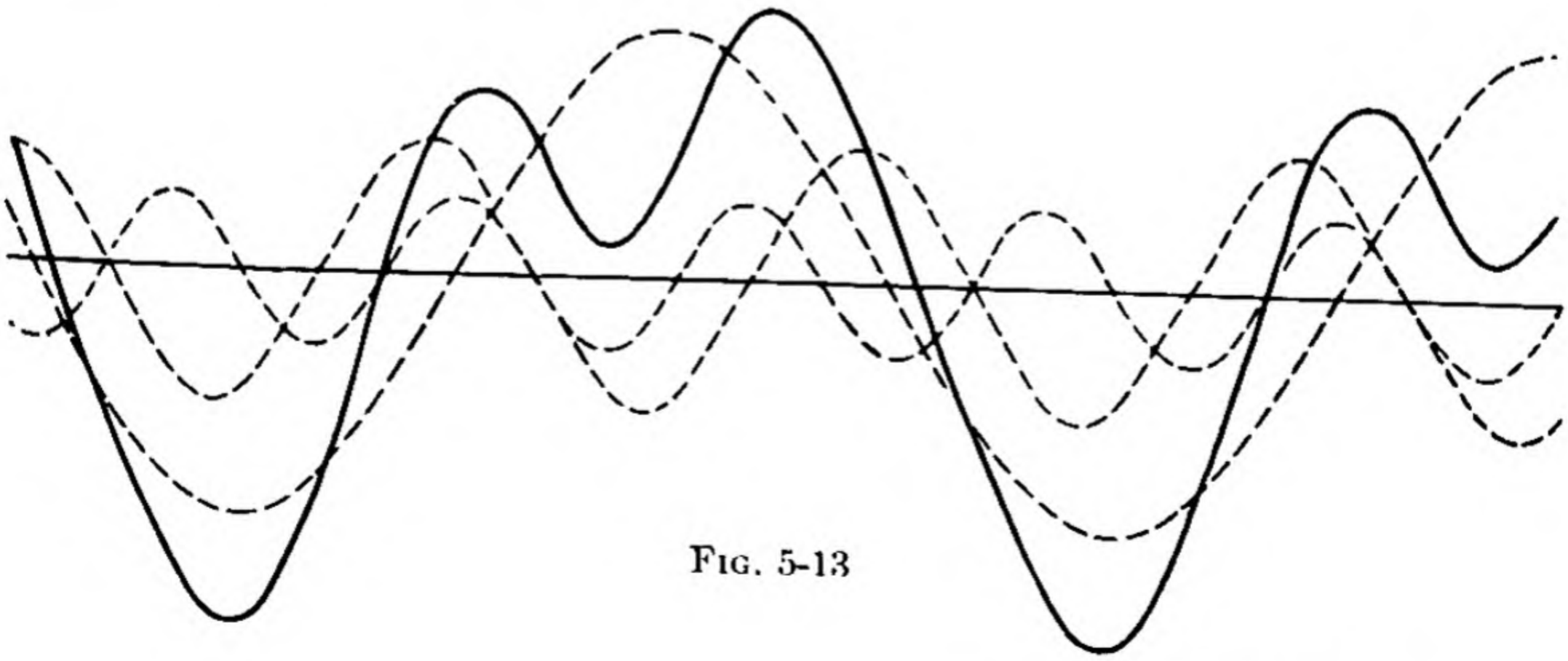


FIG. 5-13

9959

PROBLEMS

Draw the graphs of the following equations and determine their periods and amplitudes:

1. $y = 2 \sin x$.
2. $y = 3 \cos x$.
3. $y = \sin 2x$.
4. $y = 2 \cos 2x$.
5. $y = \sin \frac{1}{2}x$.
6. $y = \cos \frac{1}{2}x$.

Use the method of geometric addition of curves to obtain the graphs of the following equations:

7. $y = 1 + \sin x$.
8. $y = 2 - \cos x$.
9. $y = \sin x + 2 \cos x$.
10. $y = 2 \cos x - \sin x$.
11. $y = 2 \sin x + \cos x$.
12. $y = 2 \sin x - \cos x$.
13. $y = \sin x - 2 \cos x$.
14. $y = \sin x + \cos 2x$.
15. $y = \sin x - \cos 2x$.
16. $y = \cos x + \sin 2x$.
17. $y = 2 \sin x + \sin 2x$.
18. $y = 2 \cos x - \sin 2x$.

5-7 Function. If two variables are so related that one of them takes on a definite value when the other is given any permissible value, the variables are said to be *functionally related.* The variable to which arbitrary values are given is called the *independent variable;* and the variable whose values are then determined is called the *dependent variable.* The dependent variable, as well as its expression in terms of the independent variable, is called the *function.*

The variable coordinates of a point which moves along the curve of Fig. 5-2, for example, are related by the equation

$$y = \sin x.$$

In this equation, x is the independent variable, y the dependent variable, and y, as well as $\sin x$, the function. If we let $x = wt$ and write $y = \sin wt$, then t becomes the independent variable in both $x = wt$ and $y = \sin wt$.

5-8 Inverse function. If an equation in two variables is solved for one of the variables and then for the other, the right-hand members of the resulting equations are called the *inverse* of each other. The simpler of the two right-hand members is called the *direct function* or simply the *function*, and the other the *inverse function.*

In the equations $y = \frac{1}{8}x^3$ and $x = 2y^{1/3}$, for instance, $2y^{1/3}$ is the inverse of $\frac{1}{8}x^3$. Since both equations express the same relationship between x and y, their graphs are identical (Curve I in Fig. 5-14).

It is customary to use the same variable as the independent variable in the equations of a function and its inverse. When this is done the graph of the inverse function assumes a position different from that of the function.

If the inverse of $y = \frac{1}{8}x^3$ is written as $y = 2x^{1/3}$, for instance, its graph takes the position indicated by Curve II in Fig. 5-14.

The graph of an inverse function may be obtained *by rotating the graph of the corresponding direct function through 180° about the 45° line through the origin or by drawing a curve which is symmetric to the curve of the direct function with respect to the 45° line.**

The logarithmic function $\log_b x$ is the inverse of the exponential function b^x. The graph of $y = \log_b x$ (Curve I of Fig. 9-1, p. 91) is obtained by rotating the graph of $y = b^x$ (Curve II of the same figure) through 180° about the 45° line.

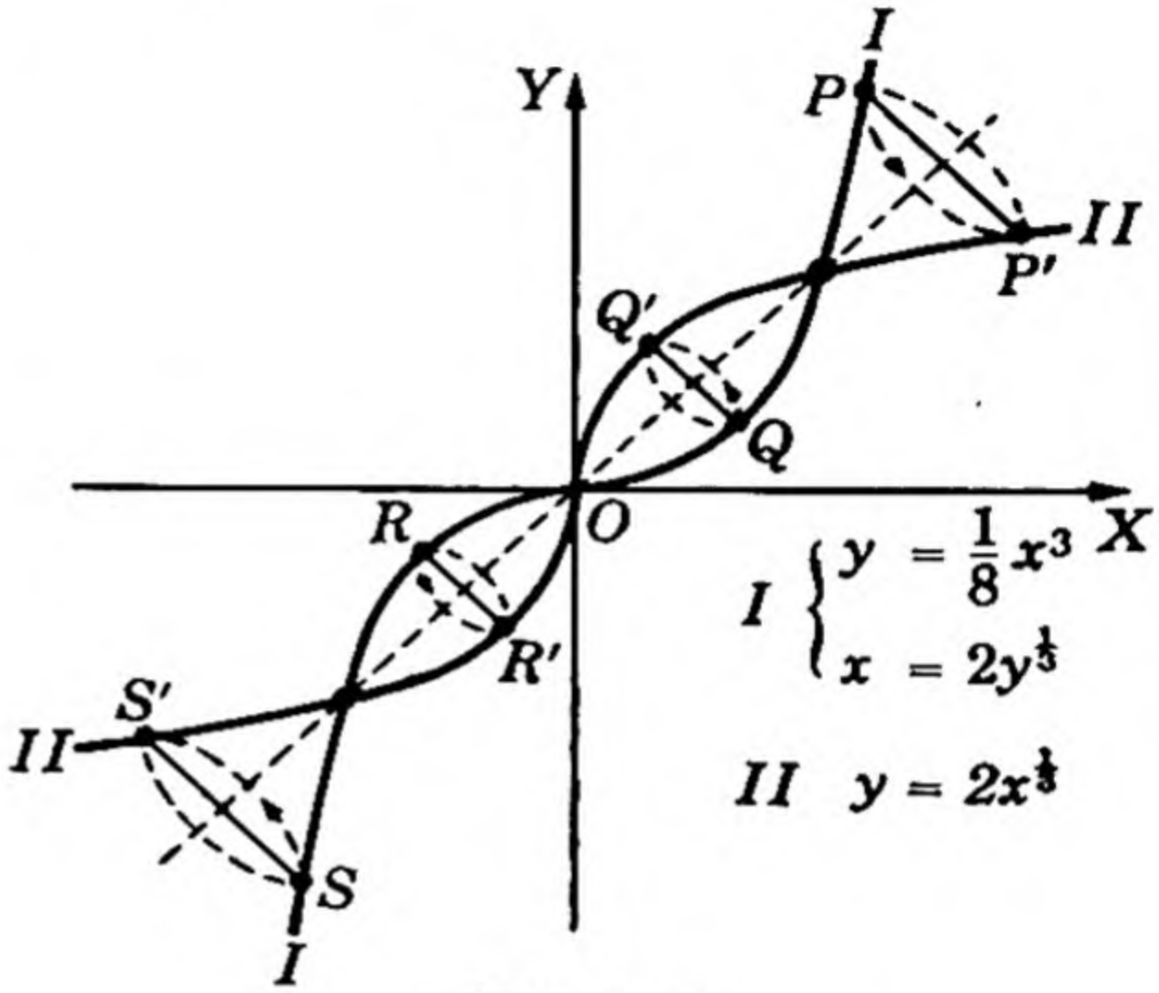


FIG. 5-14

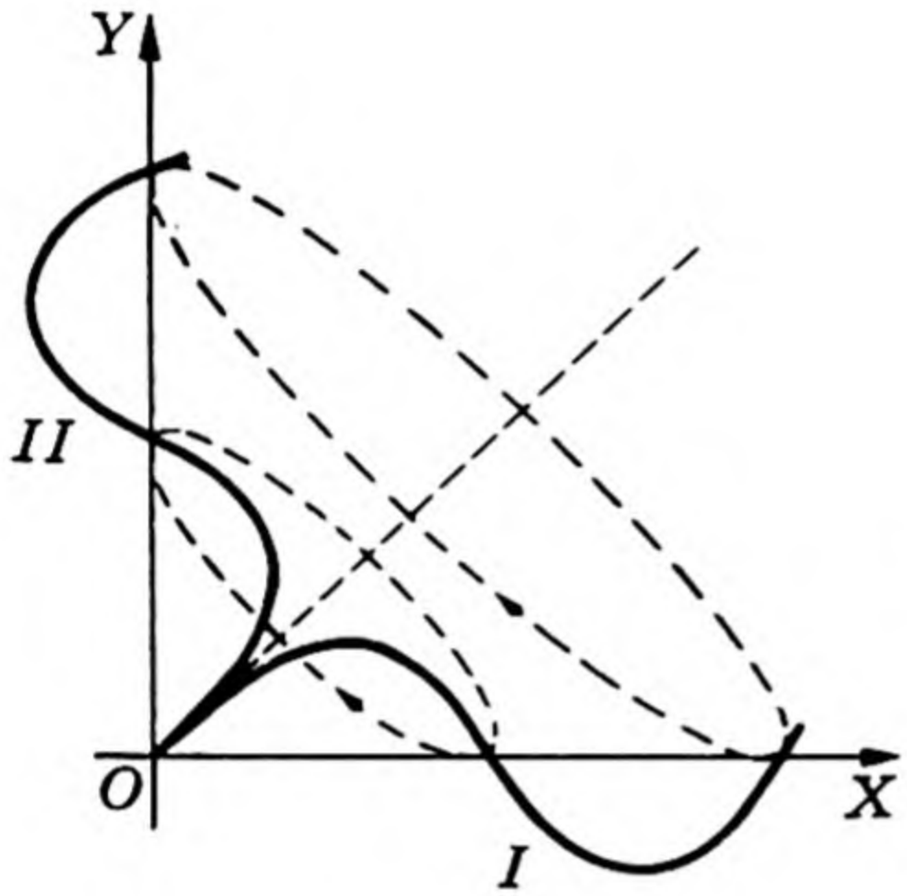
5-9 Inverse trigonometric functions. The inverse of $y = \sin x$ is written as $x = \text{arc sin } y$, which means "x is the angle whose sine is y." Thus the two equations are identical except in form and each equation amounts to reading the other backwards. When x is retained as the independent variable, the inverse of $y = \sin x$ is written as $y = \text{arc sin } x$ and its graph is obtained by rotating the graph of $y = \sin x$ about the 45° line, as indicated in Fig. 5-15.

The equation $y = \text{arc sin } x$ is sometimes written as $y = \sin^{-1} x$, and is read as "y equals arc sin x," or "y equals inverse sin x."

* Two points are said to be symmetric with respect to a line if the line is the perpendicular bisector of the line which joins the points. To draw a curve symmetric to a given curve, we locate points which are symmetric to a number of points on the given curve and then draw a smooth curve through them.

The symbol $\sin^{-1} x$ should not be confused with the reciprocal of $\sin x$; the latter should be written either as $(\sin x)^{-1}$ or $\frac{1}{\sin x}$.

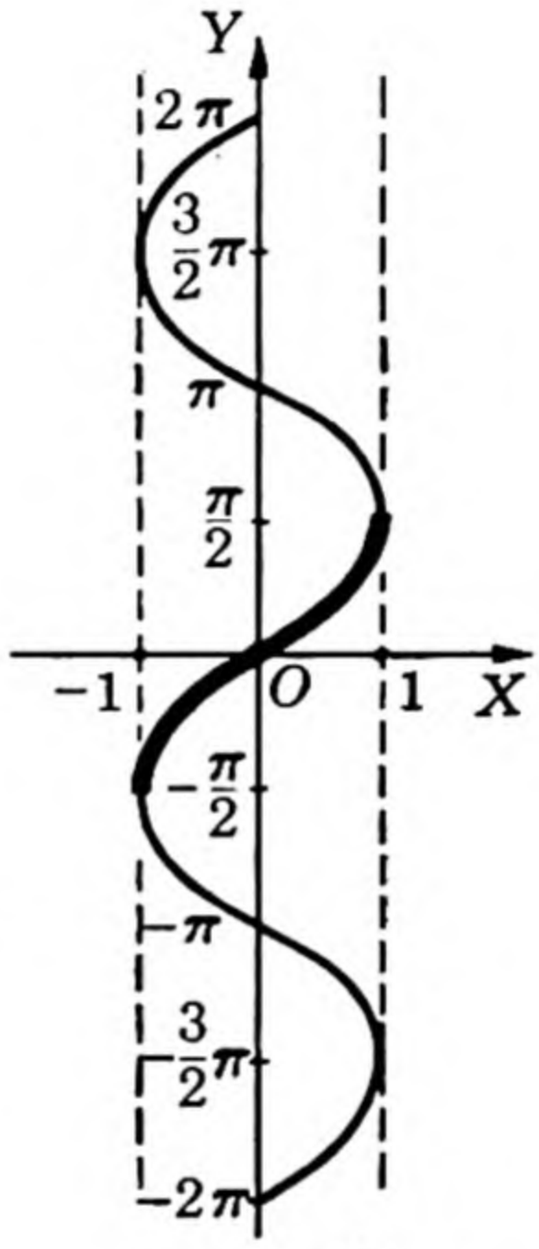
Similarly, the other inverse trigonometric functions are written as $y = \text{arc cos } x$ or $y = \cos^{-1} x$, $y = \text{arc tan } x$ or $y = \tan^{-1} x$, etc. It is important to note that in the equations of inverse trigonometric functions, x is a number and y is an angle.



$I \begin{cases} y = \sin x \\ x = \text{arc sin } y \end{cases}$

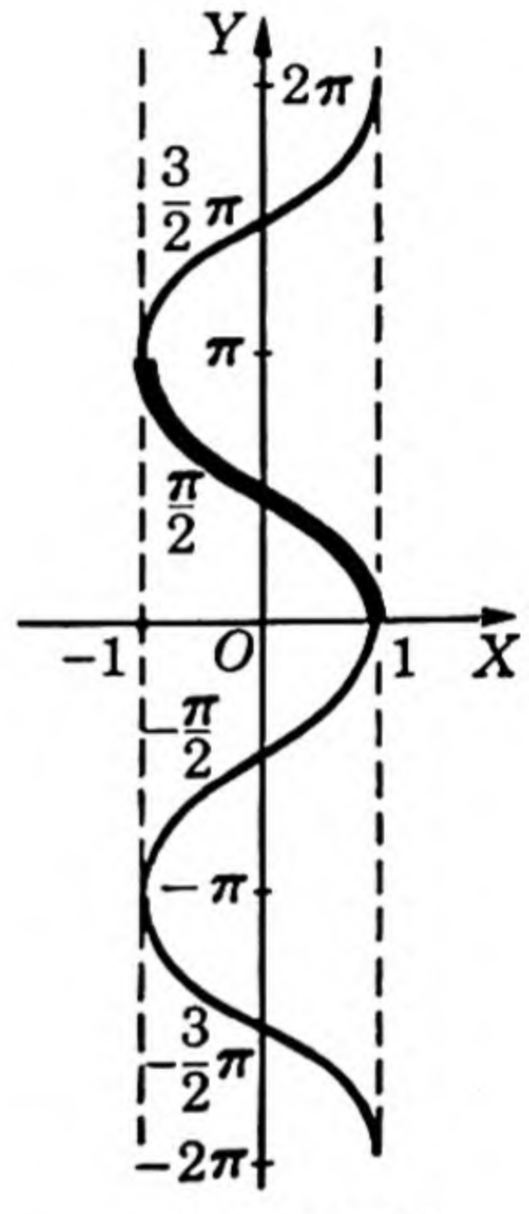
$II \quad y = \text{arc sin } x$

FIG. 5-15



$y = \text{arc sin } x$

FIG. 5-16



$y = \text{arc cos } x$

FIG. 5-17

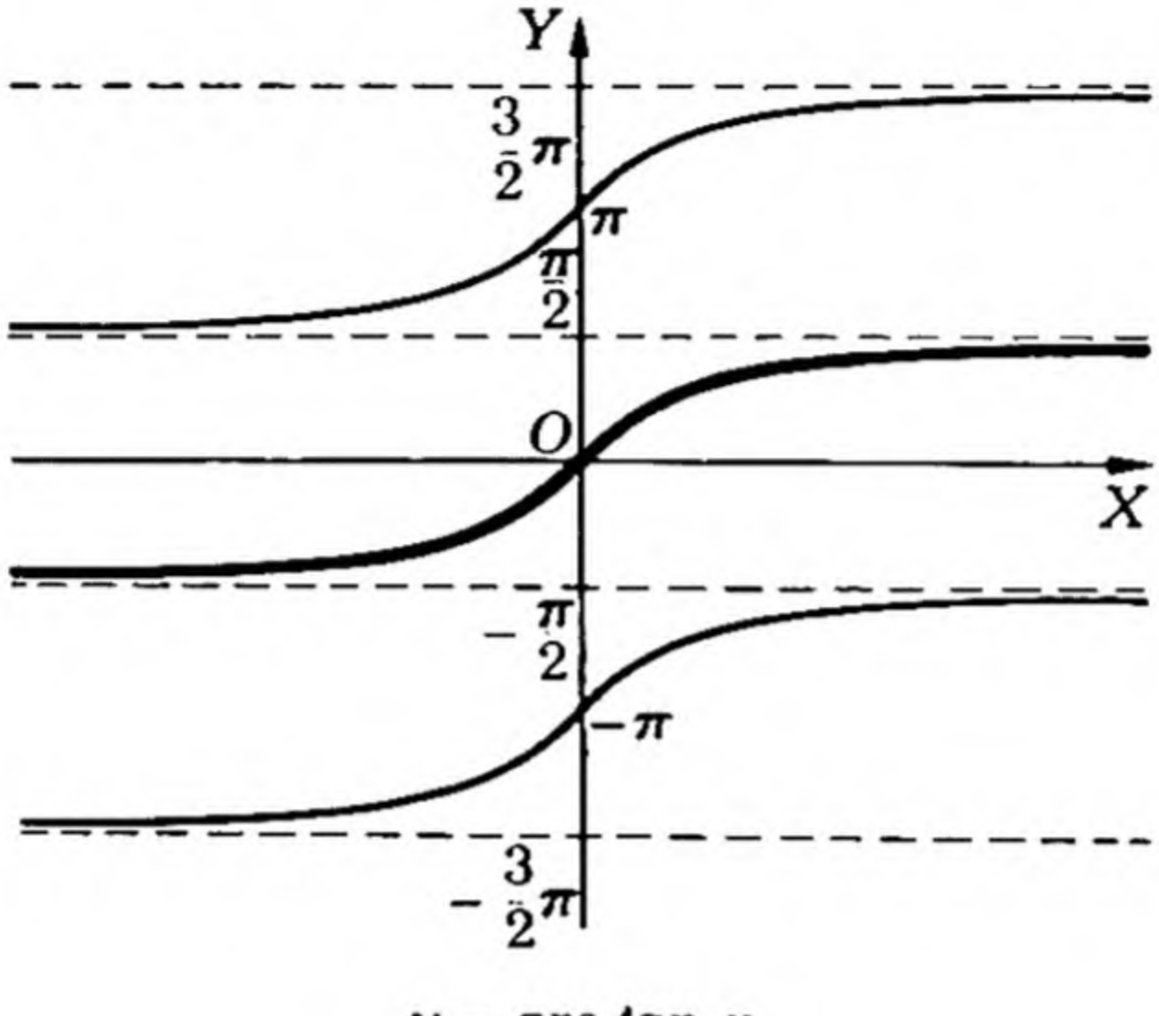
The graphs of $y = \text{arc sin } x$, $y = \text{arc cos } x$, and $y = \text{arc tan } x$ are shown in Figs. 5-16, 5-17, and 5-18 respectively.

5-10 Principal values of inverse trigonometric functions. For a given angle, each of the six trigonometric functions has a single definite value, except for angles for which some of the functions become infinite (infinity is not a definite value).

In the case of inverse trigonometric functions, however, an indefinite number of angles correspond to a given permissible number. For instance, looking at Fig. 5-16, we observe that for $x = \frac{1}{2}$ there are an indefinite number of angles represented by the ordinates of

the points whose abscissa is $\frac{1}{2}$. In other words, inverse trigonometric functions are *multiple valued*, whereas direct trigonometric functions are *single valued*.

Since for every permissible value of x, there is a value of y in the heavily drawn parts of Figs. 5-16, 5-17, 5-18, only those values of y are considered which are the ordinates of points on the heavily



$y = arc\ tan\ x$

FIG. 5-18

drawn parts of the curves. In other words, when a value of x is given, those values of the inverse functions are called for which fall within the ranges indicated by the following:

$$-90° \leq \sin^{-1} x \leq 90°,$$
$$0° \leq \cos^{-1} x \leq 180°,$$
and
$$-90° < \tan^{-1} x < 90°.$$

The values which fall within these ranges are called the *principal values* of the inverse functions. Since these values are angles, they may also be called *principal angles*.

EXAMPLES

1. Find the principal value of $\cos^{-1}(-1)$.

Solution: Let $\alpha = \cos^{-1}(-1)$; then $\cos \alpha = -1$. Therefore the principal value of α is 180°.

2. Find the principal angle of arc tan $\left(\sin \frac{\pi}{4}\right)$.

Solution: Let $\alpha = \text{arc tan } \left(\sin \frac{\pi}{4}\right)$; then $\tan \alpha = \sin \frac{\pi}{4} = 0.70711$. Therefore the principal angle is 35° 15.9′.

PROBLEMS

Find the principal angles of the following:

1. $\sin^{-1} 0$.
2. $\cos^{-1} 0$.
3. $\tan^{-1} 0$.
4. $\sin^{-1} 1$.
5. $\cos^{-1} 1$.
6. $\tan^{-1} 1$.
7. $\sin^{-1} (-1)$.
8. $\tan^{-1} (-1)$.
9. $\sin^{-1} \frac{1}{2}$.
10. $\cos^{-1} \frac{1}{2}$.
11. $\tan^{-1} \sqrt{3}$.
12. arc sin $\left(-\frac{1}{2}\right)$.
13. arc cos $\left(-\frac{1}{2}\right)$.
14. arc tan $\left(-\frac{1}{\sqrt{3}}\right)$.
15. arc sin $\left(\frac{1}{\sqrt{2}}\right)$.
16. arc cos $\left(\frac{1}{\sqrt{2}}\right)$.
17. arc tan $\left(\frac{1}{\sqrt{3}}\right)$.
18. arc sin $\left(\sin \frac{\pi}{2}\right)$.
19. arc cos $\left(\cos \frac{\pi}{2}\right)$.
20. arc tan $\left(\sin \frac{\pi}{2}\right)$.
21. arc sin $\left(\tan \frac{\pi}{4}\right)$.
22. arc cos $\left(\tan \frac{\pi}{4}\right)$.

Find the values of the following expressions:

23. sin $\left(\cos^{-1} \frac{1}{2}\right)$.
24. cos $\left(\sin^{-1} \frac{1}{2}\right)$.
25. sin $(\tan^{-1} 1)$.
26. tan $\left(\sin^{-1} \frac{1}{\sqrt{2}}\right)$.
27. cos (arc sin 0).
28. sin (arc cos 0).
29. tan (arc tan 0).
30. sin (arc cos 1).

CHAPTER 6

TRIGONOMETRIC IDENTITIES AND EQUATIONS

6-1 Equations. Equations may be divided into *definitions*, *identities*, and *equations of condition.*

In a definition, the symbol of a quantity is equated to an expression which shows how the value of the quantity can be found. For example, $E = \frac{1}{2}mv^2$ and $\sin\theta = \frac{y}{r}$ define the symbols E and $\sin\theta$, respectively.

In an equation of identity, the two members of the equation are identical except in form, and the equation is valid for all values of the variables. For instance, the two members of $(x + y)^2 = x^2 + 2xy + y^2$, and of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ are identical. The first is valid for all values of x and y, and the second for all values of θ.

An equation of condition is valid only under certain conditions. For example, $(x + 1)^2 = 1$ is valid only if $x = 0$ or -2.

A mathematical *formula* represents either a definition or an identity which is important because of its relative simplicity and frequency of application.

6-2 Fundamental identities. The following formulas are called *fundamental trigonometric identities:*

$$\csc\theta = \frac{1}{\sin\theta}. \qquad (1\text{-}1)$$

$$\sec\theta = \frac{1}{\cos\theta}. \qquad (1\text{-}2)$$

$$\cot\theta = \frac{1}{\tan\theta}. \qquad (1\text{-}3)$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}. \qquad (1\text{-}4)$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}. \qquad (1\text{-}5)$$

$$\sin^2\theta + \cos^2\theta = 1.* \qquad (6\text{-}1)$$

Formulas (1-1) to (1-5), in Chapter 1, were obtained by com-

* $\sin^2\theta$ and $\cos^2\theta$ mean $(\sin\theta)^2$ and $(\cos\theta)^2$, respectively.

paring the definitions of the trigonometric functions. Formula (6-1) may be proved as follows:

Placing θ in the standard position, Fig. 6-1, and making use of the Pythagorean theorem, and the definitions of $\sin\theta$ and $\cos\theta$, we have

$$\sin^2\theta + \cos^2\theta = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

$$= \frac{x^2 + y^2}{r^2}$$

$$= \frac{r^2}{r^2}$$

$$= 1.$$

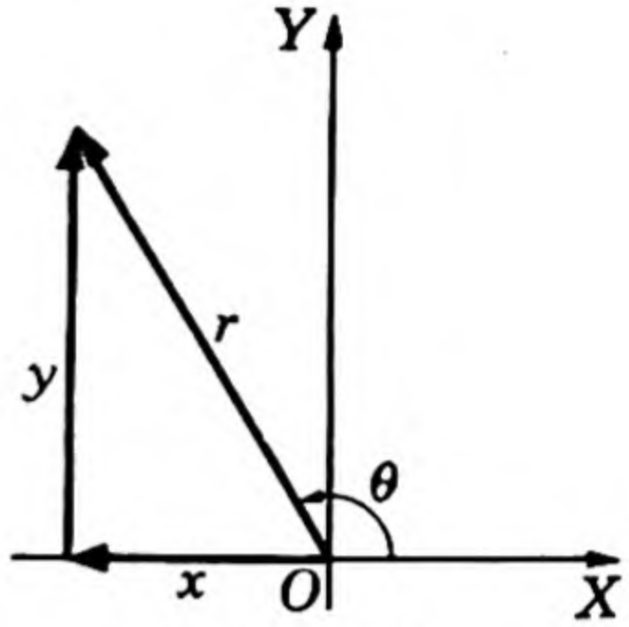


FIG. 6-1

In Fig. 6-1, θ is a positive obtuse angle but this proof is valid whatever the value of θ may be.

The following identities also may be considered fundamental.

$$1 + \tan^2\theta = \sec^2\theta, \tag{6-2}$$

$$1 + \cot^2\theta = \csc^2\theta. \tag{6-3}$$

The first is obtained by dividing (6-1) by $\cos^2\theta$, and the second by dividing (6-1) by $\sin^2\theta$.

EXAMPLES

1. *Prove* the identity $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$.

Note: In proving an equation of identity, the left-hand member should be transformed successively until the right-hand member is obtained.

Proof:

$$\begin{aligned}\sin^4\theta - \cos^4\theta &= (1 - \cos^2\theta)^2 - \cos^4\theta && \text{[by (6-1)]}\\ &= 1 - 2\cos^2\theta + \cos^4\theta - \cos^4\theta\\ &= 1 - 2\cos^2\theta.\end{aligned}$$

Alternative proof:

$$\begin{aligned}\sin^4\theta - \cos^4\theta &= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)\\ &= \sin^2\theta - \cos^2\theta && \text{[by (6-1)]}\\ &= 1 - \cos^2\theta - \cos^2\theta\\ &= 1 - 2\cos^2\theta.\end{aligned}$$

2. *Prove* the identity $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta} = \tan^2\theta$.

$$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\csc^2\theta} \qquad \text{[by (6-2) and (6-3)]}$$
$$= \frac{\sin^2\theta}{\cos^2\theta} \qquad \text{[by (1-1) and (1-2)]}$$
$$= \tan^2\theta.$$

The following alternative proof does not require the use of (6-2) and (6-3):

$$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{1+\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\cos^2\theta}{\sin^2\theta}}$$
$$= \frac{\cos^2\theta+\sin^2\theta}{\sin^2\theta+\cos^2\theta}\cdot\frac{\sin^2\theta}{\cos^2\theta}$$
$$= \tan^2\theta.$$

3. *Simplify* $\cot\theta - \sec\theta\csc\theta(1-2\sin^2\theta)$.

$$\cot\theta - \sec\theta\csc\theta(1-2\sin^2\theta) = \frac{\cos\theta}{\sin\theta} - \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}(1-2\sin^2\theta)$$
$$= \frac{\cos^2\theta - 1 + 2\sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta+\cos^2\theta-1+\sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta}{\sin\theta\cos\theta}$$
$$= \tan\theta.$$

PROBLEMS

Prove the following identities:

1. $\cos\theta + \sin\theta\tan\theta = \sec\theta.$
2. $\sin\theta\sec\theta\cot\theta = 1.$
3. $\csc\theta - \cot\theta\cos\theta = \sin\theta.$
4. $\tan\theta + \cot\theta = \sec\theta\csc\theta.$
5. $(1+\tan^2\theta)(1-\sin^2\theta) = 1.$
6. $(\cos\theta+\sin\theta)^2 + (\cos\theta-\sin\theta)^2 = 2.$
7. $(1-\cos\theta)(1+\sec\theta) = \sin\theta\tan\theta.$
8. $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1.$
9. $\sin^2\theta - \cos^2\theta = 2\sin^2\theta - 1.$
10. $(\tan\theta+\cot\theta)^2 = \sec^2\theta+\csc^2\theta.$
11. $\cos^2\theta(1-\sec^2\theta)+\sin^2\theta = 0.$
12. $\sec^2\theta - \csc^2\theta = \tan^2\theta - \cot^2\theta.$
13. $\cot^2\theta\sin^2\theta = 1-\sin^2\theta.$
14. $\dfrac{\cos\theta+\sin\theta}{\sec\theta+\csc\theta} = \sin\theta\cos\theta.$
15. $\dfrac{\cot\theta+1}{\sin\theta+\cos\theta} = \csc\theta.$
16. $\dfrac{\sin\theta+\tan\theta}{\csc\theta+\cot\theta} = \sin\theta\tan\theta.$
17. $\dfrac{\csc\theta}{\tan\theta+\cot\theta} = \cos\theta.$
18. $\dfrac{\sin\theta}{1-\cos^2\theta} = \csc\theta.$
19. $\dfrac{\sec\theta}{\tan\theta+\cot\theta} = \sin\theta.$
20. $\dfrac{\tan^2\theta+1}{\tan\theta+\cot\theta} = \tan\theta.$
21. $\dfrac{\cot^2\theta+1}{\tan\theta+\cot\theta} = \cot\theta$
22. $1 - \dfrac{\cos^2\theta}{1+\sin\theta} = \sin\theta.$

23. $1 - \dfrac{\sin^2\theta}{1+\cos\theta} = \cos\theta.$

24. $\dfrac{\tan\theta\sin\theta}{1+\cos\theta} = \sec\theta - 1.$

25. $\dfrac{\cot\theta}{\sec\theta - \tan\theta} = \csc\theta + 1.$

26. $\dfrac{1+\csc\theta}{\sec\theta} = \cos\theta + \cot\theta.$

27. $\dfrac{\cos\theta}{\sec\theta - \tan\theta} = 1 + \sin\theta.$

28. $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = 1 - 2\sin^2\theta.$

29. $\dfrac{\cos^2\theta\cot^2\theta}{\cot\theta - \cos\theta} = \cos\theta + \cot\theta.$

30. $\dfrac{1}{\sec\theta - \cos\theta} = \cot\theta\csc\theta.$

31. $\dfrac{3-\sin\theta}{\tan\theta} = 3\cot\theta - \cos\theta.$

32. $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{\sin\theta}{1-\cos\theta} = 2\csc\theta.$

33. $\dfrac{\cos\theta - \sin\theta}{\cot\theta - 1} = \sin\theta.$

34. $\dfrac{\cos\theta}{1-\sin\theta} = \tan\theta + \sec\theta.$

35. $\dfrac{\sec^2\theta - \tan^2\theta}{\sec\theta - \cos\theta} = \cot\theta\csc\theta.$

36. $\dfrac{\csc\theta - \cot\theta}{\csc\theta + \cot\theta} = \dfrac{1-\cos\theta}{1+\cos\theta}.$

37. $\dfrac{\cot\theta}{\csc\theta - 1} = \dfrac{\csc\theta + 1}{\cot\theta}.$

38. $\dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}.$

6-3 Solution of trigonometric equations. If an equation of condition contains only one kind of function, we solve it for the function as if it were an algebraic equation and then find the angles which satisfy the value of the function.

If the equation contains two or more kinds of functions, we first reduce it to an equation which contains only one kind of function or to an equation whose right-hand member is zero and whose left-hand member consists of factors each of which contains only one kind of function.

Note: Since an indefinite number of positive and negative angles correspond to a given value of a trigonometric function, we find only the angles between 0° and 360° which satisfy the equation.

EXAMPLES

1. *Solve* for α, the equation $\cos^2\alpha + \cos\alpha = 0$.

Solution: This is an algebraic equation in $\cos\alpha$. Therefore we find the values of $\cos\alpha$ as we would find the values of x from $x^2 + x = 0$; thus

$$\cos^2\alpha + \cos\alpha = \cos\alpha(\cos\alpha + 1) = 0.*$$

Therefore $\cos\alpha = 0$, or $\cos\alpha + 1 = 0$.
From $\cos\alpha = 0$, we have $\alpha = 90°$ or $\alpha = 270°$.
From $\cos\alpha + 1 = 0$, or $\cos\alpha = -1$, we have $\alpha = 180°$.

* If the product of a number of factors equals zero, at least one of the factors must equal zero.

2. *Solve* $\tan \alpha - \cot \alpha + 1 = 0$.

Solution: Since this equation contains two kinds of functions, we reduce it to an equation which contains only, say, $\tan \alpha$. Thus,

$$\tan \alpha - \frac{1}{\tan \alpha} + 1 = 0, \text{ or } \tan^2 \alpha - 1 + \tan \alpha = 0,$$

which is a quadratic equation in $\tan \alpha$. It can be solved by factoring or by the quadratic formula or by completing the square.* Using the last method, we have:

$$\tan^2 \alpha + \tan \alpha = 1,$$

$$\tan^2 \alpha + \tan \alpha + \frac{1}{4} = \frac{5}{4},$$

$$\left(\tan \alpha + \frac{1}{2}\right)^2 = \frac{5}{4},$$

$$\tan \alpha = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5}.$$

Hence (a) $\tan \alpha = \frac{1}{2}(\sqrt{5} - 1) = 0.61803,$

or (b) $\tan \alpha = -\frac{1}{2}(\sqrt{5} + 1) = -1.61803.$

From (a) we get $\alpha = 31° 43'$, or $\alpha = 211° 43'$.
To find the angles from (b), it is desirable to write

$$-\tan \alpha = \tan (-\alpha) = 1.61803,$$

which gives $-\alpha = 58° 17'$, or $-\alpha = 238° 17'$; hence $\alpha = -58° 17'$ or $\alpha = -238° 17'$. Therefore the corresponding positive angles are $301° 43'$ and $121° 43'$.

PROBLEMS

Find the angles from 0° to 360° inclusive which satisfy the following equations:

1. $\sin^2 \alpha = \frac{1}{4}$.
2. $\cos^2 \alpha = \frac{1}{4}$.
3. $\tan^2 \alpha = 3$.
4. $\sec^2 \alpha = 4$.
5. $\cos \alpha - \sqrt{3} \sin \alpha - 1 = 0$.
6. $\sin^2 \alpha - \cos^2 \alpha = 0$.

* To solve the quadratic equation $ax^2 + bx + c = 0$:
(a) Divide it by a and transfer the constant term to the right-hand member in order to put it in the form $x^2 + b_1 x = c_1$.
(b) Add $\left(\frac{b_1}{2}\right)^2$ to both members to complete the square of the left-hand member, and to put the equation in the form $\left(x + \frac{b_1}{2}\right)^2 = \left(\frac{b_1}{2}\right)^2 - c_1$.
(c) Take the square root of both members and solve for x, to obtain

$$x = \tfrac{1}{2}(-b_1 \pm \sqrt{b_1^2 - 4c_1}).$$

7. $\sin^2 \alpha - \cos^2 \alpha + \frac{1}{2} = 0.$
8. $\cot \alpha - \cos \alpha = 0.$
9. $\sin \alpha + \cos \alpha - 1 = 0.$
10. $\cos \alpha - 2 \sin \alpha = 0.$
11. $\tan \alpha + \cot \alpha - 2 = 0.$
12. $\tan \alpha + \sqrt{2} \sec \alpha + 1 = 0.$
13. $2 \cos^2 \alpha - \cos \alpha - 1 = 0.$
14. $\csc^2 \alpha - \cot \alpha - 1 = 0.$
15. $2 \sin^2 \alpha + \cos \alpha - 1 = 0.$
16. $2 \tan^2 \alpha + 3 \sec \alpha = 0.$
17. $2 \sin^2 \alpha + 3 \cos \alpha = 0.$
18. $2 \sin \alpha \cos \alpha - \sin \alpha = 0.$
19. $2 \cot \alpha \sin \alpha + \cot \alpha = 0.$
20. $2 \sin^2 \alpha - 3 \cos \alpha = 0.$
21. $\sin \alpha(\sec 2\alpha + 1) = 0.$
22. $2 \sin 2\alpha \cos \alpha + \sin 2\alpha = 0.$
23. $2 \sin^2 \alpha + \sin \alpha - 1 = 0.$
24. $\tan \alpha + \cot \alpha - 2 = 0.$

CHAPTER 7

ADDITION AND DERIVED FORMULAS

7-1 Addition formulas. The following important identities are called *addition formulas.*

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta. \tag{7-1}$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta. \tag{7-2}$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}. \tag{7-3}$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta. \tag{7-4}$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta. \tag{7-5}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}. \tag{7-6}$$

The last three of these formulas may be obtained from the first three by replacing β by $-\beta$ and remembering that $\sin(-\beta) = -\sin\beta$, $\cos(-\beta) = \cos\beta$, and $\tan(-\beta) = -\tan\beta$. Therefore we need to prove only the first three formulas.

7-2 Proof of $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$. In order to proceed from the definition of $\sin(\alpha + \beta)$ to its expression in terms of the functions of α and β, we construct Fig. 7-1 so that each right triangle which has α or β as an acute angle has a side in common with the right triangle which has $(\alpha + \beta)$ as one of its acute angles. Then, using the notation of the figure, we obtain

$$\sin(\alpha + \beta) = \frac{y}{r}$$

$$= \frac{y' + y''}{r}$$

$$= \frac{r'\sin\alpha + r''\cos\gamma}{r}$$

$$= \frac{r' \sin \alpha + r'' \cos \alpha}{r} \qquad [\gamma = \alpha^*]$$

$$= \frac{r \cos \beta \cdot \sin \alpha + r \sin \beta \cdot \cos \alpha}{r}$$

$$= \sin \alpha \cos \beta + \cos \alpha \sin \beta.$$

In this derivation and that of the next section, α and β are so chosen as to make $\alpha + \beta$ an acute angle. But it can be shown that the results are valid whatever angles α and β may be.

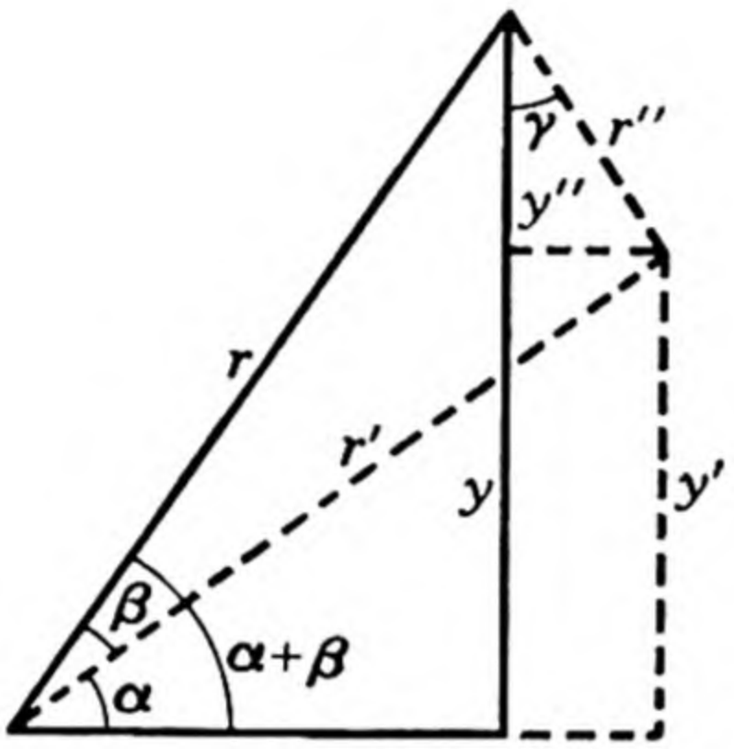


FIG. 7-1

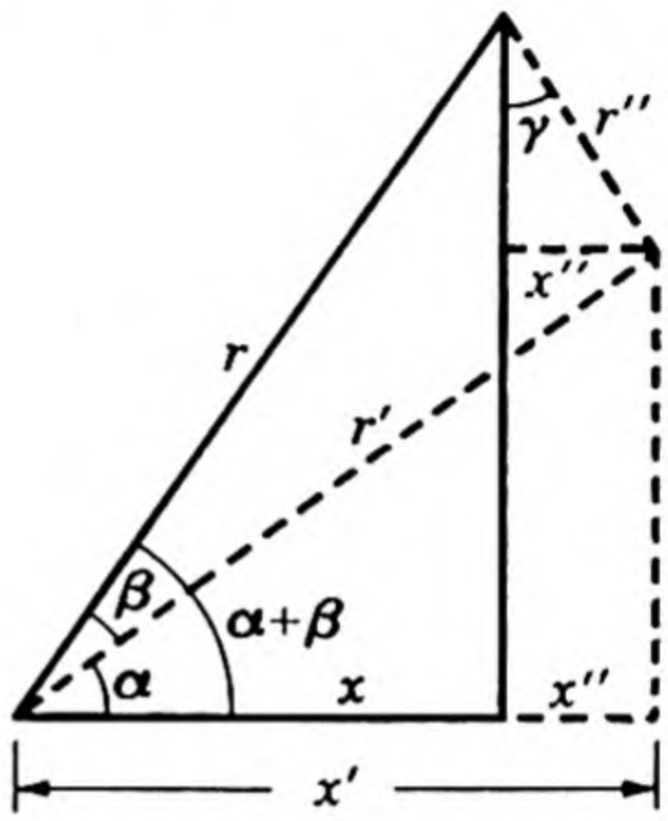


FIG. 7-2

7-3 Proof of $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$. Using the notation of Fig. 7-2, we obtain:

$$\cos(\alpha + \beta) = \frac{x}{r}$$

$$= \frac{x' - x''}{r}$$

$$= \frac{r' \cos \alpha - r'' \sin \gamma}{r}$$

$$= \frac{r \cos \beta \cdot \cos \alpha - r \sin \beta \cdot \sin \alpha}{r}$$

$$= \cos \alpha \cos \beta - \sin \alpha \sin \beta.$$

---

* Since each side of γ is perpendicular to one side of α, $\gamma = \alpha$.

7-4 Proof of $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$. In the following proof, the third right-hand member is obtained by dividing both the numerator and the denominator of the second expression by $\cos\alpha\cos\beta$.

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

$$=\frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}.$$

EXAMPLES

1. Considering 75° as the sum of 30° and 45°, *find* the exact value of sin 75°:

$$\sin 75^\circ=\sin(30^\circ+45^\circ)$$
$$=\sin 30^\circ\cos 45^\circ+\cos 30^\circ\sin 45^\circ$$
$$=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$$
$$=\frac{1+\sqrt{3}}{2\sqrt{2}}$$
$$=\frac{\sqrt{2}+\sqrt{6}}{4}.$$

2. *Prove* the identity, $\cos(x+y)\cos(x-y)=\cos^2 x-\sin^2 y$.

$$\cos(x+y)\cos(x-y)=(\cos x\cos y-\sin x\sin y)(\cos x\cos y+\sin x\sin y)$$
$$=(\cos x\cos y)^2-(\sin x\sin y)^2\ *$$
$$=\cos^2 x\cos^2 y-(1-\cos^2 x)(1-\cos^2 y)$$
$$=\cos^2 x\cos^2 y-(1-\cos^2 x)+\cos^2 y-\cos^2 x\cos^2 y$$
$$=\cos^2 y-\sin^2 x.$$

---

* The product of the sum and the difference of two expressions equals the difference of their squares, that is,

$$(a+b)(a-b)=a^2-b^2.$$

3. *Simplify*, $\sin(\theta + 60°) - \cos(\theta + 30°)$.

$$\begin{aligned}\sin(\theta + 60°) - \cos(\theta + 30°) &= \sin\theta\cos 60° + \cos\theta\sin 60° - (\cos\theta\cos 30° - \sin\theta\sin 30°)\\ &= \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta - \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\\ &= \sin\theta.\end{aligned}$$

PROBLEMS

In each of the following problems, express the given angle in terms of one or more of the angles 30°, 45° and 60°, and then find the exact value of the function.

1. $\cos 75°$.
2. $\tan 75°$.
3. $\sin 90°$.
4. $\sin 15°$.
5. $\cos 15°$.
6. $\tan 15°$.
7. $\sin 120°$.
8. $\cos 120°$.
9. $\tan 120°$.
10. $\sin 105°$.
11. $\cos 105°$.
12. $\tan 105°$.

Using the addition formulas, simplify the following:

13. $\sin\left(\frac{\pi}{2} + \theta\right)$.
14. $\sin\left(\frac{\pi}{2} - \theta\right)$.
15. $\cos\left(\frac{\pi}{2} + \theta\right)$.
16. $\cos\left(\frac{\pi}{2} - \theta\right)$.
17. $\sin(\pi + \theta)$.
18. $\sin(\pi - \theta)$.
19. $\cos(\pi + \theta)$.
20. $\cos(\pi - \theta)$.
21. $\tan(\pi + \theta)$.
22. $\tan(\pi - \theta)$.
23. $\sin\left(\frac{3\pi}{2} + \theta\right)$.
24. $\sin\left(\frac{3\pi}{2} - \theta\right)$.
25. $\cos\left(\frac{3\pi}{2} + \theta\right)$.
26. $\cos\left(\frac{3\pi}{2} - \theta\right)$.
27. $\sin(2\pi + \theta)$.
28. $\sin(2\pi - \theta)$.
29. $\cos(2\pi + \theta)$.
30. $\cos(2\pi - \theta)$.
31. $\tan(2\pi + \theta)$.
32. $\tan(2\pi - \theta)$.

Prove the following identities:

33. $\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta$.
34. $\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta$.
35. $\dfrac{\tan\alpha - \tan(\alpha - \beta)}{1 + \tan\alpha\tan(\alpha - \beta)} = \tan\beta$.
36. $\dfrac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$.
37. $\dfrac{\sin(\alpha + \beta)}{\sin\alpha\sin\beta} = \cot\alpha + \cot\beta$.
38. $\dfrac{\cos(\alpha + \beta)}{\cos\alpha\cos\beta} = 1 - \tan\alpha\tan\beta$.

39. $\dfrac{\cos(\alpha+\beta)}{\sin\alpha\sin\beta} = \cot\alpha\cot\beta - 1.$

7-5 Double-angle formulas. Replacing β by α in formulas (7-1) to (7-3) and simplifying, we obtain the following *double-angle* formulas:

$$\sin 2\alpha = 2\sin\alpha\cos\alpha, \tag{7-7}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha, \tag{7-8}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}. \tag{7-9}$$

Making use of $\sin^2\alpha + \cos^2\alpha = 1$ to eliminate either $\sin^2\alpha$ or $\cos^2\alpha$ from (7-8) and simplifying, we get

$$\cos 2\alpha = 2\cos^2\alpha - 1, \tag{7-8a}$$

or

$$\cos 2\alpha = 1 - 2\sin^2\alpha. \tag{7-8b}$$

7-6 Half-angle formulas. From the last two formulas we obtain

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2},$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2},$$

$$\tan^2\alpha = \frac{1-\cos 2\alpha}{1+\cos 2\alpha}.$$

Replacing α by $\frac{1}{2}\theta$ in the last three equations, and taking the square roots, we get

$$\sin\frac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{2}}, \tag{7-10}$$

$$\cos\frac{1}{2}\theta = \pm\sqrt{\frac{1+\cos\theta}{2}}, \tag{7-11}$$

$$\tan\frac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}. \tag{7-12}$$

Note: If the value of θ is given, the sign of the left-hand member of each of the last three formulas is determined and the same sign is given to the right-hand member, since the value of both members of an equation must have the same sign.

The following are alternative formulas for $\tan \frac{1}{2}\theta$:

$$\tan \frac{1}{2}\theta = \frac{1 - \cos \theta}{\sin \theta}, \tag{7-12a}$$

$$\tan \frac{1}{2}\theta = \frac{\sin \theta}{1 + \cos \theta}. \tag{7-12b}$$

These formulas may be proved as follows:

$$\tan \tfrac{1}{2}\theta = \frac{\sin \frac{1}{2}\theta}{\cos \frac{1}{2}\theta}$$
$$= \frac{\sin \frac{1}{2}\theta}{\cos \frac{1}{2}\theta} \cdot \frac{2 \sin \frac{1}{2}\theta}{2 \sin \frac{1}{2}\theta}$$
$$= \frac{2 \sin^2 \frac{1}{2}\theta}{2 \sin \frac{1}{2}\theta \cos \frac{1}{2}\theta}$$
$$= \frac{1 - \cos \theta}{\sin \theta}.$$
[by (7-10) & (7-7)]

$$\tan \tfrac{1}{2}\theta = \frac{\sin \frac{1}{2}\theta}{\cos \frac{1}{2}\theta}$$
$$= \frac{\sin \frac{1}{2}\theta}{\cos \frac{1}{2}\theta} \cdot \frac{2 \cos \frac{1}{2}\theta}{2 \cos \frac{1}{2}\theta}$$
$$= \frac{2 \sin \frac{1}{2}\theta \cos \frac{1}{2}\theta}{2 \cos^2 \frac{1}{2}\theta}$$
$$= \frac{\sin \theta}{1 + \cos \theta}.$$
[by (7-7) & (7-11)]

EXAMPLES

1. Find the value of $\tan 15°$ without the use of tables.

Solution:
$$\tan 15° = \tan \frac{30°}{2}$$
$$= \frac{1 - \cos 30°}{\sin 30°} \qquad \text{[by (7-12a)]}$$
$$= \frac{1 - \sqrt{3}/2}{1/2}$$
$$= 2 - \sqrt{3}.$$

2. Find the values of α between $0°$ and $360°$ which satisfy the equation $\sin 2\alpha + 2 \sin \frac{1}{2}\alpha \cos \frac{1}{2}\alpha = 0$.

Solution: In order to solve such an equation, we must transform it so that it contains only one angle, that is, we must express $\sin 2\alpha$ in terms of functions of $\frac{1}{2}\alpha$, or $\sin \frac{1}{2}\alpha$ and $\cos \frac{1}{2}\alpha$ in terms of functions of 2α, or all three in terms of functions of α. The last alternative is the most convenient in this problem.

$$\sin 2\alpha + 2 \sin \frac{1}{2}\alpha \cos \frac{1}{2}\alpha = 2 \sin \alpha \cos \alpha + \sin \alpha$$
$$= \sin \alpha(2 \cos \alpha + 1).$$

Therefore $\sin \alpha(2 \cos \alpha + 1) = 0.$

Hence $\sin \alpha = 0$; or $2 \cos \alpha + 1 = 0,$

and $\alpha = 0$, or $180°$; or $\alpha = 120°$, or $240°$.

7-7 Product and sum formulas. Adding (7-1) and (7-4), subtracting (7-4) from (7-1), adding (7-2) and (7-5), and subtracting (7-5) from (7-2), we obtain the following *product formulas.*

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta, \tag{7-13}$$

$$\sin(\alpha+\beta)-\sin(\alpha-\beta)=2\cos\alpha\sin\beta, \tag{7-14}$$

$$\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta, \tag{7-15}$$

$$\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta. \tag{7-16}$$

Letting $\alpha+\beta=A$ and $\alpha-\beta=B$, we have $\alpha=\frac{1}{2}(A+B)$ and $\beta=\frac{1}{2}(A-B)$. Making these substitutions in (7-13) to (7-16), we obtain the following *sum formulas.*

$$\sin A+\sin B=2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B), \tag{7-17}$$

$$\sin A-\sin B=2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B), \tag{7-18}$$

$$\cos A+\cos B=2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B), \tag{7-19}$$

$$\cos A-\cos B=-2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B). \tag{7-20}$$

EXAMPLE

Prove the identity $\dfrac{\sin 3\alpha+\sin\alpha}{\cos 3\alpha+\cos\alpha}=\tan 2\alpha$.

Solution: Making use of formulas (7-17) and (7-19), we have

$$\begin{aligned}\frac{\sin 3\alpha+\sin\alpha}{\cos 3\alpha+\cos\alpha}&=\frac{2\sin\frac{1}{2}(3\alpha+\alpha)\cos\frac{1}{2}(3\alpha-\alpha)}{2\cos\frac{1}{2}(3\alpha+\alpha)\cos\frac{1}{2}(3\alpha-\alpha)}\\&=\frac{\sin 2\alpha\cos\alpha}{\cos 2\alpha\cos\alpha}\\&=\tan 2\alpha.\end{aligned}$$

PROBLEMS

Prove the following identities.

1. $\dfrac{1-\cos 2\theta}{\sin 2\theta}=\tan\theta.$
2. $\dfrac{\sin 2\theta}{1+\cos 2\theta}=\tan\theta.$

3. $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta.$

4. $\dfrac{\sin 2\theta}{1-\cos 2\theta} = \cot\theta.$

5. $\dfrac{\sin 2\theta + \sin\theta}{1+\cos 2\theta+\cos\theta} = \tan\theta.$

6. $\dfrac{\sec 2\theta + 1}{\sec 2\theta - 1} = \cot^2\theta.$

7. $\dfrac{1+\sin 2\theta - \cos 2\theta}{1+\sin 2\theta+\cos 2\theta} = \tan\theta.$

8. $\dfrac{2\tan 2\theta}{\tan 2\theta + \sin 2\theta} = \sec^2\theta.$

9. $\dfrac{2\sin 2\theta - \sin 4\theta}{2\sin 2\theta + \sin 4\theta} = \tan^2\theta.$

10. $\csc 2\theta - \cot 2\theta = \tan\theta.$

11. $\tan^2\theta + 2\cot 2\theta\tan\theta = 1.$

12. $\left(\sin\dfrac{\theta}{2}+\cos\dfrac{\theta}{2}\right)^2 = 1+\sin\theta.$

13. $\cot\tfrac{1}{2}\theta - \tan\tfrac{1}{2}\theta = 2\cot\theta.$

14. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$

15. $\sin 3\theta - \sin\theta = 2\cos 2\theta\sin\theta.$

16. $\dfrac{\sin 3\theta - \cos 3\theta}{\sin\theta+\cos\theta} = 2\sin 2\theta - 1.$

17. $\dfrac{2\tan\tfrac{1}{2}\theta}{1+\tan^2\tfrac{1}{2}\theta} = \sin\theta.$

18. $\sin 4\theta = 2\sin 2\theta(1-2\sin^2\theta).$

19. $\sin(45^\circ+\alpha)\sin(45^\circ-\alpha) = \tfrac{1}{2}\cos 2\alpha.$

20. $\dfrac{\sin 2\alpha+\sin 2\beta}{\sin 2\alpha-\sin 2\beta} = \dfrac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}.$

21. $\dfrac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta} = \tan\dfrac{1}{2}(\alpha+\beta)\cot\dfrac{1}{2}(\alpha-\beta).$

22. $\dfrac{\cos\alpha+\cos\beta}{\cos\beta-\cos\alpha} = \cot\dfrac{1}{2}(\alpha+\beta)\cot\dfrac{1}{2}(\alpha-\beta).$

23. $\dfrac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta} = \tan\dfrac{1}{2}(\alpha+\beta).$

24. $\dfrac{\sin\alpha-\sin\beta}{\cos\beta-\cos\alpha} = \cot\dfrac{1}{2}(\alpha+\beta).$

Find the angles from 0° and 360° inclusive which satisfy the following equations:

25. $\sin 2\alpha + \sin\alpha = 0.$

26. $\sin 2\alpha - \sin\alpha = 0.$

27. $\sin 2\alpha + \cos\alpha = 0.$

28. $\cos 2\alpha - \sin^2\alpha = 0.$

29. $\cos 2\alpha + \cos\alpha = 0.$

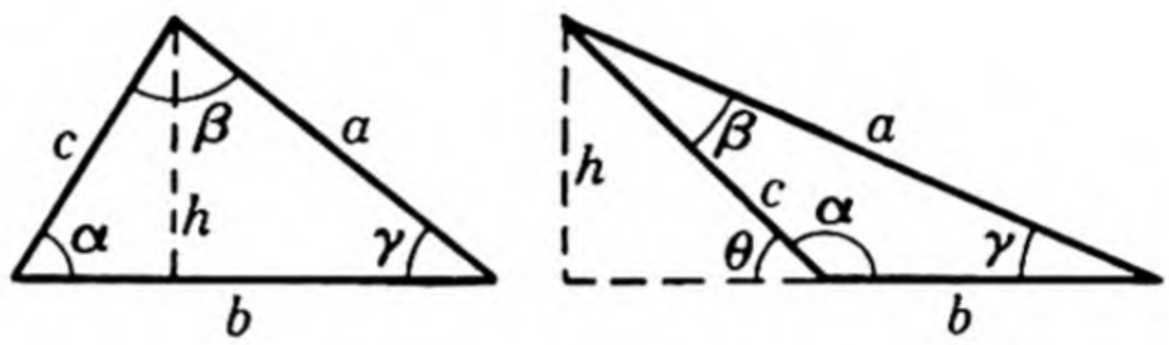
30. $\tan 2\alpha - \cot\alpha = 0.$

31. $\cos 2\alpha + 2\cos\alpha = 0.$
32. $\sin^2 2\alpha + 3\cos^2\alpha = 3.$
33. $\sin 2\alpha + 2\sin^2 \tfrac{1}{2}\alpha = 1.$
34. $\sin 2\alpha - \cos\alpha(1 + \cos\alpha) = 0.$
35. $\cos\alpha + \sin \tfrac{1}{2}\alpha - 1 = 0.$
36. $\cos\alpha - \cos \tfrac{1}{2}\alpha + 1 = 0.$
37. $\cos\alpha - \sin \tfrac{1}{2}\alpha + 1 = 0.$
38. $2\cos 2\alpha - \cos\alpha - 1 = 0.$

CHAPTER 8

OBLIQUE TRIANGLES

8-1 Solution of oblique triangles. An oblique triangle can be solved by the method of right triangles, after two right triangles have been obtained by dropping a perpendicular from one vertex onto the opposite side. But this method is not convenient for solving an oblique triangle completely, except when the triangle is isosceles. In this chapter we shall develop more convenient methods based on the *law of sines*, the *law of cosines*, and the *laws of tangents*.



(*a*) (*b*)

FIG. 8-1

8-2 Law of sines. The law of sines states: *The ratio of each side of a triangle to the sine of the opposite angle equals the corresponding ratios of the other sides and angles*, that is,

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}. \tag{8-1}$$

Proof: From Fig. 8-1a, we have

(a) $$h = c \sin \alpha, \text{ and } h = a \sin \gamma.$$

From Fig. 8-1b, we get

(b) $h = a \sin \gamma$, and $h = c \sin \theta = c \sin (180° - \alpha) = c \sin \alpha$.

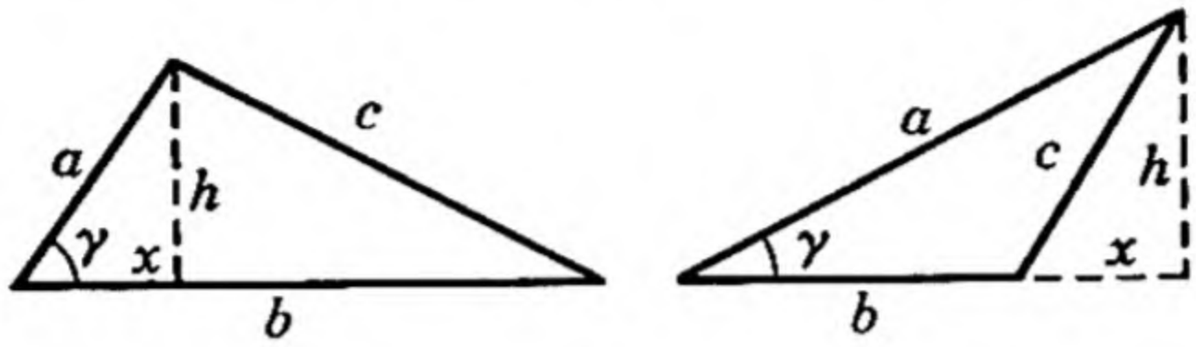
Eliminating h between each of these pairs of equations, we get

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma}.$$

If the perpendicular is dropped on the side a from the vertex at angle α, similar considerations yield

$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}.$$

Combining the last two equations, we obtain the law of sines.



(a) (b)

FIG. 8-2

8-3 The law of cosines. This law states: *In any triangle, the square of one side equals the sum of the squares of the other two sides minus twice the product of these two sides and the cosine of the included angle;* that is,

$$c^2 = a^2 + b^2 - 2ab \cos \gamma. \tag{8-2}$$

Proof: From Fig. 8-2a, we get

$$\begin{aligned} c^2 &= h^2 + (b - x)^2 \\ &= h^2 + b^2 - 2bx + x^2 \\ &= (h^2 + x^2) + b^2 - 2bx \\ &= a^2 + b^2 - 2ba \cos \gamma. \end{aligned}$$

The same result is obtained if Fig. 8-2b is used.

Note: If $\gamma = 90°$, (8-2) reduces to $c^2 = a^2 + b^2$; therefore *the Pythagorean theorem is a special case of the law of cosines.*

If we interchange the lettering of the sides and the angles of the triangle and solve for a^2 and b^2, we obtain

$$a^2 = b^2 + c^2 - 2bc \cos \alpha, \tag{8-2'}$$

and

$$b^2 = a^2 + c^2 - 2ac \cos \beta. \tag{8-2''}$$

Since the given sides and the included angle may be labeled a, b and γ in a given problem, (8-2) is sufficient to express the law of cosines. However, if the data are labeled differently in a problem it is convenient to use (8-2′) or (8-2″).

8-4 Four cases. The solution of an oblique triangle comes under one of the following four cases:

1. Given one side and two angles.
2. Given two sides and the angle opposite one of them.
3. Given two sides and the included angle.
4. Given three sides.

8-5 Case 1. Given one side and two angles. When two angles and one side are given, the third angle can be found from the relation

$$\alpha + \beta + \gamma = 180°,$$

and the required sides by the application of the law of sines.

EXAMPLES

1. *Given* $b = 73.16$, $\alpha = 132° 47.1'$, and $\gamma = 21° 36.3'$; *find* the other parts and the area of the triangle.

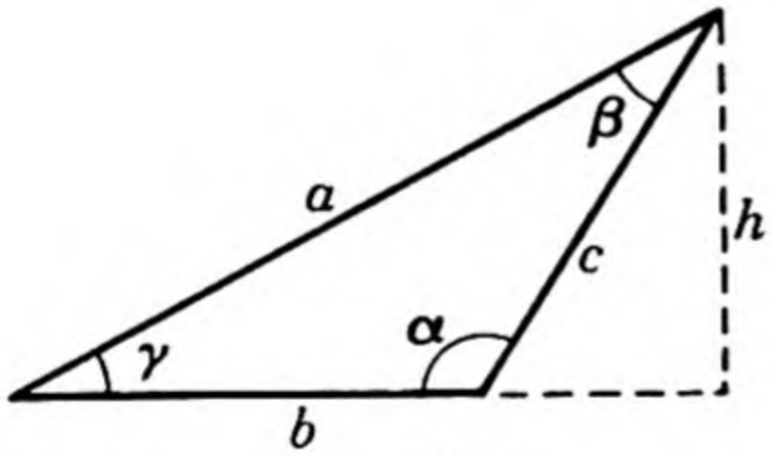


FIG. 8-3

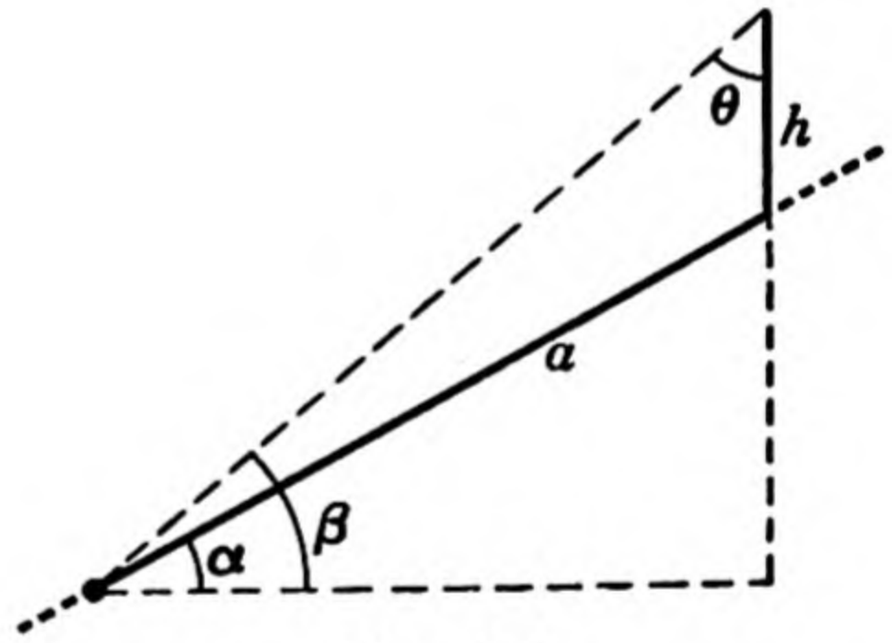


FIG. 8-4

Solution: $\beta = 180° - (\alpha + \gamma) = 25° 36.6'$, Fig. 8-3.

$$a = \frac{b \sin \alpha}{\sin \beta}. \qquad c = \frac{b \sin \gamma}{\sin \beta}.$$

$$\begin{aligned} \log b &= 1.86427 \\ \log \sin \alpha &= \underline{9.86564 - 10} \\ &\quad 11.72991 - 10 \\ \log \sin \beta &= \underline{9.63573 - 10} \\ \log a &= 2.09418 \end{aligned}$$

$$a = 124.2.$$

$$\begin{aligned} \log b &= 1.86427 \\ \log \sin \gamma &= \underline{9.56609 - 10} \\ &\quad 11.43036 - 10 \\ \log \sin \beta &= \underline{9.63573 - 10} \\ \log c &= 1.79463 \end{aligned}$$

$$c = 62.32.$$

$$A = \tfrac{1}{2}bh$$
$$= \tfrac{1}{2}ba \sin \gamma.$$

$$\log \tfrac{1}{2} = 9.69897 - 10$$
$$\log b = 1.86427$$
$$\log a = 2.09418$$
$$\log \sin \gamma = \underline{9.56609 - 10}$$
$$\log A = 3.22351$$
$$A = 1673.$$

2. A flagpole stands on a hillside which has an angle of inclination 29° 14.1′; at a point 573 feet (downhill) from the base of the pole, the angle of elevation of the top of the pole is 34° 35.7′. Find the height of the flagpole.

Given:

$\alpha = 29° 14.1'$ $\qquad a = 573$ ft
$\beta = 34° 35.7'$ $\qquad h = ?$

Solution: Using the notation of Fig. 8-4, we have

$$\frac{h}{\sin (\beta - \alpha)} = \frac{a}{\sin \theta}, \qquad \theta = 90° - \beta.$$

$$h = \frac{a \sin (\beta - \alpha)}{\sin (90° - \beta)}$$
$$= \frac{a \sin (\beta - \alpha)}{\cos \beta}$$
$$= \frac{573 \sin 5° 21.6'}{\cos 34° 35.7'} \text{ ft.}$$

$$\log a = 2.75815$$
$$\log \sin (\beta - \alpha) = \underline{8.97041 - 10}$$
$$11.72856 - 10$$
$$\log \cos \beta = \underline{9.91550 - 10}$$
$$\log h = 1.81306$$
$$h = 65.02 \text{ ft.}$$

PROBLEMS

Note: In these problems, it is advisable to draw the figures approximately to scale, in order to see whether or not the results obtained are reasonable.

Solve the following triangles:

1. $a = 24.7$	$\alpha = 65° 10'$	$\beta = 42° 20'$
2. $b = 67.31$	$\beta = 107° 20'$	$\gamma = 34° 15'$
3. $c = 7890$	$\alpha = 26° 50'$	$\beta = 51° 10'$
4. $a = 0.2675$	$\alpha = 77° 20'$	$\gamma = 42° 14'$
5. $b = 0.472$	$\alpha = 31° 50'$	$\beta = 66° 20'$
6. $c = 24.89$	$\alpha = 28° 15'$	$\gamma = 110° 28'$
7. $c = 32.45$	$\beta = 133° 21.8'$	$\gamma = 25° 30.5'$
8. $b = 5.784$	$\alpha = 28° 30.3'$	$\beta = 69° 15.6'$
9. $b = 283.14$	$\alpha = 22° 42.7'$	$\gamma = 16° 46.8'$
10. $a = 0.03157$	$\beta = 78° 46.2'$	$\gamma = 83° 36.1'$

11. $c = 55.42$	$\alpha = 56° 29'$	$\beta = 86° 13'$
12. $a = 9314$	$\beta = 129° 76.8'$	$\gamma = 31° 11.4'$
13. $b = 13.048$	$\beta = 13° 44.7'$	$\gamma = 43° 10.1'$
14. $c = 2.741$	$\alpha = 123° 14.5'$	$\gamma = 27° 45.8'$
15. $a = 105$	$\alpha = 23° 27.1'$	$\gamma = 25° 50.5'$
16. $b = 5280$	$\beta = 82° 16.7'$	$\gamma = 42° 38.4'$
17. $a = 5.158$	$\beta = 61° 45'$	$\gamma = 17° 50'$
18. $c = 34.12$	$\alpha = 18° 27.2'$	$\beta = 54° 31.6'$
19. $b = 36.74$	$\alpha = 70° 58'$	$\gamma = 72° 6'$
20. $c = 401.46$	$\beta = 19° 31.3'$	$\gamma = 53° 44.5'$
21. $a = 2.6904$	$\beta = 19° 47.4'$	$\gamma = 95° 25.4'$
22. $b = 87.015$	$\alpha = 32° 57.1'$	$\beta = 84° 51.7'$

8-6 Case 2. The ambiguous case. If a, c and α are given, angle α is constructed with c as its terminal side and an arc of a circle is drawn about the free end of c; then the result will be one of the figures in Fig. 8-5, depending on one of the following conditions.

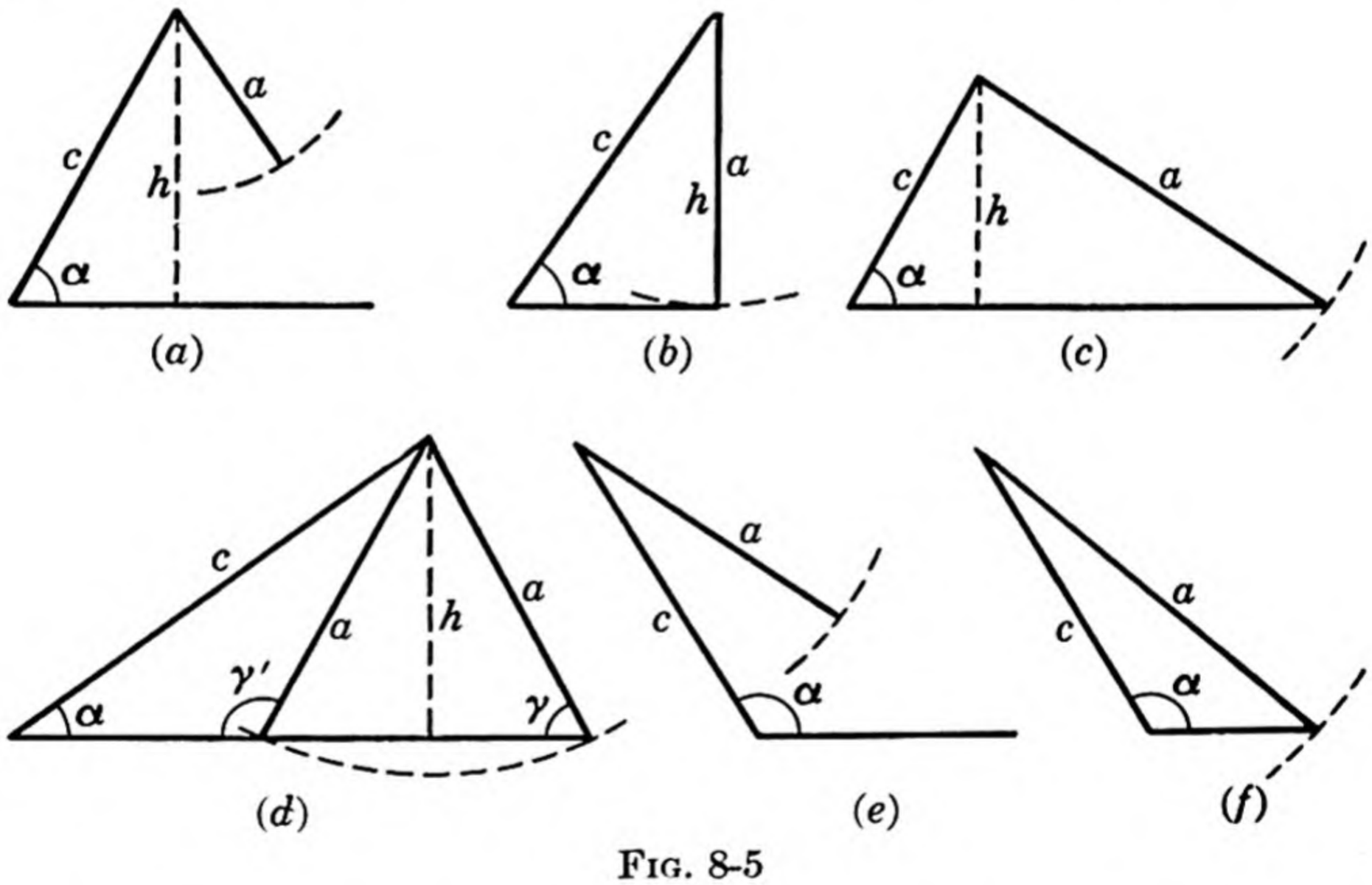


FIG. 8-5

(a) $\alpha < 90°$, and $a < c \sin \alpha$ $(= h)$, Fig. 8-5a. Line a is too short to complete the triangle; there is no solution.

(b) $\alpha < 90°$, and $a = c \sin \alpha$, Fig. 8-5b. Line a is just long enough to form a right triangle; there is a solution.

(c) $\alpha < 90°$, $a \geq c$, Fig. 8-5c. An oblique triangle is formed; there is a solution.

(d) $\alpha < 90°$, and $c \sin \alpha < a < c$, Fig. 8-5d. Two oblique triangles are formed; there are two solutions.

(e) $\alpha > 90°$, and $a \leq c$, Fig. 8-5e. Line a is too short to complete the triangle; there is no solution.

(f) $\alpha > 90°$, and $a > c$, Fig. 8-5f. An oblique triangle is formed; there is a solution.

Conditions (a) and (e) may arise only when a, c, and α are assigned arbitrary values; they do not arise in problems, such as problems of surveying, in which a, c and α represent parts of an actual triangle.

Conditions (b), (c) and (f) involve no ambiguity.

If the values of a, c and α in condition (d) represent parts of an actual triangle, only one of the two solutions can apply to the triangle. In one of the solutions $\gamma > 90°$, and in the other $\gamma < 90°$; therefore the right solution is selected according to whether γ is greater or less than 90° in the actual triangle.

EXAMPLES

Given: $a = 681.4$, $c = 978.7$, and $\alpha = 38° 14'$; *find* the other parts of the triangle.

Solution: In this problem, $\alpha < 90°$, and $c \sin \alpha < a < c$; hence there are two solutions, Fig. 8-6. Applying the law of sines, we have:

$$\sin \gamma = \frac{c \sin \alpha}{a}.$$

$$\begin{aligned} \log c &= 2.99065 \\ \log \sin \alpha &= \underline{9.79160 - 10} \\ &\quad 2.78225 \\ \log a &= \underline{2.83340} \\ \log \sin \gamma &= 9.94885 - 10 \end{aligned}$$

$$\gamma = 62° 44'.$$

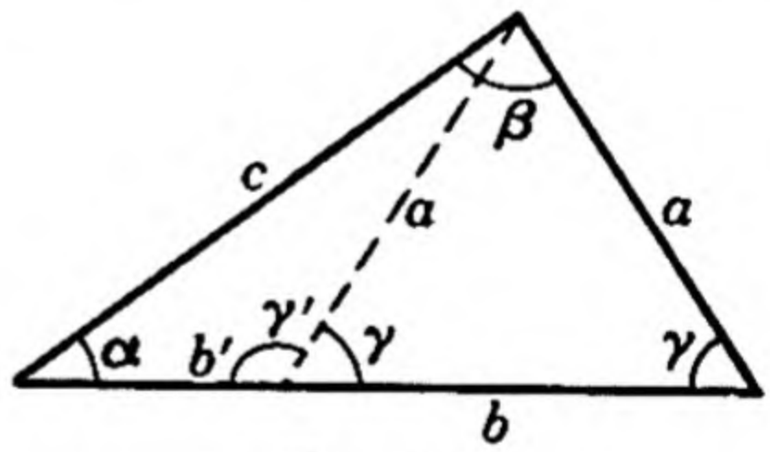


FIG. 8-6

$$\beta = 180° - \alpha - \gamma = 79° 2'.$$

$$\gamma' = 180° - \gamma = 117° 16'.$$

$$\beta' = 180° - \alpha - \gamma' = 24° 30'.$$

$$b = \frac{a \sin \beta}{\sin \alpha}.$$

$$\begin{aligned} \log a &= 2.83340 \\ \log \sin \beta &= \underline{9.99200 - 10} \\ &\quad 2.82540 \\ \log \sin \alpha &= \underline{9.79160 - 10} \\ \log b &= 3.03380 \end{aligned}$$

$$b = 1087.$$

$$b' = \frac{a \sin \beta'}{\sin \alpha}.$$

$$\begin{aligned} \log a &= 2.83340 \\ \log \sin \beta' &= \underline{9.61773 - 10} \\ &\quad 2.45113 \\ \log \sin \alpha &= \underline{9.79160 - 10} \\ \log b' &= 2.65953 \end{aligned}$$

$$b' = 456.6.$$

PROBLEMS

Solve the following triangles:

1. $a = 738$	$b = 118$	$\alpha = 79° 40'$
2. $b = 42.97$	$c = 23.46$	$\beta = 44° 48'$
3. $a = 16.7$	$b = 18.9$	$\beta = 56° 25'$
4. $a = 564$	$b = 607$	$\alpha = 62° 51'$
5. $a = 0.189$	$c = 0.294$	$\gamma = 67° 32'$
6. $b = 234.1$	$c = 835.3$	$\gamma = 13° 37'$
7. $b = 35.6$	$c = 40.71$	$\gamma = 72° 5'$
8. $a = 3.913$	$c = 9.824$	$\alpha = 6° 43'$
9. $b = 104.68$	$c = 452.16$	$\gamma = 68° 54.2'$
10. $b = 35.42$	$c = 22.18$	$\beta = 61° 26'$
11. $a = 817.54$	$b = 785.24$	$\beta = 54° 50.9'$
12. $b = 3.424$	$c = 5.241$	$\beta = 40° 32'$
13. $a = 10.547$	$c = 4.0032$	$\gamma = 20° 31.7'$
14. $a = 36.51$	$b = 23.42$	$\alpha = 38° 52'$
15. $b = 4067.7$	$c = 3284.0$	$\beta = 37° 50.4'$
16. $b = 0.5292$	$c = 0.6411$	$\beta = 46° 7'$
17. $a = 1665.4$	$b = 1268.1$	$\beta = 47° 43.5'$
18. $b = 10.432$	$c = 9.7463$	$\gamma = 67° 20.8'$
19. $a = 19.017$	$c = 90.168$	$\alpha = 11° 45.1'$
20. $b = 28.497$	$c = 30.808$	$\beta = 41° 1.2'$
21. $a = 6.8351$	$b = 4.6369$	$\alpha = 51° 34.6'$
22. $b = 398.04$	$c = 749.21$	$\gamma = 37° 25.3'$

8-7 Cases 3 and 4. The law of cosines can be used to find the third side when two sides and the included angle are given (Case 3) and to find one of the angles when three sides are given (Case 4). In either case, the remaining parts can be found by means of either the law of cosines or the law of sines; however, it is preferable to use the law of sines when the numbers involved are large, since the law of cosines does not lend itself conveniently to logarithms.

The laws of tangents, taken up in Sec. 8-8 and 8-10, are better adapted to the logarithmic solution of Cases 3 and 4 when large numbers are involved.

EXAMPLE

An airplane is moving toward the East with a velocity of 120 miles per hour relative to the ground. The wind is blowing from N 30° E with a velocity of 40 miles per hour. Find the magnitude and the direction of the velocity of the wind relative to the plane.*

* The velocity of one body relative to another is the vector sum of the velocity of the first (referred body) and the reversed velocity of the second (reference body).

Solution: From Fig. 8-7b, where the velocity of the plane is reversed, we have

$$\begin{aligned} v_{12} &= \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos \gamma} \\ &= \sqrt{120^2 + 40^2 - 2 \times 120 \times 40 \cos 120^\circ} \text{ mi/hr} \\ &= 40\sqrt{9 + 1 + 6 \cos 60^\circ} \text{ mi/hr} \\ &= 40\sqrt{13} \text{ mi/hr} \\ &= 144.2 \text{ mi/hr.} \end{aligned}$$

$$\sin \alpha = \frac{v_2}{v_{12}} \sin \gamma = \frac{v_2}{v_{12}} \sin 60^\circ.$$

$$\alpha = 13^\circ\ 54'.$$

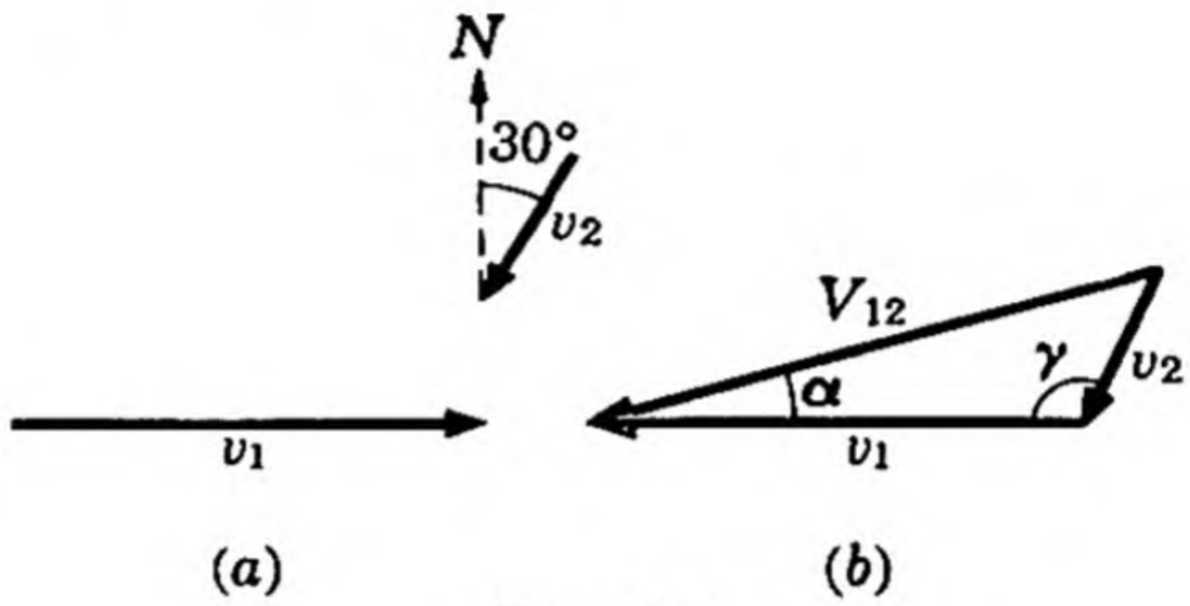


FIG. 8-7

Therefore the wind appears, to the pilot, to be blowing from N 76° 6′ E with a velocity of 144.2 miles per hour.

PROBLEMS

Note: In the following problems, find each of the required angles independently, in order to be able to use the relation $\alpha + \beta + \gamma = 180^\circ$ as a check.

Solve the following triangles:

1. $a = 20$	$b = 30$	$\gamma = 104^\circ\ 35'$
2. $a = 45$	$b = 35$	$\gamma = 82^\circ\ 37'$
3. $a = 15$	$b = 27$	$\gamma = 46^\circ$
4. $a = 36$	$b = 48$	$\gamma = 37^\circ\ 56'$
5. $a = 200$	$b = 150$	$\gamma = 50^\circ\ 21'$
6. $a = 24$	$b = 32$	$\gamma = 12^\circ\ 32'$
7. $a = 120$	$b = 170$	$\gamma = 59^\circ\ 17'$
8. $a = 90$	$b = 270$	$\gamma = 93^\circ\ 44'$
9. $a = 7.25$	$b = 6.25$	$\gamma = 74^\circ\ 55.8'$
10. $a = 12$	$b = 15$	$\gamma = 123^\circ\ 46'$
11. $a = 130$	$b = 100$	$\gamma = 51^\circ\ 49'$
12. $a = 80$	$b = 60$	$\gamma = 19^\circ\ 57'$
13. $a = 12$	$b = 14$	$c = 10$
14. $a = 6$	$b = 9$	$c = 12$

15. $a = 15$	$b = 12$	$c = 17$
16. $a = 40$	$b = 30$	$c = 50$
17. $a = 50$	$b = 60$	$c = 40$
18. $a = 10$	$b = 40$	$c = 30$
19. $a = 20$	$b = 25$	$c = 30$
20. $a = 25$	$b = 15$	$c = 30$
21. $a = 50$	$b = 30$	$c = 70$
22. $a = 10$	$b = 20$	$c = 25$
23. $a = 20$	$b = 30$	$c = 40$
24. $a = 60$	$b = 40$	$c = 80$

MISCELLANEOUS PROBLEMS

1. From one bank of a river the angle of elevation of the top of a tree on the opposite bank is 60°. From a point 150 feet back the angle of elevation is 30°. Find the width of the river.

2. Using the data but not the answer of the preceding problem, find the height of the tree.

3. In order to determine the range of a target, the angles of the triangle formed by the target and two points on a battleship are found. If the distance between the points is 500 feet and the angles at the points are 46° 37′ and 123° 11′, find the distance of the target from the vertex of the larger angle.

4. Using the data but not the answer of the preceding problem, find the distance of the target from the vertex of the smaller angle.

5. The angle of elevation of the top of a flagpole is 25° 54′ from a certain point; from another point 200 feet nearer, it is 63° 5.2′. Find the height of the flagpole.

6. Using the data but not the answer of the preceding problem, find the distance of the first point from the flagpole.

7. A point A is at one bank of a river and a point B is on a small island. In order to determine the distance AB, a distance AC is measured along the river, and the angles of the triangle ABC at A and C are found to be 65° 16′ and 70° 3′, respectively. Find the distance AB, if AC equals 400 feet.

8. Using the data but not the answer of the preceding problem, find the distance CB.

9. From a point on the plain east of Pikes Peak, the angle of elevation of the summit is found to be 6° 25′; from a point 5 miles nearer, the angle of elevation is found to be 9° 35′. Find the height of the mountain above the plain.

10. Using the data but not the answer of the preceding problem find the distance of the summit from the nearer point.

11. In order to determine a distance AB across a swamp, a point C is selected, and AC, CB, and the angle CAB are found to be 1690 feet, 1960 feet, and 58° 25′, respectively. Find the distance AB.

12. In order to find the length of a proposed tunnel between two points A and B, two distances AC and BC, and the angle ACB are measured and found to be 1426 feet, 1723 feet, and 49° 12.6′. Find the length of the tunnel.

13. The angles of elevation to a captive balloon are measured from two points 3000 feet apart, and found to be 43° 27′ and 74° 38′. If the two points are on the same side of the balloon and in the same vertical plane, find the height of the balloon.

14. In the preceding problem, take the two points to be on opposite sides of the balloon.

15. From a point, the angle of elevation of the top of a tower is 59° 36′. On moving 100 feet back, the angle of elevation becomes 29° 14′. Find the height of the tower.

16. Two adjacent sides of a parallelogram are 10 inches and 15 inches, and the included angle is 123° 47′. Find the lengths of the diagonals.

17. From the base and the top of a lighthouse on a cliff, the angles of depression to a ship at anchor are 3° 11′ and 4° 28′, respectively. If the lighthouse is 75 feet high, find the distance of the ship.

18. Using the data but not the answer of the preceding problem, find the height of the cliff.

19. A tower stands on a uniformly sloping ground with an angle of inclination of 13° 20′. From two points on the ground, the angles of elevation to the top of the tower are 29° 12′ and 44° 26′. If the points are 100 feet apart and in the same vertical plane as the top of the tower, find the height of the tower.

20. From a window, the angle of elevation of the top of a flagpole is 14° 11′; from the street 40 feet below the window, the angle of elevation is 31° 16′. Find the height of the flagpole.

21. Using the data but not the answer of the preceding problem, find the horizontal distance of the flagpole from the window.

22. A flyer is instructed to fly north as far as he can, for observation purposes, and to return to his aircraft carrier which is moving in a northeasterly direction at the rate of 20 miles per hour. If the speed of the plane is 150 miles per hour and if it has gas enough to fly one hour, how far north should he fly?

23. In order to determine the height of a mountain above a plain, a surveyor takes a base line a on the plain and finds the angles α, β and γ, Fig. 8-8. Prove that the height of the mountain is given by

$$h = \frac{\sin \alpha \sin \gamma}{\sin (\beta + \gamma)} a.$$

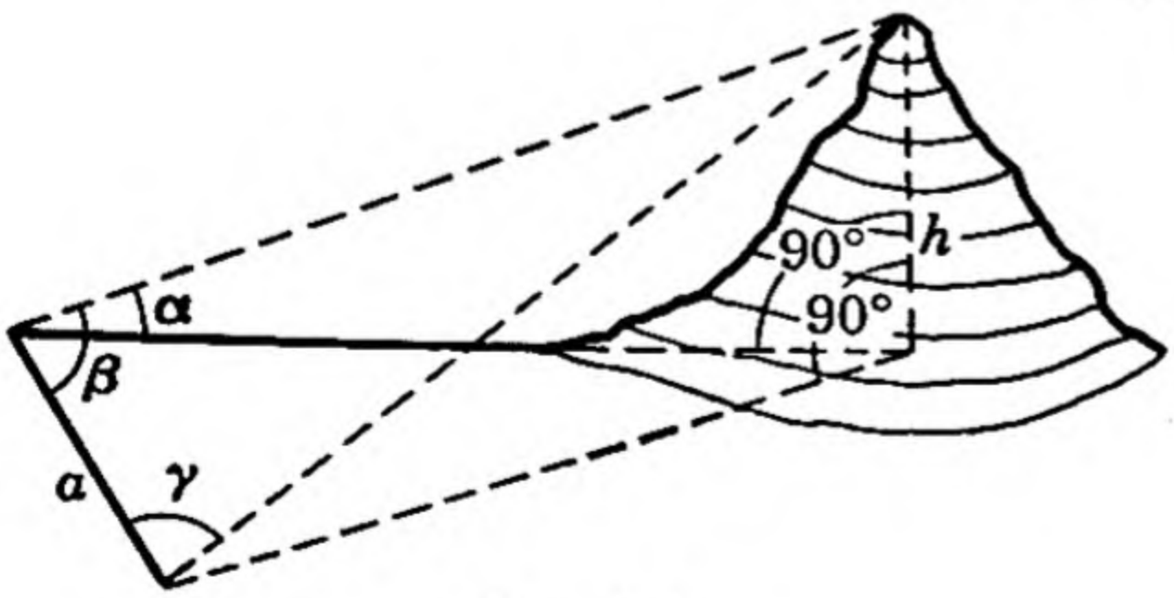


FIG. 8-8

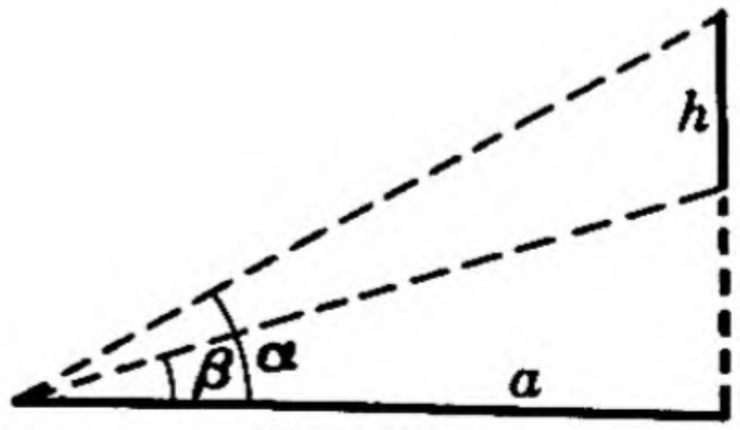


FIG. 8-9

24. Prove that in Fig. 8-9, $h = \dfrac{\sin(\alpha - \beta)}{\cos\alpha \cos\beta} a$.

25. Prove that in Fig. 8-10, $h = \dfrac{\sin\beta}{\cos\alpha \cos(\alpha + \beta)} a$.

26. Prove that in Fig. 8-11, $h = \dfrac{\sin\alpha \sin\beta}{\sin(\alpha - \beta)} a$.

27. Prove that in Fig. 8-12, $h = \dfrac{\sin(\beta - \alpha)\sin\gamma}{\sin(\gamma - \beta)} a$.

28. Prove that in Fig. 8-13, $h = \dfrac{\sin\beta \sin\gamma}{\sin(\gamma - \beta)\cos\alpha} a$.

29. Prove that in Fig. 8-14, $x = r\cos\theta + \sqrt{l^2 - r^2\sin^2\theta}$.

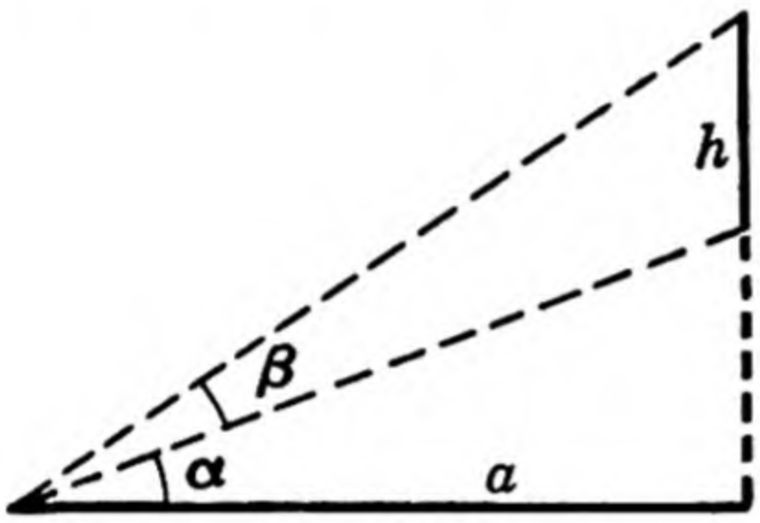


FIG. 8-10

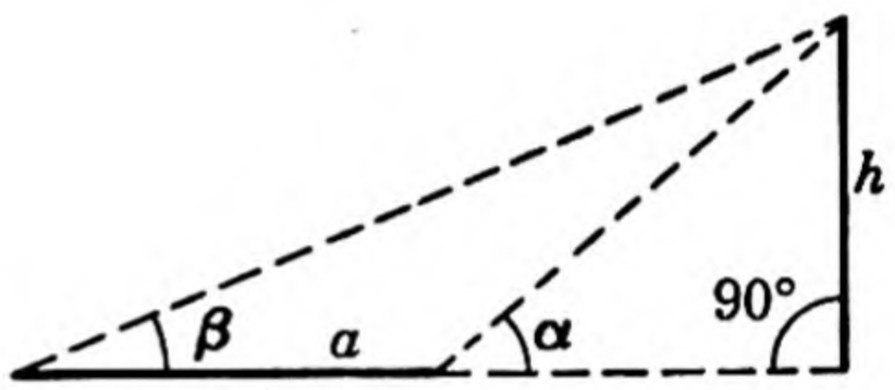


FIG. 8-11

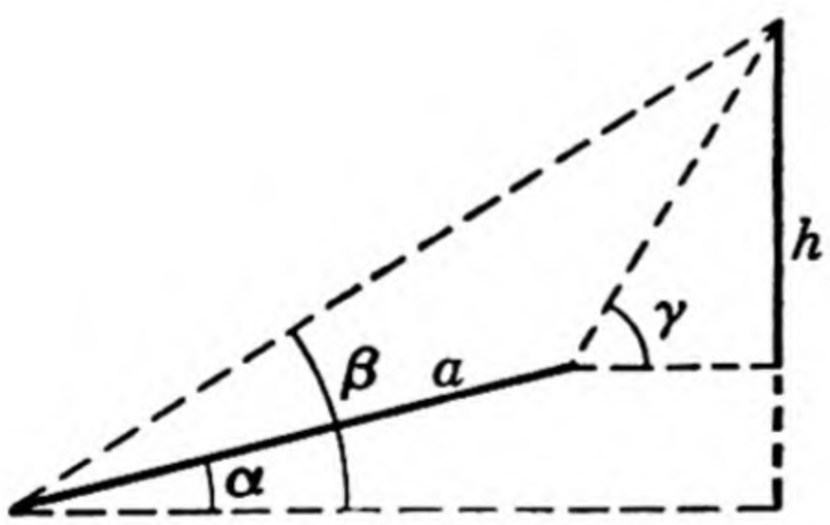


FIG. 8-12

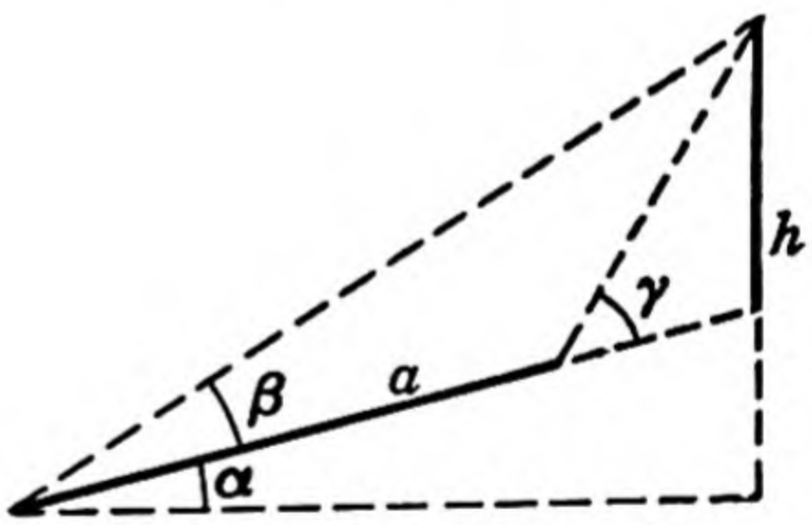


FIG. 8-13

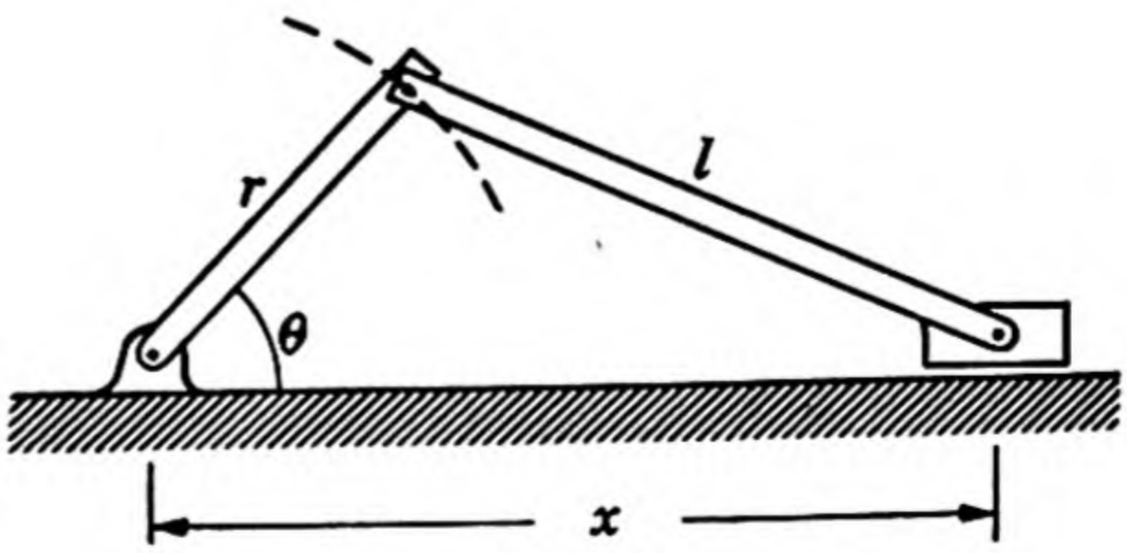


FIG. 8-14

8-8 First law of tangents. Since logarithms do not apply to a sum and since the right-hand member of the law of cosines consists of the sum of three terms, the method described in Sec. 8-7 is not well suited to solutions by means of logarithms. It is desirable, therefore, to obtain formulas which lend themselves to the solution of Cases 3 and 4 by the use of logarithms. This is especially true when the values of the sides of a triangle contain more than three significant figures. The formulas which do lend themselves to the solution of Cases 3 and 4 may be called, respectively, the *first law of tangents* and the *second law of tangents*.

If a and b denote two sides of a triangle and α and β the angles opposite them, the first law of tangents states:

$$\frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)} = \frac{a - b}{a + b}. \tag{8-3}$$

Proof of (8-3): Subtracting 1 from, or adding 1 to, each member of

$$\frac{a}{b} = \frac{\sin \alpha}{\sin \beta},$$

and reducing each member to a common denominator, we get

$$\frac{a - b}{b} = \frac{\sin \alpha - \sin \beta}{\sin \beta}, \tag{a}$$

or

$$\frac{a + b}{b} = \frac{\sin \alpha + \sin \beta}{\sin \beta}. \tag{b}$$

Dividing (a) by (b), we have:

$$\frac{a - b}{a + b} = \frac{\sin \alpha - \sin \beta}{\sin \alpha + \sin \beta}$$

$$= \frac{2 \cos \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)}{2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)} \text{ [by (7-18) and (7-17)]}$$

$$= \frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)}.$$

8-9 Area of a triangle. If the data fall under Case 3, that is, if two sides a and b, and the included angle γ are given, the area of a

triangle, Fig. 8-15, can be found most conveniently by the use of the equation

$$A = \tfrac{1}{2}bh,$$

or

$$A = \tfrac{1}{2}ab \sin \gamma.$$

This equation can be used also in the other cases, after determining

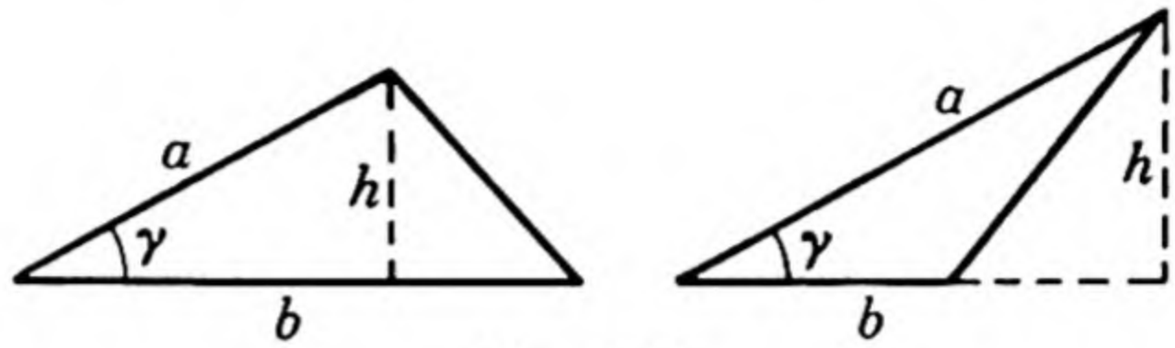


FIG. 8-15

a side or an angle that is not given. The following formulas are more convenient, however, when all three sides are given:

$$A = \sqrt{s(s-a)(s-b)(s-c)} \qquad (8\text{-}4)$$

and

$$A = rs, \qquad (8\text{-}4')$$

where $s = \frac{1}{2}(a + b + c)$ and r is the radius of the circle inscribed in the triangle.

Proof of (8-4):

$$\begin{aligned} A^2 &= \tfrac{1}{4}a^2b^2 \sin^2 \gamma \\ &= \tfrac{1}{4}a^2b^2(1 - \cos^2 \gamma) \\ &= \tfrac{1}{4}a^2b^2(1 + \cos \gamma)\,(1 - \cos \gamma) \\ &= \tfrac{1}{4}a^2b^2\left(1 + \frac{a^2 + b^2 - c^2}{2ab}\right)\left(1 - \frac{a^2 + b^2 - c^2}{2ab}\right) \quad \text{[by (8-2)]} \\ &= \tfrac{1}{16}[(a + b)^2 - c^2]\,[c^2 - (a - b)^2] \\ &= \tfrac{1}{16}(a + b + c)\,(a + b - c)\,(c + a - b)\,(c - a + b) \\ &= \tfrac{1}{16}\,2s(2s - 2c)\,(2s - 2b)\,(2s - 2a) \\ &= s(s - a)\,(s - b)\,(s - c). \end{aligned}$$

$$A = \sqrt{s(s - a)\,(s - b)\,(s - c)}.$$

Proof of (8-4′): The area of a triangle equals the sum of the areas of the three triangles obtained by connecting the center of the inscribed circle with the vertices of the triangle, Fig. 8-16; that is,

$$\begin{aligned} A &= \tfrac{1}{2}ar + \tfrac{1}{2}br + \tfrac{1}{2}cr \\ &= \tfrac{1}{2}(a + b + c)r \\ &= rs. \end{aligned}$$

Equating this expression for A with that in (8-4), we have:

$$r = \sqrt{\frac{(s-a)\,(s-b)\,(s-c)}{s}}. \tag{8-5}$$

8-10 Second law of tangents. This law is given by the following formulas:

$$\tan \frac{1}{2}\alpha = \frac{r}{s-a}, \tag{8-6a}$$

$$\tan \frac{1}{2}\beta = \frac{r}{s-b}, \tag{8-6b}$$

$$\tan \frac{1}{2}\gamma = \frac{r}{s-c}, \tag{8-6c}$$

where s and r have the same meanings as in the preceding section.

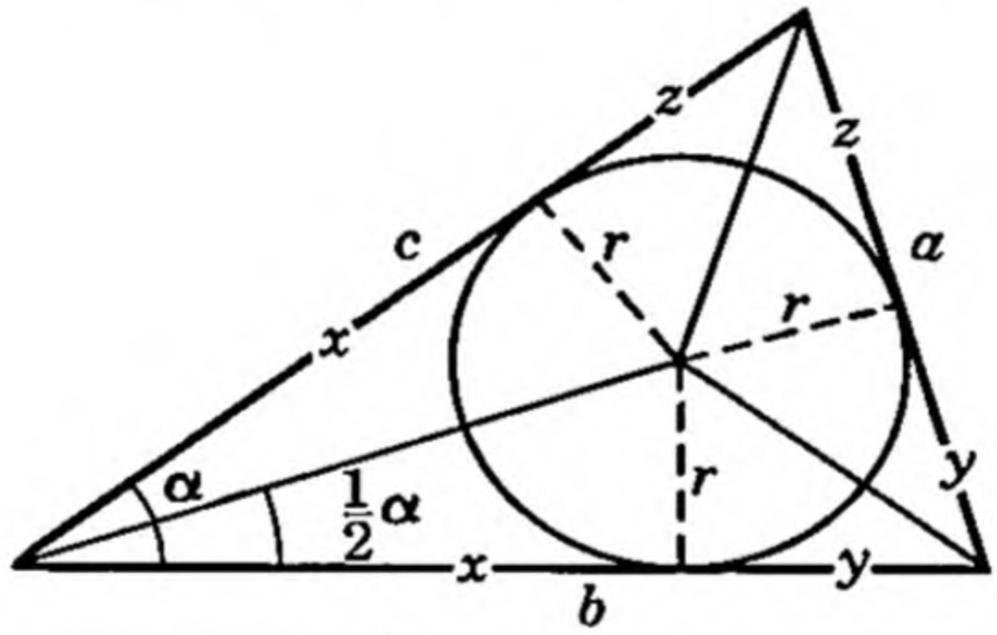


FIG. 8-16

Proof of (8-6a): From Fig. 8-16, we have

(a) $\tan \frac{1}{2}\alpha = \frac{r}{x}$,

(b) $s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}[(y + z) + (x + y) + (x + z)]$

$= x + y + z$

$= x + a.$

Eliminating x between equations (a) and (b) we get formula (8-6a). Formulas (8-6b) and (8-6c) are obtained in a similar manner.

EXAMPLES

1. Two sides of a triangle are 299.7 and 221.2, and the included angle is 118° 25.2′; find the other parts of the triangle.

Solution: $\tan \frac{1}{2}(\alpha - \beta) = \frac{a-b}{a+b} \tan \frac{1}{2}(\alpha + \beta)$, by (8-3).

$$\begin{aligned} a &= 299.7 \\ b &= \underline{221.2} \\ a - b &= \ 78.5 \end{aligned} \qquad \begin{aligned} 180° &= 179°\ 60.0' \\ \gamma &= \underline{118°\ 25.2'} \\ \alpha + \beta &= \ 61°\ 34.8' \end{aligned}$$

$$a + b = 520.9 \qquad \frac{1}{2}(\alpha + \beta) = \ 30°\ 47.4'$$

$$\begin{aligned} \log (a - b) &= 1.89487 \\ \log (a + b) &= \underline{2.71675} \\ &\overline{9.17812 - 10} \\ \log \tan \tfrac{1}{2}(\alpha + \beta) &= \underline{9.77516 - 10} \\ \log \tan \tfrac{1}{2}(\alpha - \beta) &= 8.95328 - 10 \end{aligned}$$

$$\alpha = \tfrac{1}{2}(\alpha + \beta) + \tfrac{1}{2}(\alpha - \beta) = 35°\ 55.3'.$$

$$\beta = \tfrac{1}{2}(\alpha + \beta) - \tfrac{1}{2}(\alpha - \beta) = 25°\ 39.5'$$

$$\tfrac{1}{2}(\alpha - \beta) = 5°\ 7.9'$$

$$\begin{aligned} c &= \frac{a \sin \gamma}{\sin \alpha} \\ &= \frac{a \sin (180° - \gamma)}{\sin \alpha} \\ &= \frac{a \sin (\alpha + \beta)}{\sin \alpha} \end{aligned}$$

$$\begin{aligned} \log a &= \ 2.47669 \\ \log \sin (\alpha + \beta) &= \ \underline{9.94423 - 10} \\ &\overline{12\ .42092 - 10} \\ \log \sin \alpha &= \ \underline{9.76840 - 10} \\ \log c &= \ 2.65252 \\ c &= 449.3 \end{aligned}$$

Note: The relation $\alpha + \beta + \gamma = 180°$ is of no use for checking the results obtained by this method. Even when the value obtained for $\frac{1}{2}(\alpha - \beta)$ and, consequently, the values of α and β are incorrect, the relation is satisfied; because the equation $\alpha + \beta + \gamma = \frac{1}{2}(\alpha + \beta) + \frac{1}{2}(\alpha - \beta) + \frac{1}{2}(\alpha + \beta) - \frac{1}{2}(\alpha - \beta) + \gamma$ is an equation of identity and is true for all values of the angles. The values of α and β can be checked, however, by the application of the law of sines, as follows:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}; \qquad \begin{array}{rr} \log a = & 2.47669 \\ \log \sin\alpha = & \underline{9.76840 - 10} \\ & 2.70829 \end{array} \qquad \begin{array}{rr} \log b = & 2.34479 \\ \log \sin\beta = & \underline{9.63649 - 10} \\ & 2.70830 \end{array}.$$

Hence the values of α and β satisfy the law of sines, as they should.

2. Find the angles and the area of the triangle with sides 624.13, 489.32, and 903.45.

Solution:

$s = \frac{1}{2}(a + b + c)$ | $s - a$ | $s - b$ | $s - c$

624.13	1008.45	1008.45	1008.45
489.32	624.13	489.32	903.45
903.45	384.32	519.13	105.00
2|2016.90			
$s = 1008.45$			

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$\log (s - a) = 2.58470$
$\log (s - b) = 2.71527$
$\log (s - c) = 2.02119$
7.32116
$\log s = 3.00364$
$2)4.31752$
$\log r = 2.15876$

$$\tan\tfrac{1}{2}\alpha = \frac{r}{s-a}$$

$\log r = 2.15876$
$\log (s - a) = 2.58470$
$\log \tan \frac{1}{2}\alpha = 9.57406 - 10$

$\frac{1}{2}\alpha = 20^\circ\ 33.4'$

$\alpha = 41^\circ\ 6.8'$

$$\tan\tfrac{1}{2}\beta = \frac{r}{s-b}$$

$\log r = 2.15876$
$\log (s - b) = 2.71527$
$\log \tan \frac{1}{2}\beta = 9.44349 - 10$

$\frac{1}{2}\beta = 15^\circ\ 31'$

$\beta = 31^\circ\ 2'$

$$\tan\tfrac{1}{2}\gamma = \frac{r}{s-c}$$

$\log r = 2.15876$
$\log (s - c) = 2.02119$
$\log \tan \frac{1}{2}\gamma = 0.13757$

$\frac{1}{2}\gamma = 53^\circ\ 55.6'$

$\gamma = 107^\circ\ 51.2'$

$$A = rs$$

$\log r = 2.15876$
$\log s = 3.00364$
$\log A = 5.16240$

$A = 143{,}345$

PROBLEMS

In each of the following problems, the values denote two sides and the included angle of a triangle. Using the first law of tangents, solve the triangle.

1. 26.85, 28.34, 63° 38′.
2. 3.761, 5.925, 15° 27′.
3. 6.682, 5.028, 101° 6.6′.
4. 10.41, 16.78, 97° 26′.
5. 43.62, 39.81, 60° 16′.
6. 1.234, 4.912, 76° 31′.
7. 0.2540, 0.3429, 85° 26′.
8. 0.761, 0.952, 101° 37′.
9. 599.3, 543.7, 56° 34′.
10. 27.68, 35.41, 19° 28′.
11. 53.360, 22.734, 92° 51.5′.
12. 117.6, 289.4, 23° 3.5′.
13. 332.1, 229.5, 29° 10′.
14. 18.28, 23.45, 54° 18′.

Using the second law of tangents, find the angles of the triangles which have the following sides:

15. 912, 831, 264.
16. 0.732, 0.314, 0.681.
17. 45.9, 56.3, 64.3.
18. 7.95, 12.17, 19.64.
19. 12.95, 10.18, 12.43.
20. 13.04, 17.61, 25.89.

Find the angles and areas of the triangles which have the following sides:

21. 66.06, 58.96, 42.36.
22. 10.62, 14.78, 20.25.
23. 120.1, 82.37, 102.5.
24. 8.33, 6.41, 11.89.
25. 180.1, 282.0, 268.6.
26. 102.3, 97.6, 151.2.

CHAPTER 9

LOGARITHMS

9-1 Logarithms. Whenever the mathematician finds it necessary to perform a new kind of mathematical operation, he introduces a new symbol to denote the operation and gives it a name. For instance, the operation by means of which $y = x^3$ is solved for x is expressed by the symbol $\sqrt[3]{\ }$ and is called "cube root." The operation by means of which the *exponential* equation

$$N = b^x \tag{a}$$

is solved for x is written as

$$x = \log_b N, \tag{b}$$

and the right-hand member is called the *logarithm* of N to the *base* b. Eqs. (a) and (b) are identical except in form. Eq. (a) states, "N is the number obtained when b is raised to the power x." Eq. (b) states, "x is the number to which b must be raised in order to obtain N." In other words Eq. (b) is the same as Eq. (a) read backwards. Hence the definition:

The logarithm of N *to the base* b *is that power to which* b *must be raised in order to obtain* N.

Illustrations: since $8 = 2^3$, $\log_2 8 = 3$. Similarly $\log_3 9 = 2$; $\log_5 125 = 3$; $\log_{10} 10{,}000 = 4$; $\log_{10} 0.0001 = -4$.

9-2 Laws of logarithms. The following laws show how the operations of multiplication, division, and raising to a power may be reduced to the simpler operations of addition, subtraction, and multiplication, respectively. This simplification is one of the important features of logarithms. The logarithms in each of the following laws are to the same base.

I. The logarithm of a product equals the sum of the logarithms of the factors, that is,

$$\log MN = \log M + \log N. \tag{9-1}$$

II. The logarithm of a quotient equals the logarithm of the numerator minus the logarithm of the denominator, that is

$$\log \frac{M}{N} = \log M - \log N. \tag{9-2}$$

III. The logarithm of a number raised to a power equals the product of the exponent and the logarithm of the number, that is,

$$\log N^p = \mathrm{p} \log N. \tag{9-3}$$

These laws follow from the definition of a logarithm and the laws of *exponents*. Thus, if

$$x = \log_b M, \text{ and } y = \log_b N,$$

then by the definition of a logarithm,

$$M = b^x, \text{ and } N = b^y.$$

Hence,
$$MN = b^x \cdot b^y = b^{x+y}, \tag{a}$$

$$\frac{M}{N} = \frac{b^x}{b^y} = b^{x-y}, \tag{b}$$

and
$$N^p = (b^y)^p = b^{py}. \tag{c}$$

Applying the definition of a logarithm to the first and last expressions in each of the equations (a) to (c), we obtain

$$\log_b MN = x + y = \log_b M + \log_b N,$$

$$\log_b \left(\frac{M}{N}\right) = x - y = \log_b M - \log_b N,$$

$$\log_b (N^p) = py = p \log_b N.$$

The following are important points about logarithms.

(a) *The base* b *may be any positive number which is not 1.* This statement excludes zero also, since zero is neither positive nor negative.

(b) *The logarithm of a negative number is not defined* here, because no value of x can make b^x a negative *real number*.* However, in higher mathematics the logarithm of a negative number is defined in terms of complex numbers.

* A number which does not contain an even root of a negative number is called a real number. A number which consists of even roots of negative numbers is called an *imaginary number*. A number which contains both real and imaginary terms is called a *complex number*.

(c) *The logarithm of 1 is zero.* Taking the logarithm of both sides of $b^0 = 1$, we have $0 \times \log b = \log 1$, or $\log 1 = 0$.

(d) *The logarithm of the base equals 1.* Solving $b = b^1$ for the exponent, we have $1 = \log_b b$, or $\log_b b = 1$.

(e) *The logarithm is positive if the number is greater than 1, and negative if the number is between 0 and 1;* $b > 1$.

(f) *The logarithm of a variable number increases less and less rapidly as the value of the number increases;* $b > 1$. Thus $\log_{10} 10 = 1$, $\log_{10} 100 = 2$, $\log_{10} 1000 = 3$.

(g) *As a variable number approaches zero, the negative of the logarithm increases indefinitely;* $b > 1$.

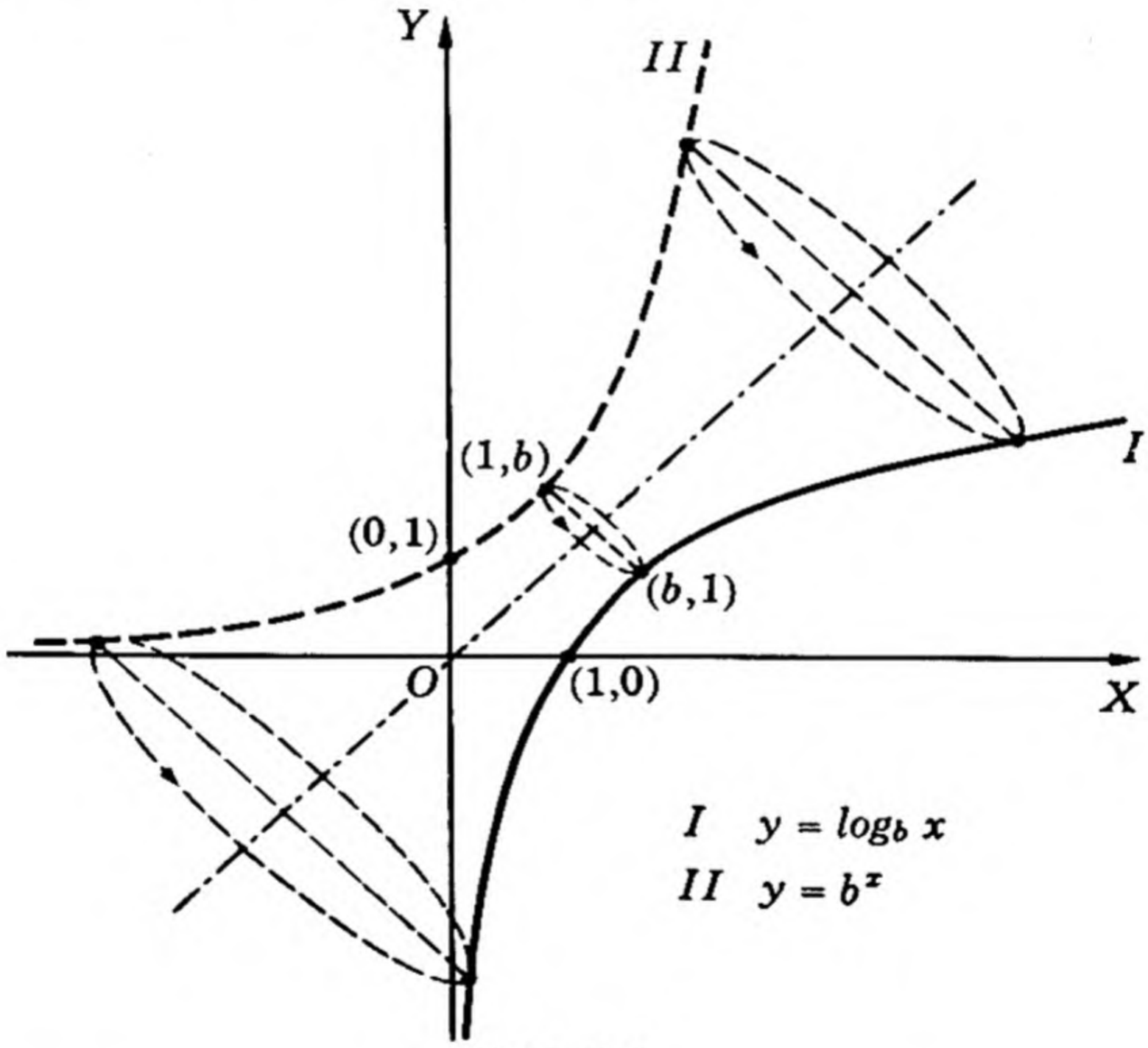


FIG. 9-1

These points are brought out by Curve I in Fig. 9-1, the graph of $y = \log_b x$. Curve II is the graph of $y = b^x$. The two curves are identical except for their positions relative to the coordinate axes. They are symmetric* with respect to the 45° line. When one of the curves is rotated through 180° about the 45° line, it coincides with the other curve.

* See footnote on p. 53.

PROBLEMS

Write each of the following as the algebraic sum of logarithms:

1. $\log pv$.
2. $\log pv^n$.
3. $\log \frac{pv}{T}$.
4. $\log \pi r^2$.
5. $\log \frac{4}{3}\pi r^3$.
6. $\log \frac{1}{3}\pi r^2 h$.
7. $\log 2\pi\sqrt{\frac{l}{g}}$.
8. $\log P(1 + r)^n$.
9. $\log \sqrt{\frac{ab^2}{c^3}}$.
10. $\log \left(\frac{7 \times 19^2}{25}\right)^{\frac{1}{2}}$.
11. $\log \frac{\sqrt[3]{13 \times \sqrt{17}}}{15 \times \sqrt{2}}$.
12. $\log \left(\frac{22 \times (15)^2}{\sqrt{29}}\right)^{\frac{1}{5}}$.
13. $\log \frac{a(b - c)^2}{c\sqrt{a - b}}$.
14. $\log \sqrt{\frac{(s - a)(s - b)(s - c)}{s}}$.

Write each of the following as the logarithm of a single expression:

15. $\log a + \log b^2 - \log c$.
16. $\log \sqrt{a} - \log 2 + \log b$.
17. $\log a + 2 \log b$.
18. $\log a^2 - 2 \log b + \log c$.
19. $\log (a - b) - \log (a + b)$.
20. $\log (a + b) + \log (a - b)$.
21. $\log 100 + \log 15 - \log 20$.
22. $\log 15 - 3 \log 5 + 2 \log 10$.
23. $\log \sqrt{30} - \frac{1}{2} \log 15 + \log 8$.
24. $\log \sqrt{50} + \frac{1}{2} \log 16 - \frac{1}{3} \log 27$.

9-3 The two bases. As has been stated, any positive number other than 1 may be used as the base of a set of logarithms. But the number 10 and the *irrational* number e ($= 2.7182818 \cdots$) are the most important bases. The first is important because, being the basic number of the decimal system, it makes tables of logarithms shorter and numerical computations simpler. The second is important because, being a basic number of nature, it makes theoretical work simpler. The number 10 is called the *common base* and logarithms based on it are called *common logarithms*. The number e is called the *natural base* and logarithms based on it are called *natural logarithms*.

Unless the base is indicated, it will be taken for granted, in the rest of this chapter, that logarithms are to the base 10.

9-4 Characteristic and mantissa. The following table illustrates the fact that the common logarithm of a number which is an integral power of 10 is a positive or a negative integer. The logarithm of a number which is not an integral power of 10 consists of an integral part, called the *characteristic*, and of a decimal part, called the *mantissa*.

N	$\log N$
$1{,}000{,}000 = 10^{6}$	6
$100{,}000 = 10^{5}$	5
$10{,}000 = 10^{4}$	4
$1{,}000 = 10^{3}$	3
$100 = 10^{2}$	2
$10 = 10^{1}$	1
$1 = 10^{0}$	0
$0.1 = 10^{-1}$	-1
$0.01 = 10^{-2}$	-2
$0.001 = 10^{-3}$	-3
$0.0001 = 10^{-4}$	-4
$0.00001 = 10^{-5}$	-5
$0.000001 = 10^{-6}$	-6

Illustration: The logarithm of 5306, which falls between 1000 and 10,000, lies between 3 and 4, and equals 3.72477, to five decimals. In this case the characteristic is 3 and the mantissa 0.72477. On the other hand, the logarithm of 0.005306, which falls between 0.001 and 0.01, lies between -3 and -2. To find its logarithm, we write 0.005306 as 5.306×10^{-3}, take the logarithm of both sides, and obtain

$$\log 0.005306 = \log 5.306 + \log 10^{-3} = 0.72477 - 3.$$

To avoid the negative characteristic, we add 10 to the last expression and subtract 10 from it, and write

$$\log 0.005306 = 7.72477 - 10.$$

9-5 Standard position of the decimal point. All numbers expressed in the decimal form which have the same sequence of

digits, except for zeros at either end, have the same mantissa. The great advantage of 10 as a base is due to this fact.

For instance, wherever the position of the decimal point may be in 000053060000, this number may be written in the form 5.306×10^n, where n is a positive or negative integer or zero and depends on the position of the decimal point. But

$$\begin{aligned} \log (5.306 \times 10^n) &= \log 5.306 + \log 10^n \\ &= n + \log 5.306 \\ &= n + 0.72477. \end{aligned}$$

Thus the characteristic n changes as the decimal point is shifted, but the mantissa remains the same.

When the decimal point immediately follows the first digit which is not zero, it is said to be in the *standard position*. The characteristic of the logarithm of a number may be determined by the following rules:

The characteristic is zero if the decimal point is at the standard position.

The characteristic equals the number of digits between the standard position and the actual position of the decimal point, if the decimal point is at the right of the standard position; it equals the negative of that number if the decimal point is at the left.

Thus $\log 53060.0 = 4.72477$

and $\log 0.0005306 = -4 + 0.72477 = 6.72477 - 10.$

9-6 Table of logarithms. The characteristic of the logarithm of any number which is in the decimal form can be found readily and for this reason tables of the logarithms of numbers give only the mantissas. The mantissa of a number can be found to any number of decimals by methods of advanced mathematics. A table which gives mantissas to n decimals is called an *n-place table* of logarithms. Table I is a five-place table.

9-7 Significant figures. Numbers used in science, engineering, and other fields of knowledge represent, with rare exceptions, approximate values. The accuracy of the value represented by such a number is indicated by the number of *significant figures* it con-

tains. The number of significant figures in a properly written number equals the number of digits it contains exclusive of zeros at either end. For instance, the numbers 0.007049, 0.02704, 0.9214, 3.142, 4,368,000 have four significant figures unless otherwise indicated. One or more zeros at the right end of a number may be significant; in such a case either the estimated error should be given or the number should be expressed in powers of 10. Thus, if 4,368,000 is written as 4.368×10^6, 4.3680×10^6, 4.36800×10^6 or 4.368000×10^6, it is supposed to have 4, 5, 6, or 7 significant figures, respectively.

Arithmetical operations on a number do not increase its accuracy. Therefore the number of significant figures of a computed value should be, at most, one more than the number of significant figures in the least accurate number involved in the computation.

9-8 Interpolation. The mantissa of a number which has four or fewer significant figures can be read from Table I. To find the mantissa of a number which has more than four significant figures the method of *interpolation* is used. This method is based on the following considerations.

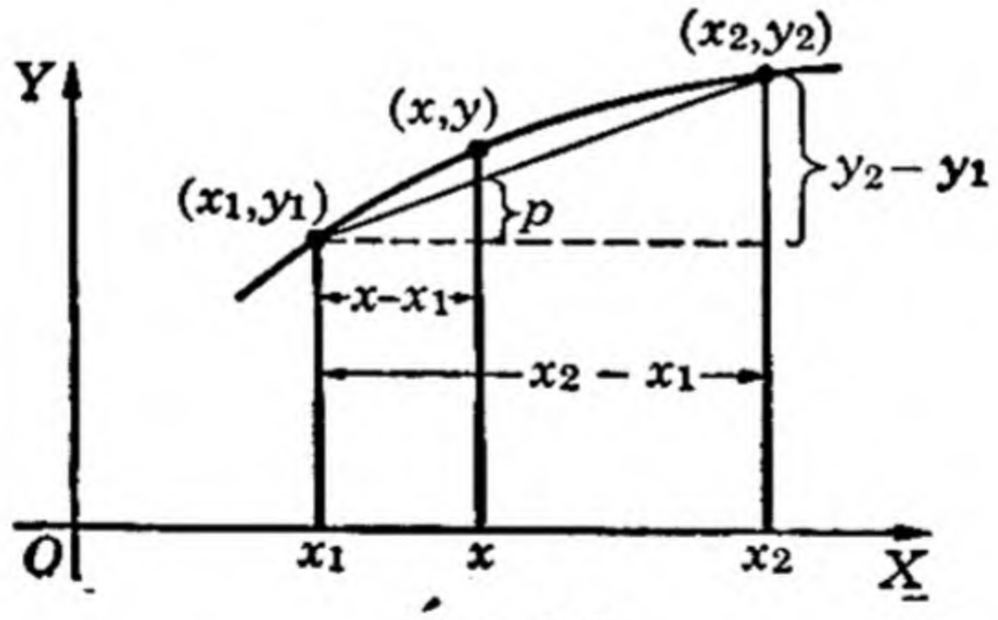


FIG. 9-2

If (x_1,y_1) and (x_2,y_2) are two closely placed points on a curve, Fig. 9-2, the ordinate of an intermediate point (x,y) very nearly equals the ordinate of the point on the secant line whose abscissa is x. Therefore an approximate value of y can be obtained by finding the value of p and adding it to y_1.

Comparing the sides of the two similar right triangles, we have

$$\frac{p}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}, \quad \text{or } p = \frac{x - x_1}{x_2 - x_1}(y_2 - y_1).$$

The number $(y_2 - y_1)$ is called the *tabular difference*, and p the *proportional part* of $(y_2 - y_1)$.

EXAMPLES

1. Find the logarithm of 60.974.

This number lies between the numbers 60.970 and 60.980, and is $\frac{4}{10}$ of the way from the first number to the second. From the table, we get

$$\left.\begin{aligned}\log 60.970 &= 1.78512\\ \log 60.980 &= 1.78519\end{aligned}\right| 7$$

where 7 is the tabular difference. The whole number nearest to the proportional part, $\frac{4}{10} \times 7$, is 3. Adding 3 to the last digit of the first logarithm, we have

$$\log 60.974 = 1.78515.$$

2. Find the *antilogarithm* of $9.38145 - 10$, that is, the number x whose logarithm is $9.38145 - 10$.

The characteristic is $9 - 10$. The mantissa 38145 falls, in Table I, between the mantissas of 24060 and 24070. Therefore we write

$$\left.\begin{aligned}\log 0.24060 &= 9.38130 - 10\\ \log x &= 9.38145 - 10\\ \log 0.24070 &= 9.38148 - 10\end{aligned}\right.\begin{matrix}\Big]15\\ \Big]18\end{matrix}$$

Hence x is about $\frac{15}{18}$ of the way from 0.24060 to 0.24070. Therefore the proportional part to be added is the whole number nearest to $\frac{15}{18} \times 10$. Thus $x = 0.24068$.

Note: In most cases the numbers involved in the computation of the proportional part are small. In such cases the computation can be carried out mentally. But when the numbers involved are relatively large, it is convenient to make use of Table VI.

9-9 Computation by logarithms. The following examples illustrate computation by means of logarithms.

EXAMPLES

1. Find the value of $\sqrt[3]{\dfrac{43.2 \times 0.0370}{226 \times (0.362)^2}}$.

Let x denote the number, N the numerator under the radical and D the denominator. Then

$$\begin{aligned}
\log 43.2 &= 1.63548 \\
\log 0.0370 &= 8.56820 - 10 \\
\log N &= 10.20368 - 10 \\
\log D &= 1.47153 \\
3)\,&28.73215 - 30 \\
\log x &= 9.57738 - 10
\end{aligned}$$

$$\begin{aligned}
\log 226 &= 2.35411 \\
\log 0.362 &= 9.55871 - 10 \\
\log 0.362 &= 9.55871 - 10 \\
\log D &= 1.47153
\end{aligned}$$

$$x = 0.3779$$

Note: The logarithm of the squared number was written twice instead of multiplying it by 2. This procedure makes the work simpler if the logarithm of the square is to be added to other logarithms.

Before dividing the logarithm of N/D by 3, 20 was added to it and subtracted from it in order to avoid having $-\frac{10}{3}$ at the end of the logarithm of x.

2. Find the value of $\dfrac{\sqrt{87.625}\,(-3.4912)^3}{(21.694)^3}$.

Before applying logarithms to this number it is desirable to put it in the form $-(87.625)^{1/2}\left(\dfrac{3.4912}{21.694}\right)^3$.

$$\begin{aligned}
\log (87.625)^{1/2} &= \tfrac{1}{2} \times 1.94262 \\
&= 0.97131 \\
&\quad \underline{7.61989 - 10} \\
&\quad 8.59120 - 10
\end{aligned}$$

$$\begin{aligned}
\log 3.4912 &= 0.54297 \\
\log 21.694 &= \underline{1.33634} \\
&\quad 9.20663 - 10 \\
&\quad \underline{\qquad 3} \\
&\quad 7.61989 - 10
\end{aligned}$$

$$x = -0.039012.$$

Note: Observe that the negative sign was disregarded during the use of logarithms.

An error of 1 in the characteristic of a logarithm makes the corresponding number ten times too large or too small. For this reason, it is desirable to check the result obtained by making a rough estimate of the number. In this example the factor under the root sign is about 9, and the cubed factor is about $-(\frac{1}{8})^3$; therefore a rough estimate gives about -0.04, which is near the computed value.

3. If $(2.74)^x = 0.381$, find x.

Taking the logarithms of both sides, we have

$$\begin{aligned}
x \log 2.74 &= \log 0.381. \\
x &= \frac{\log 0.381}{\log 2.74} \\
&= \frac{9.58092 - 10}{0.43775} \\
&= -\frac{0.41908}{0.43775}.
\end{aligned}$$

$$\begin{aligned}
\log 0.41908 &= 19.62229 - 20 \\
\log 0.43775 &= \underline{9.64123 - 10} \\
\log (-x) &= 9.98106 - 10 \\
-x &= 0.9573
\end{aligned}$$

$$x = -0.9573.$$

9-10 Logarithm to the base *e*. Whenever a logarithm appears in pure mathematics or in the application of mathematics to science, the base is invariably the natural base, e $(= 2.71828\cdots)$. To find the value of a number in which a logarithm to the base e is involved, it is convenient to convert it to a logarithm to the base 10 by the use of the formula

$$\log_e N = 2.3026 \log_{10} N. \tag{9-4}$$

This formula may be obtained as follows. Let

$$x = \log_e N, \text{ then } e^x = N.$$

Taking the logarithm to the base 10, we get

$$x \log_{10} e = \log_{10} N,$$

or
$$x = \frac{\log_{10} N}{\log_{10} e} = 2.3026 \log_{10} N.$$

Hence
$$\log_e N = 2.3026 \log_{10} N.$$

PROBLEMS

Find the values of the following numbers:

1. $(3.1416)^2$.
2. $(0.7859)^2$.
3. $(3.1416)^3$.
4. $(46.125)^3$.
5. $(82.934)^2$.
6. $\sqrt[3]{8.2534}$.
7. $\sqrt{52.231}$.
8. $\sqrt{93.15 \times 0.2761}$.
9. $(36.84)^2 \sqrt{0.0245}$.
10. $\dfrac{25.37 \sqrt{0.07354}}{\sqrt{4.968}}$.
11. $\left(\dfrac{0.463 \times 38.47}{0.037 \times 576}\right)^{3/2}$
12. $\dfrac{(1.576)^3 \sqrt{20.423}}{(3.1416)^3}$.
13. $\dfrac{\sqrt[3]{-7.142}}{6.42\ (0.742)^3}$.
14. $\dfrac{24.51 \sqrt[3]{-27.348}}{72.985}$.
15. $\dfrac{36.3 \sqrt{87.61}}{\sqrt{39.47}}$.
16. $\dfrac{(0.0093141)^2}{(0.7129)^2 \sqrt{3.712}}$.
17. $\sqrt[3]{\dfrac{(3.16)^2\ (-3.241)}{24.63 \times 78.4}}$.
18. $\dfrac{(867.1)^3 \sqrt{10.26}}{(32.14)^3 \sqrt{0.8249}}$.
19. $\dfrac{(6.4545)^2}{(8.9262)^2 \times \sqrt[3]{32.492}}$.
20. $\dfrac{(2.837)^5 \sqrt[3]{73.15}}{(60.13)^5}$.
21. $\dfrac{(2.79)^2 \sqrt{6.073}}{\sqrt{51.97}}$.
22. $\sqrt[3]{\dfrac{-1.3672 \times (24.176)^3}{(0.08345)^2}}$.
23. $\dfrac{(-23.67)^2 \times 0.925 \sqrt{7256}}{(1.2789)^3 \sqrt[3]{92.786}}$.
24. $\dfrac{(4.8179)^2 \sqrt[4]{0.1476}}{0.5219 \sqrt[4]{10.76}}$.

In each of the following problems, find the value of x:

25. $(14.73)^x = 2.316$.
26. $(0.8941)^x = 0.07835$.
27. $(0.21735)^x = 13.945$.
28. $(758.6)^x = 3.165$.

LIST OF FORMULAS

1-1. $$\csc\ \theta = \frac{1}{\sin\ \theta}.$$

1-2. $$\sec\ \theta = \frac{1}{\cos\ \theta}.$$

1-3. $$\cot\ \theta = \frac{1}{\tan\ \theta}.$$

1-4. $$\tan\ \theta = \frac{\sin\ \theta}{\cos\ \theta}.$$

1-5. $$\cot\ \theta = \frac{\cos\ \theta}{\sin\ \theta}.$$

4-1. $$\theta = \frac{s}{r}.$$

4-2. $$A = \tfrac{1}{2}r^2\,\theta.$$

4-3. $$v = r\omega.$$

6-1. $$\sin^2\theta + \cos^2\theta = 1.$$

6-2. $$1 + \tan^2\theta = \sec^2\theta.$$

6-3. $$1 + \cot^2\theta = \csc^2\theta.$$

7-1. $$\sin\ (\alpha + \beta) = \sin\ \alpha\cos\beta + \cos\alpha\sin\beta.$$

7-2. $$\cos\ (\alpha + \beta) = \cos\alpha\cos\beta - \sin\ \alpha\sin\beta.$$

7-3. $$\tan\ (\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}.$$

7-4. $$\sin\ (\alpha - \beta) = \sin\ \alpha\cos\beta - \cos\alpha\sin\beta.$$

7-5. $$\cos\ (\alpha - \beta) = \cos\alpha\cos\beta + \sin\ \alpha\sin\beta.$$

7-6. $$\tan\ (\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}.$$

7-7. $\sin 2\alpha = 2 \sin \alpha \cos \alpha.$

7-8. $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha.$

7-8a. $\cos 2\alpha = 2 \cos^2 \alpha - 1.$

7-8b. $\cos 2\alpha = 1 - 2 \sin^2 \alpha.$

7-9. $\tan 2\alpha = \dfrac{2 \tan \alpha}{1 - \tan^2 \alpha}.$

7-10. $\sin \frac{1}{2}\theta = \pm\sqrt{\dfrac{1 - \cos \theta}{2}}.$

7-11. $\cos \frac{1}{2}\theta = \pm\sqrt{\dfrac{1 + \cos \theta}{2}}.$

7-12. $\tan \frac{1}{2}\theta = \pm\sqrt{\dfrac{1 - \cos \theta}{1 + \cos \theta}}.$

7-12a. $\tan \frac{1}{2}\theta = \dfrac{1 - \cos \theta}{\sin \theta}.$

7-12b. $\tan \frac{1}{2}\theta = \dfrac{\sin \theta}{1 + \cos \theta}.$

7-13. $\sin (\alpha + \beta) + \sin (\alpha - \beta) = 2 \sin \alpha \cos \beta.$

7-14. $\sin (\alpha + \beta) - \sin (\alpha - \beta) = 2 \cos \alpha \sin \beta.$

7-15. $\cos (\alpha + \beta) + \cos (\alpha - \beta) = 2 \cos \alpha \cos \beta.$

7-16. $\cos (\alpha + \beta) - \cos (\alpha - \beta) = -2 \sin \alpha \sin \beta.$

7-17. $\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B).$

7-18. $\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B).$

7-19. $\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B).$

7-20. $\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B).$

8-1. $\dfrac{a}{\sin \alpha} = \dfrac{b}{\sin \beta} = \dfrac{c}{\sin \gamma}.$

8-2. $c^2 = a^2 + b^2 - 2ab \cos \gamma.$

8-3. $$\frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)} = \frac{a - b}{a + b}.$$

8-4. $$A = \sqrt{s(s - a)\ (s - b)\ (s - c)}.$$

8-4′. $$A = rs. \qquad [s = \tfrac{1}{2}(a + b + c)]$$

8-5. $$r = \sqrt{\frac{(s - a)\ (s - b)\ (s - c)}{s}}.$$

8-6a. $$\tan \tfrac{1}{2}\alpha = \frac{r}{s - a}.$$

8-6b. $$\tan \tfrac{1}{2}\beta = \frac{r}{s - b}.$$

8-6c. $$\tan \tfrac{1}{2}\gamma = \frac{r}{s - c}.$$

9-1. $$\log MN = \log M + \log N.$$

9-2. $$\log \frac{M}{N} = \log M - \log N.$$

9-3. $$\log N^p = p \log N.$$

9-4. $$\log_e N = 2.3026 \log_{10} N.$$

INDEX

Abscissa, 3
Absolute value, 34
Addition formulas, 63
Ambiguous case, 76
Amplitude, 50
Angle(s), 1
  circular measure of, 37
  complementary, 9
  coterminal, 2
  initial sides of, 1
  of depression, 18
  of elevation, 18
  of inclination, 18
  positive and negative, 2
  reference, 34
  standard position of, 4
  terminal side of, 1
  units of, 2
  vertex of, 1
Angular velocity, 39
Antilogarithm, 96
Application to physics, 22
Area, of a sector of circle, 39
  of a triangle, 83

Base, of a logarithm, 89
  common, 92
  natural, 92
Bearing, 18

Characteristic of a logarithm, 92
Cofunction, 10
Components of a vector, 23
Coordinate(s), 2
  axes, 3
  origin of, 3
  rectangular, 2
Cosecant, graph of, 48
Cosine(s), graph of, 44
  law of, 73
  period of, 45
  range of, 44
  variation of, 44
Cotangent, graph of, 48
Curve(s), exponential, 91
  geometric addition of, 48
  logarithmic, 91
  symmetric, 53

Degree, 2
Directed distance, 3

Equation(s), 57
  exponential, 89
  of condition, 57
  of definition, 57
  of identity, 57
  quadratic, 61
  trigonometric, 60
Equilibrium, 27
  condition of, 27

Force, frictional, 27
  normal, 27
  tangential, 27
Formula(s), 57
  addition, 63
  double-angle, 67
  half-angle, 67
  list of, 100
  product, 69
  sum, 69
Friction, 27
  coefficient of, 27
Frictional force, 27
Function(s), 5, 52
  exponential, 89
  inverse, 52
  logarithmic, 89
  multiple valued, 55
  natural trigonometric, 15
  of a positive acute angle, 8
  of any angle, 5
  of complementary angles, 9
  of 0° and 90°, 12
  of 30°, 45°, and 60°, 11
  of $(90° \pm \theta)$, $(180° \pm \theta)$, $(270° \pm \theta)$ and $(360° \pm \theta)$, 36
  periodic, 44, 50
  quadratic, 61
  single valued, 55
  trigonometric, 1, 4

Harmonic analyzer, 51

Identities, 57
  trigonometric, 57

Infinity, 14
Interpolation, 95
Inverse functions, 52
  trigonometric, 53
  principal values of, 54

Knot, 19

Latitude, 40
Law(s), of cosines, 73
  of exponents, 90
  of logarithms, 89
  of sines, 72
  of tangents, 83, 85
Light-year, 41
Logarithm(s), 89
  base of, 89
  characteristic of, 93
  common, 93
  mantissa of, 93
  natural, 93, 98
  table of, 94
Longitude, 40

Mantissa, 93
Mil, 2
Motion, in a circle, 39
  simple harmonic, 50

Nautical mile, 19
Number(s), absolute value of, 34
  complex, 90
  imaginary, 90
  numerical value of, 34
  real, 90

Ordinate, 3
Origin, of a vector, 23
  of coordinate axes, 3
Overtones, 51

Parallax, 41
Particle, 27
  equilibrium of a, 27
Parts of a triangle, 17
Period, 44
Periodic phenomena, 50
Principal, angle, 55
  value, 55
Projection, 23
Proportional part, 96
Pythagorean theorem, 73

Quadrants, 3
Quadratic equation, 61

Radian, 2, 38
Radius of inscribed circle, 85
Radius vector, 5
Rectangular coordinates, 2
Reduction, rules, 34
  to positive acute angle, 34
Reference angle, 34
Resultant of vectors, 23

Scalars, 22
Secant, graph of, 48
Sector of a circle, 39
Significant figures, 94
Simple harmonic motion, 50
Sine(s), graph of, 43
  law of, 72
  period of, 44
  range of, 42
  variation of, 42
Standard position, of angles, 4
  of the decimal point, 93
Symmetry, 53

Tabular difference, 96
Tangent(s), first law of, 83
  graph of, 47
  period of, 47
  range of, 46
  second law of, 85
  variation of, 46
Triangle(s), area of, 83
  parts of, 17
  radius of inscribed circle in, 85
  solution of right, 17
  solution of oblique, 72

Variable, dependent, 52
  independent, 52
Vector(s), 22
  analytical addition of, 24
  components of, 23
  geometric addition of, 22
  origin of, 23
  resolution of, 24
  resultant of, 23
  terminus of, 23
Velocity, angular, 39
  linear, 39
  relative, 27, 78

ANSWERS

(Answers to even-numbered problems and to problems which do not require computation or analytical work are not given.)

PAGE 16

9. 0.62169, 9.79357 − 10
11. 0.69765, 9.84363 − 10
13. 0.96309, 9.98366 − 10
15. 7.5214, 0.87630
17. 24° 3.1′
19. 32° 22.5′
21. 56° 6.7′
23. 85° 18.7′
25. 27° 17.3′
27. 36° 51.4′
29. 53° 42.6′
31. 54° 47.5′

PAGES 20-21

1. 5, $5\sqrt{2}$, $12\frac{1}{2}$
3. $5\sqrt{2}$, $5\sqrt{2}$, 25
5. $\sqrt{3}$, $2\sqrt{3}$, $\frac{3}{2}\sqrt{3}$
7. $15\sqrt{3}$, 30, $\frac{225}{2}\sqrt{3}$
9. $2\sqrt{3}$, $4\sqrt{3}$, $6\sqrt{3}$
11. 463.33
13. 57,280
15. 12,895
17. 1.593, 1.424
19. 82.40, 90.00
21. 11.035, 16.17
23. 41.93 ft.
25. 1320 ft.
27. 4198 ft.
29. 16.53 m.
31. $nr^2 \sin \frac{180°}{n} \cos \frac{180°}{n}$
33. 7214 ft.

PAGES 26-27

1. 25.9 $\frac{\text{m.}}{\text{hr.}}$, 96.6 $\frac{\text{m.}}{\text{hr.}}$
3. 249.7 lb., N 70° 8′ E
5. 300 ft.
7. 10.8 $\frac{\text{m.}}{\text{hr.}}$, 21° 48′
9. 45 min. 19 sec.
11. 1.485 P, N 28° 28′ E

PAGES 31-33

1. 55 lb.
3. 192.3 lb., 199.3 lb.
5. 937.8 lb.
7. $F = \frac{a}{\sqrt{a^2 - l^2}} W$, $T = \frac{l}{\sqrt{a^2 - l^2}} W$
9. $F = \frac{W \sin \beta}{\sin \beta \cos \alpha - \cos \beta \sin \alpha}$,
$T = \frac{W \sin \alpha}{\sin \beta \cos \alpha - \cos \beta \sin \alpha}$
11. $F = \frac{W \sin \alpha}{\sin \alpha \cos \beta - \cos \alpha \sin \beta}$,
$T = \frac{W \sin \beta}{\sin \alpha \cos \beta - \cos \alpha \sin \beta}$
13. $N_1 = \frac{W \sin \beta}{\sin \alpha \cos \beta + \cos \alpha \sin \beta}$,
$N_2 = \frac{W \sin \alpha}{\sin \alpha \cos \beta + \cos \alpha \sin \beta}$
15. $W \tan \alpha$

17. $\dfrac{\mu W}{\cos \alpha + \mu \sin \alpha}$

19. $(\sin \alpha - \mu \cos \alpha)W$
Note: When $\alpha = 0$, not only $\sin \alpha = 0$, but also $\mu W \cos \alpha\ (= F) = 0$

21. $\dfrac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha} W$

23. $\dfrac{\sin \alpha - \mu \cos \alpha}{\cos \alpha + \mu \sin \alpha} W$

25. $\dfrac{\sin \alpha + \mu \cos \alpha}{1 + \cos \alpha - \mu \sin \alpha} W$

PAGES 39-41

1. 4.24 in.
3. 13.89 in.
5. 25.03 in.
7. 32.09 in^2.
9. 2803 m.
11. 3295 m.
13. 815.6 m.
15. 34.22 $\dfrac{\text{rad.}}{\text{sec.}}$
5.446 $\dfrac{\text{rev.}}{\text{sec.}}$
17. 18.5 $\dfrac{\text{m.}}{\text{sec.}}$
19. 239,500 m.
21. 4.28 l. yr.

PAGE 56

1. 0°
3. 90°
5. 0°
7. −90°
9. 30°
11. 60°
13. 120°
15. 45°
17. 30°
19. 90°
21. 90°
23. $\frac{1}{2}\sqrt{3}$ or $-\frac{1}{2}\sqrt{3}$
25. $\frac{1}{2}\sqrt{2}$ or $-\frac{1}{2}\sqrt{2}$
27. 1 or −1
29. 0

PAGES 61-62

1. 30°, 150°, 210°, 330°
3. 60°, 120°, 240°, 300°
5. 0°, 240°, 360°
7. 30°, 150°, 210°, 330°
9. 0°, 90°, 180°, 360°
11. 45°, 225°
13. 0°, 120°, 240°, 360°
15. 0°, 120°, 240°, 360°
17. 120°, 240°
19. 90°, 210°, 270°, 330°
21. 0°, 90°, 180°, 270°, 360°
23. 30°, 150°, 270°

PAGE 66

1. $\frac{1}{4}(\sqrt{6} - \sqrt{2})$
3. 1
5. $\frac{1}{4}(\sqrt{6} + \sqrt{2})$
7. $\frac{1}{2}\sqrt{3}$
9. $-\sqrt{3}$
11. $-\frac{1}{4}(\sqrt{6} - \sqrt{2})$
13. $\cos \theta$
15. $-\sin \theta$
17. $-\sin \theta$
19. $-\cos \theta$
21. $\tan \theta$
23. $-\cos \theta$
25. $\sin \theta$
27. $\sin \theta$
29. $\cos \theta$
31. $\tan \theta$

PAGES 70-71

25. 0°, 120°, 180°, 240°, 360°
27. 90°, 210°, 270°, 330°
29. 60°, 180°, 300°
31. 68° 31.9′, 291° 28.1′
33. 30°, 90°, 150°, 270°
35. 0°, 60°, 300°, 360°
37. 102° 39.8′, 257° 20.2′

PAGES 75-76

1. $t = 18.33$,	$c = 25.96$,	$\gamma = 72° 30'$
3. $a = 3641$,	$b = 6284$,	$\gamma = 102°$
5. $a = 0.2718$,	$c = 0.5101$,	$\gamma = 81° 50'$
7. $a = 27.16$,	$b = 54.78$,	$\alpha = 21° 7.7'$
9. $a = 171.83$,	$c = 128.41$,	$\beta = 140° 30.5'$
11. $a = 76.25$,	$b = 91.26$,	$\gamma = 37° 18'$
13. $a = 46.01$,	$c = 37.57$,	$\alpha = 123° 5.2'$
15. $b = 200.0$,	$c = 115.0$,	$\beta = 130° 42.4'$
17. $b = 4.618$,	$c = 1.606$,	$\alpha = 100° 25'$
19. $a = 57.80$,	$c = 58.18$,	$\beta = 36° 56'$
21. $b = 1.0068$,	$c = 2.9604$,	$\alpha = 64° 47.2'$

PAGE 78

1. $c = 750$,	$\beta = 9° 3'$,	$\gamma = 91° 17'$
3. $c = 22.03$,	$\alpha = 47° 24'$,	$\gamma = 76° 11'$
5. $b = 0.309$,	$\alpha = 36° 27'$,	$\beta = 76° 1'$
7. $a = 33.55$,	$\alpha = 51° 37.6'$,	$\beta = 56° 17.4'$
9. $a = 479.15$,	$\alpha = 98° 37.4'$,	$\beta = 12° 28.4'$
11. $c = 882.74$,	$\alpha = 58° 21'$,	$\gamma = 66° 48'$
13. $b = 8.3454$,	$\alpha = 112° 30'$,	$\beta = 46° 58.3'$
15. $a = 6127.3$,	$\alpha = 112° 27.7'$,	$\gamma = 29° 41.9'$
17. $c = 820.96$,	$\alpha = 103° 39.3'$,	$\gamma = 28° 37.2'$
$c' = 1521.2$,	$\alpha' = 76° 20.7'$	$\gamma' = 55° 55.8'$
19. $b = 83.339$,	$\beta = 63° 11.6'$,	$\gamma = 105° 3.3'$
21. $c = 8.6716$,	$\beta = 32° 6.3'$,	$\gamma = 96° 19.1'$

PAGES 79-80

1. $c = 40$,	$\alpha = 28° 56'$,	$\beta = 46° 29'$
3. $c = 19.8$,	$\alpha = 33° 1'$,	$\beta = 100° 59'$
5. $c = 155.6$,	$\alpha = 81° 45'$,	$\beta = 46° 54'$
7. $c = 149.87$,	$\alpha = 43° 30'$,	$\beta = 77° 13'$
9. $c = 8.247$,	$\alpha = 58° 5.1'$,	$\beta = 46° 59.1'$
11. $c = 104.1$,	$\alpha = 79° 6'$,	$\beta = 49° 5'$
13. $\alpha = 57° 7.3'$,	$\beta = 78° 27.8'$,	$\gamma = 44° 24.9'$
15. $\alpha = 59° 21'$,	$\beta = 43° 29.4'$,	$\gamma = 77° 9.7'$
17. $\alpha = 55° 46.4'$,	$\beta = 82° 49.2'$,	$\gamma = 41° 24.4'$
19. $\alpha = 41° 25.5'$,	$\beta = 55° 45.3'$,	$\gamma = 82° 49.2'$
21. $\alpha = 38° 12.8'$,	$\beta = 21° 47.2'$,	$\gamma = 120°$
23. $\alpha = 28° 57.3'$,	$\beta = 46° 34.1'$,	$\gamma = 104° 28.6'$

PAGES 80-81

1. 75 ft.
3. 2047 ft.
5. 128.9 ft.
7. 534.7 ft.
9. 8892 ft.
11. 2215 ft.
13. 3842 ft.
15. 83.3 ft.
17. 3334 ft.
19. 55.29 ft.
21. 111.8 ft.

PAGE 88

1. 55° 41.5′,	60° 40.5′,	29.124
3. 46° 4.4′,	32° 49.0′,	9.104
5. 64° 29.1′,	55° 22.1′,	41.97
7. 38° 7.2′,	56° 26.8′,	0.4102
9. 66° 53′,	56° 33′,	543.7
11. 64° 31.3′,	22° 37.2′,	59.035
13. 110° 49.6′,	40° 20.4′,	173.14
15. 99° 22.4′,	64° 1.5′,	16° 35.6′
17. 44° 7.4′,	58° 38.6′,	77° 14.0′
19. 69° 3.9′,	47° 14.4′,	63° 41.7′
21. 79° 32.5′,	61° 22′,	39° 5.5′, 1228
23. 80° 13.4′,	42° 31.4′,	57° 15.2′, 4160.4
25. 38° 5.2′,	74° 59.4′,	66° 55.4′, 2336.2

PAGE 92

1. $\log p + \log v$
3. $\log p + \log v - \log T$
5. $\log 4 + \log \pi + 3 \log r - \log 3$
7. $\log 2 + \log \pi + \frac{1}{2} (\log l - \log g)$
9. $\frac{1}{2} \log a + 2 \log b - \frac{1}{2} \log c$
11. $\frac{1}{3} \log 13 + \frac{1}{6} \log 17 - \log 15 - \frac{1}{2} \log 2$
13. $\log a + 2 \log (b - c) - \log c - \frac{1}{2} \log (a - b)$
15. $\log \dfrac{ab^2}{c}$
17. $\log ab^2$
19. $\log \dfrac{a - b}{a + b}$
21. $\log 75$
23. $\log 8\sqrt{2}$

PAGE 98

1. 9.6960
3. 31.074
5. 6878.0
7. 7.2270
9. 212.43
11. 0.76405
13. −0.7343
15. 54.08
17. −0.2559
19. 0.16386
21. 2.661
23. 4661.7

LIST OF TABLES

TABLE		PAGE
I.	Common Logarithms of Numbers	1
II.	Logarithms of Constants	20
III.	Logarithms of Trigonometric Functions	21
IV.	Natural Trigonometric Functions	50
V.	Powers, Roots, Reciprocals	74
VI.	Proportional Parts of Differences	77
VII.	Conversion of Degrees to Radians	80
VIII.	Conversion of Minutes to Radians	80
IX.	Conversion of Radians to Degrees and Minutes	80
X.	Conversion of Minutes to Decimal Parts of a Degree	80
XI.	Natural Haversines and their Logarithms	81

These tables are reproduced from *Plane Trigonometry* by Ballou and Steen with the permission of the authors and of Ginn and Company.

No use is made of Table XI in this book. The haversine and certain allied trigonometric functions are defined as follows:

$$\text{versed sine } \theta \text{ (vers } \theta) = 1 - \cos\theta,$$
$$\text{coversed sine } \theta \text{ (covers } \theta \text{ or cov } \theta) = 1 - \sin\theta,$$
$$\text{haversine } \theta \text{ (havers } \theta \text{ or hav } \theta) = \tfrac{1}{2}(1 - \cos\theta).$$
$$\text{external secant } \theta \text{ (exsec } \theta) = \sec\theta - 1.$$

TABLE I

COMMON LOGARITHMS OF FUNCTIONS

Table I gives the mantissas of the logarithms of all numbers with four digits, exclusive of zeros at either end. An underlined figure 5 indicates that the figure in the fifth decimal place was 4 and the next figure was 5 or more, and that the two figures were rounded off to 5.

100–150

N	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	377	412	447	482	517	552	587	621	656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	727	760	793	826	860	893	926	959	992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	333	364	395	426	457	489	520	551	582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	922	953	983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	319	348	377	406	435	464	493	522	551	580
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
N	0	1	2	3	4	5	6	7	8	9

150–200

N	0	1	2	3	4	5	6	7	8	9
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	898	926	955	984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	276	303	330	358	385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	254	279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	261
179	285	310	334	358	382	406	431	455	479	503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	951	975	998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	30 016	30 038	30 060	30 081
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
N	0	1	2	3	4	5	6	7	8	9

200–250

N	0	1	2	3	4	5	6	7	8	9
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	203	224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	203	223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
N	0	1	2	3	4	5	6	7	8	9

250–300

N	0	1	2	3	4	5	6	7	8	9
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
251	967	985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	824	841	858	875	892	909	926	943	960	976
257	993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	42 012	42 029	42 045	42 062	42 078	42 095	42 111	42 127	42 144
264	42 160	177	193	210	226	243	259	275	292	308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
270	43 136	43 152	43 169	43 185	43 201	43 217	43 233	43 249	43 265	43 281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	44 122	44 138	44 154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	44 716	44 731	44 747	44 762	44 778	44 793	44 809	44 824	44 840	44 855
281	871	886	902	917	932	948	963	979	994	45 010
282	45 025	45 040	45 056	45 071	45 086	45 102	45 117	45 133	45 148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	150	165	180	195	210	225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
N	0	1	2	3	4	5	6	7	8	9

300–350

N	0	1	2	3	4	5	6	7	8	9
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	49 010	49 024	49 038	49 052	49 066	49 080	49 094	49 108	49 122
310	49 136	49 150	49 164	49 178	49 192	49 206	49 220	49 234	49 248	49 262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	49 831	49 845	49 859	49 872	49 886	49 900	49 914	49 927	49 941	49 955
316	969	982	996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	50 515	50 529	50 542	50 556	50 569	50 583	50 596	50 610	50 623	50 637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	51 001	51 014	51 028	51 041
324	51 055	51 068	51 081	51 095	51 108	51 121	135	148	162	175
325	51 188	51 202	51 215	51 228	51 242	51 255	51 268	51 282	51 295	51 308
326	322	335	348	362	375	388	402	415	428	441
327	455	468	481	495	508	521	534	548	561	574
328	587	601	614	627	640	654	667	680	693	706
329	720	733	746	759	772	786	799	812	825	838
330	51 851	51 865	51 878	51 891	51 904	51 917	51 930	51 943	51 957	51 970
331	983	996	52 009	52 022	52 035	52 048	52 061	52 075	52 088	52 101
332	52 114	52 127	140	153	166	179	192	205	218	231
333	244	257	270	284	297	310	323	336	349	362
334	375	388	401	414	427	440	453	466	479	492
335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
336	634	647	660	673	686	699	711	724	737	750
337	763	776	789	802	815	827	840	853	866	879
338	892	905	917	930	943	956	969	982	994	53 007
339	53 020	53 033	53 046	53 058	53 071	53 084	53 097	53 110	53 122	135
340	53 148	53 161	53 173	53 186	53 199	53 212	53 224	53 237	53 250	53 263
341	275	288	301	314	326	339	352	364	377	390
342	403	415	428	441	453	466	479	491	504	517
343	529	542	555	567	580	593	605	618	631	643
344	656	668	681	694	706	719	732	744	757	769
345	53 782	53 794	53 807	53 820	53 832	53 845	53 857	53 870	53 882	53 895
346	908	920	933	945	958	970	983	995	54 008	54 020
347	54 033	54 045	54 058	54 070	54 083	54 095	54 108	54 120	133	145
348	158	170	183	195	208	220	233	245	258	270
349	283	295	307	320	332	345	357	370	382	394
350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518
N	0	1	2	3	4	5	6	7	8	9

350–400

N	0	1	2	3	4	5	6	7	8	9
350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518
351	531	543	555	568	580	593	605	617	630	642
352	654	667	679	691	704	716	728	741	753	765
353	777	790	802	814	827	839	851	864	876	888
354	900	913	925	937	949	962	974	986	998	55 011
355	55 023	55 035	55 047	55 060	55 072	55 084	55 096	55 108	55 121	55 133
356	145	157	169	182	194	206	218	230	242	255
357	267	279	291	303	315	328	340	352	364	376
358	388	400	413	425	437	449	461	473	485	497
359	509	522	534	546	558	570	582	594	606	618
360	55 630	55 642	55 654	55 666	55 678	55 691	55 703	55 715	55 727	55 739
361	751	763	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	967	979
363	991	56 003	56 015	56 027	56 038	56 050	56 062	56 074	56 086	56 098
364	56 110	122	134	146	158	170	182	194	205	217
365	56 229	56 241	56 253	56 265	56 277	56 289	56 301	56 312	56 324	56 336
366	348	360	372	384	396	407	419	431	443	455
367	467	478	490	502	514	526	538	549	561	573
368	585	597	608	620	632	644	656	667	679	691
369	703	714	726	738	750	761	773	785	797	808
370	56 820	56 832	56 844	56 855	56 867	56 879	56 891	56 902	56 914	56 926
371	937	949	961	972	984	996	57 008	57 019	57 031	57 043
372	57 054	57 066	57 078	57 089	57 101	57 113	124	136	148	159
373	171	183	194	206	217	229	241	252	264	276
374	287	299	310	322	334	345	357	368	380	392
375	57 403	57 415	57 426	57 438	57 449	57 461	57 473	57 484	57 496	57 507
376	519	530	542	553	565	576	588	600	611	623
377	634	646	657	669	680	692	703	715	726	738
378	749	761	772	784	795	807	818	830	841	852
379	864	875	887	898	910	921	933	944	955	967
380	57 978	57 990	58 001	58 013	58 024	58 035	58 047	58 058	58 070	58 081
381	58 092	58 104	115	127	138	149	161	172	184	195
382	206	218	229	240	252	263	274	286	297	309
383	320	331	343	354	365	377	388	399	410	422
384	433	444	456	467	478	490	501	512	524	535
385	58 546	58 557	58 569	58 580	58 591	58 602	58 614	58 625	58 636	58 647
386	659	670	681	692	704	715	726	737	749	760
387	771	782	794	805	816	827	838	850	861	872
388	883	894	906	917	928	939	950	961	973	984
389	995	59 006	59 017	59 028	59 040	59 051	59 062	59 073	59 084	59 095
390	59 106	59 118	59 129	59 140	59 151	59 162	59 173	59 184	59 195	59 207
391	218	229	240	251	262	273	284	295	306	318
392	329	340	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	506	517	528	539
394	550	561	572	583	594	605	616	627	638	649
395	59 660	59 671	59 682	59 693	59 704	59 715	59 726	59 737	59 748	59 759
396	770	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	934	945	956	966	977
398	988	999	60 010	60 021	60 032	60 043	60 054	60 065	60 076	60 086
399	60 097	60 108	119	130	141	152	163	173	184	195
400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304
N	0	1	2	3	4	5	6	7	8	9

400–450

N	0	1	2	3	4	5	6	7	8	9
400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	735
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
406	853	863	874	885	895	906	917	927	938	949
407	959	970	981	991	61 002	61 013	61 023	61 034	61 045	61 055
408	61 066	61 077	61 087	61 098	109	119	130	140	151	162
409	172	183	194	204	215	225	236	247	257	268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	384	395	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	61 805	61 815	61 826	61 836	61 847	61 857	61 868	61 878	61 888	61 899
416	909	920	930	941	951	962	972	982	993	62 003
417	62 014	62 024	62 034	62 045	62 055	62 066	62 076	62 086	62 097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	655	665	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
426	941	951	961	972	982	992	63 002	63 012	63 022	63 033
427	63 043	63 053	63 063	63 073	63 083	63 094	104	114	124	134
428	144	155	165	175	185	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	63 347	63 357	63 367	63 377	63 387	63 397	63 407	63 417	63 428	63 438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	63 849	63 859	63 869	63 879	63 889	63 899	63 909	63 919	63 929	63 939
436	949	959	969	979	988	998	64 008	64 018	64 028	64 038
437	64 048	64 058	64 068	64 078	64 088	64 098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	64 345	64 355	64 365	64 375	64 385	64 395	64 404	64 414	64 424	64 434
441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	64 836	64 846	64 856	64 865	64 875	64 885	64 895	64 904	64 914	64 924
446	933	943	953	963	972	982	992	65 002	65 011	65 021
447	65 031	65 040	65 050	65 060	65 070	65 079	65 089	099	108	118
448	128	137	147	157	167	176	186	196	205	215
449	225	234	244	254	263	273	283	292	302	312
450	65 321	65 331	65 341	65 350	65 360	65 369	65 379	65 389	65 398	65 408
N	0	1	2	3	4	5	6	7	8	9

450–500

N	0	1	2	3	4	5	6	7	8	9
450	65 321	65 331	65 341	65 350	65 360	65 369	65 379	65 389	65 398	65 408
451	418	427	437	447	456	466	475	485	495	504
452	514	523	533	543	552	562	571	581	591	600
453	610	619	629	639	648	658	667	677	686	696
454	706	715	725	734	744	753	763	772	782	792
455	65 801	65 811	65 820	65 830	65 839	65 849	65 858	65 868	65 877	65 887
456	896	906	916	925	935	944	954	963	973	982
457	992	66 001	66 011	66 020	66 030	66 039	66 049	66 058	66 068	66 077
458	66 087	096	106	115	124	134	143	153	162	172
459	181	191	200	210	219	229	238	247	257	266
460	66 276	66 285	66 295	66 304	66 314	66 323	66 332	66 342	66 351	66 361
461	370	380	389	398	408	417	427	436	445	455
462	464	474	483	492	502	511	521	530	539	549
463	558	567	577	586	596	605	614	624	633	642
464	652	661	671	680	689	699	708	717	727	736
465	66 745	66 755	66 764	66 773	66 783	66 792	66 801	66 811	66 820	66 829
466	839	848	857	867	876	885	894	904	913	922
467	932	941	950	960	969	978	987	997	67 006	67 015
468	67 025	67 034	67 043	67 052	67 062	67 071	67 080	67 089	099	108
469	117	127	136	145	154	164	173	182	191	201
470	67 210	67 219	67 228	67 237	67 247	67 256	67 265	67 274	67 284	67 293
471	302	311	321	330	339	348	357	367	376	385
472	394	403	413	422	431	440	449	459	468	477
473	486	495	504	514	523	532	541	550	560	569
474	578	587	596	605	614	624	633	642	651	660
475	67 669	67 679	67 688	67 697	67 706	67 715	67 724	67 733	67 742	67 752
476	761	770	779	788	797	806	815	825	834	843
477	852	861	870	879	888	897	906	916	925	934
478	943	952	961	970	979	988	997	68 006	68 015	68 024
479	68 034	68 043	68 052	68 061	68 070	68 079	68 088	097	106	115
480	68 124	68 133	68 142	68 151	68 160	68 169	68 178	68 187	68 196	68 205
481	215	224	233	242	251	260	269	278	287	296
482	305	314	323	332	341	350	359	368	377	386
483	395	404	413	422	431	440	449	458	467	476
484	485	494	502	511	520	529	538	547	556	565
485	68 574	68 583	68 592	68 601	68 610	68 619	68 628	68 637	68 646	68 655
486	664	673	681	690	699	708	717	726	735	744
487	753	762	771	780	789	797	806	815	824	833
488	842	851	860	869	878	886	895	904	913	922
489	931	940	949	958	966	975	984	993	69 002	69 011
490	69 020	69 028	69 037	69 046	69 055	69 064	69 073	69 082	69 090	69 099
491	108	117	126	135	144	152	161	170	179	188
492	197	205	214	223	232	241	249	258	267	276
493	285	294	302	311	320	329	338	346	355	364
494	373	381	390	399	408	417	425	434	443	452
495	69 461	69 469	69 478	69 487	69 496	69 504	69 513	69 522	69 531	69 539
496	548	557	566	574	583	592	601	609	618	627
497	636	644	653	662	671	679	688	697	705	714
498	723	732	740	749	758	767	775	784	793	801
499	810	819	827	836	845	854	862	871	880	888
500	69 897	69 906	69 914	69 923	69 932	69 940	69 949	69 958	69 966	69 975
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

500–550

N	0	1	2	3	4	5	6	7	8	9
500	69 897	69 906	69 914	69 923	69 932	69 940	69 949	69 958	69 966	69 975
501	984	992	70 001	70 010	70 018	70 027	70 036	70 044	70 053	70 062
502	70 070	70 079	088	096	105	114	122	131	140	148
503	157	165	174	183	191	200	209	217	226	234
504	243	252	260	269	278	286	295	303	312	321
505	70 329	70 338	70 346	70 355	70 364	70 372	70 381	70 389	70 398	70 406
506	415	424	432	441	449	458	467	475	484	492
507	501	509	518	526	535	544	552	561	569	578
508	586	595	603	612	621	629	638	646	655	663
509	672	680	689	697	706	714	723	731	740	749
510	70 757	70 766	70 774	70 783	70 791	70 800	70 808	70 817	70 825	70 834
511	842	851	859	868	876	885	893	902	910	919
512	927	935	944	952	961	969	978	986	995	71 003
513	71 012	71 020	71 029	71 037	71 046	71 054	71 063	71 071	71 079	088
514	096	105	113	122	130	139	147	155	164	172
515	71 181	71 189	71 198	71 206	71 214	71 223	71 231	71 240	71 248	71 257
516	265	273	282	290	299	307	315	324	332	341
517	349	357	366	374	383	391	399	408	416	425
518	433	441	450	458	466	475	483	492	500	508
519	517	525	533	542	550	559	567	575	584	592
520	71 600	71 609	71 617	71 625	71 634	71 642	71 650	71 659	71 667	71 675
521	684	692	700	709	717	725	734	742	750	759
522	767	775	784	792	800	809	817	825	834	842
523	850	858	867	875	883	892	900	908	917	925
524	933	941	950	958	966	975	983	991	999	72 008
525	72 016	72 024	72 032	72 041	72 049	72 057	72 066	72 074	72 082	72 090
526	099	107	115	123	132	140	148	156	165	173
527	181	189	198	206	214	222	230	239	247	255
528	263	272	280	288	296	304	313	321	329	337
529	346	354	362	370	378	387	395	403	411	419
530	72 428	72 436	72 444	72 452	72 460	72 469	72 477	72 485	72 493	72 501
531	509	518	526	534	542	550	558	567	575	583
532	591	599	607	616	624	632	640	648	656	665
533	673	681	689	697	705	713	722	730	738	746
534	754	762	770	779	787	795	803	811	819	827
535	72 835	72 843	72 852	72 860	72 868	72 876	72 884	72 892	72 900	72 908
536	916	925	933	941	949	957	965	973	981	989
537	997	73 006	73 014	73 022	73 030	73 038	73 046	73 054	73 062	73 070
538	73 078	086	094	102	111	119	127	135	143	151
539	159	167	175	183	191	199	207	215	223	231
540	73 239	73 247	73 255	73 263	73 272	73 280	73 288	73 296	73 304	73 312
541	320	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	616	624	632
545	73 640	73 648	73 656	73 664	73 672	73 679	73 687	73 695	73 703	73 711
546	719	727	735	743	751	759	767	775	783	791
547	799	807	815	823	830	838	846	854	862	870
548	878	886	894	902	910	918	926	933	941	949
549	957	965	973	981	989	997	74 005	74 013	74 020	74 028
550	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107
N	0	1	2	3	4	5	6	7	8	9

550–600

N	0	1	2	3	4	5	6	7	8	9
550	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107
551	115	123	131	139	147	155	162	170	178	186
552	194	202	210	218	225	233	241	249	257	265
553	273	280	288	296	304	312	320	327	335	343
554	351	359	367	374	382	390	398	406	414	421
555	74 429	74 437	74 445	74 453	74 461	74 468	74 476	74 484	74 492	74 500
556	507	515	523	531	539	547	554	562	570	578
557	586	593	601	609	617	624	632	640	648	656
558	663	671	679	687	695	702	710	718	726	733
559	741	749	757	764	772	780	788	796	803	811
560	74 819	74 827	74 834	74 842	74 850	74 858	74 865	74 873	74 881	74 889
561	896	904	912	920	927	935	943	950	958	966
562	974	981	989	997	75 005	75 012	75 020	75 028	75 035	75 043
563	75 051	75 059	75 066	75 074	082	089	097	105	113	120
564	128	136	143	151	159	166	174	182	189	197
565	75 205	75 213	75 220	75 228	75 236	75 243	75 251	75 259	75 266	75 274
566	282	289	297	305	312	320	328	335	343	351
567	358	366	374	381	389	397	404	412	420	427
568	435	442	450	458	465	473	481	488	496	504
569	511	519	526	534	542	549	557	565	572	580
570	75 587	75 595	75 603	75 610	75 618	75 626	75 633	75 641	75 648	75 656
571	664	671	679	686	694	702	709	717	724	732
572	740	747	755	762	770	778	785	793	800	808
573	815	823	831	838	846	853	861	868	876	884
574	891	899	906	914	921	929	937	944	952	959
575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
576	76 042	76 050	76 057	76 065	76 072	080	087	095	103	110
577	118	125	133	140	148	155	163	170	178	185
578	193	200	208	215	223	230	238	245	253	260
579	268	275	283	290	298	305	313	320	328	335
580	76 343	76 350	76 358	76 365	76 373	76 380	76 388	76 395	76 403	76 410
581	418	425	433	440	448	455	462	470	477	485
582	492	500	507	515	522	530	537	545	552	559
583	567	574	582	589	597	604	612	619	626	634
584	641	649	656	664	671	678	686	693	701	708
585	76 716	76 723	76 730	76 738	76 745	76 753	76 760	76 768	76 775	76 782
586	790	797	805	812	819	827	834	842	849	856
587	864	871	879	886	893	901	908	916	923	930
588	938	945	953	960	967	975	982	989	997	77 004
589	77 012	77 019	77 026	77 034	77 041	77 048	77 056	77 063	77 070	078
590	77 085	77 093	77 100	77 107	77 115	77 122	77 129	77 137	77 144	77 151
591	159	166	173	181	188	195	203	210	217	225
592	232	240	247	254	262	269	276	283	291	298
593	305	313	320	327	335	342	349	357	364	371
594	379	386	393	401	408	415	422	430	437	444
595	77 452	77 459	77 466	77 474	77 481	77 488	77 495	77 503	77 510	77 517
596	525	532	539	546	554	561	568	576	583	590
597	597	605	612	619	627	634	641	648	656	663
598	670	677	685	692	699	706	714	721	728	735
599	743	750	757	764	772	779	786	793	801	808
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
N	0	1	2	3	4	5	6	7	8	9

550–600

600–650

N	0	1	2	3	4	5	6	7	8	9
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
601	887	895	902	909	916	924	931	938	945	952
602	960	967	974	981	988	996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	075	082	089	097
604	104	111	118	125	132	140	147	154	161	168
605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	247	254	262	269	276	283	290	297	305	312
607	319	326	333	340	347	355	362	369	376	383
608	390	398	405	412	419	426	433	440	447	455
609	462	469	476	483	490	497	504	512	519	526
610	78 533	78 540	78 547	78 554	78 561	78 569	78 576	78 583	78 590	78 597
611	604	611	618	625	633	640	647	654	661	668
612	675	682	689	696	704	711	718	725	732	739
613	746	753	760	767	774	781	789	796	803	810
614	817	824	831	838	845	852	859	866	873	880
615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	958	965	972	979	986	993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	071	078	085	092
618	099	106	113	120	127	134	141	148	155	162
619	169	176	183	190	197	204	211	218	225	232
620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	309	316	323	330	337	344	351	358	365	372
622	379	386	393	400	407	414	421	428	435	442
623	449	456	463	470	477	484	491	498	505	511
624	518	525	532	539	546	553	560	567	574	581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	657	664	671	678	685	692	699	706	713	720
627	727	734	741	748	754	761	768	775	782	789
628	796	803	810	817	824	831	837	844	851	858
629	865	872	879	886	893	900	906	913	920	927
630	79 934	79 941	79 948	79 955	79 962	79 969	79 975	79 982	79 989	79 996
631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
632	072	079	085	092	099	106	113	120	127	134
633	140	147	154	161	168	175	182	188	195	202
634	209	216	223	229	236	243	250	257	264	271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	346	353	359	366	373	380	387	393	400	407
637	414	421	428	434	441	448	455	462	468	475
638	482	489	496	502	509	516	523	530	536	543
639	550	557	564	570	577	584	591	598	604	611
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	686	693	699	706	713	720	726	733	740	747
642	754	760	767	774	781	787	794	801	808	814
643	821	828	835	841	848	855	862	868	875	882
644	889	895	902	909	916	922	929	936	943	949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	070	077	084
647	090	097	104	111	117	124	131	137	144	151
648	158	164	171	178	184	191	198	204	211	218
649	224	231	238	245	251	258	265	271	278	285
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
N	0	1	2	3	4	5	6	7	8	9

650–700

N	0	1	2	3	4	5	6	7	8	9
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
651	358	365	371	378	385	391	398	405	411	418
652	425	431	438	445	451	458	465	471	478	485
653	491	498	505	511	518	525	531	538	544	551
654	558	564	571	578	584	591	598	604	611	617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	690	697	704	710	717	723	730	737	743	750
657	757	763	770	776	783	790	796	803	809	816
658	823	829	836	842	849	856	862	869	875	882
659	889	895	902	908	915	921	928	935	941	948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	066	073	079
662	086	092	099	105	112	119	125	132	138	145
663	151	158	164	171	178	184	191	197	204	210
664	217	223	230	236	243	249	256	263	269	276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	347	354	360	367	373	380	387	393	400	406
667	413	419	426	432	439	445	452	458	465	471
668	478	484	491	497	504	510	517	523	530	536
669	543	549	556	562	569	575	582	588	595	601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	672	679	685	692	698	705	711	718	724	730
672	737	743	750	756	763	769	776	782	789	795
673	802	808	814	821	827	834	840	847	853	860
674	866	872	879	885	892	898	905	911	918	924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	065	072	078	085	091	097	104	110	117
678	123	129	136	142	149	155	161	168	174	181
679	187	193	200	206	213	219	225	232	238	245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	315	321	327	334	340	347	353	359	366	372
682	378	385	391	398	404	410	417	423	429	436
683	442	448	455	461	467	474	480	487	493	499
684	506	512	518	525	531	537	544	550	556	563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	632	639	645	651	658	664	670	677	683	689
687	696	702	708	715	721	727	734	740	746	753
688	759	765	771	778	784	790	797	803	809	816
689	822	828	835	841	847	853	860	866	872	879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	948	954	960	967	973	979	985	992	998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	067
693	073	080	086	092	098	105	111	117	123	130
694	136	142	148	155	161	167	173	180	186	192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	261	267	273	280	286	292	298	305	311	317
697	323	330	336	342	348	354	361	367	373	379
698	386	392	398	404	410	417	423	429	435	442
699	448	454	460	466	473	479	485	491	497	504
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
N	0	1	2	3	4	5	6	7	8	9

700–750

N	0	1	2	3	4	5	6	7	8	9
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	572	578	584	590	597	603	609	615	621	628
702	634	640	646	652	658	665	671	677	683	689
703	696	702	708	714	720	726	733	739	745	751
704	757	763	770	776	782	788	794	800	807	813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	985	991	997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	065	071	077	083	089	095	101	107	114	120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	187	193	199	205	211	217	224	230	236	242
712	248	254	260	266	272	278	285	291	297	303
713	309	315	321	327	333	339	345	352	358	364
714	370	376	382	388	394	400	406	412	418	425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	491	497	503	509	516	522	528	534	540	546
717	552	558	564	570	576	582	588	594	600	606
718	612	618	625	631	637	643	649	655	661	667
719	673	679	685	691	697	703	709	715	721	727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	794	800	806	812	818	824	830	836	842	848
722	854	860	866	872	878	884	890	896	902	908
723	914	920	926	932	938	944	950	956	962	968
724	974	980	986	992	998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	094	100	106	112	118	124	130	136	141	147
727	153	159	165	171	177	183	189	195	201	207
728	213	219	225	231	237	243	249	255	261	267
729	273	279	285	291	297	303	308	314	320	326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	392	398	404	410	415	421	427	433	439	445
732	451	457	463	469	475	481	487	493	499	504
733	510	516	522	528	534	540	546	552	558	564
734	570	576	581	587	593	599	605	611	617	623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	688	694	700	705	711	717	723	729	735	741
737	747	753	759	764	770	776	782	788	794	800
738	806	812	817	823	829	835	841	847	853	859
739	864	870	876	882	888	894	900	906	911	917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	982	988	994	999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	064	070	075	081	087	093
743	099	105	111	116	122	128	134	140	146	151
744	157	163	169	175	181	186	192	198	204	210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	274	280	286	291	297	303	309	315	320	326
747	332	338	344	349	355	361	367	373	379	384
748	390	396	402	408	413	419	425	431	437	442
749	448	454	460	466	471	477	483	489	495	500
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
N	0	1	2	3	4	5	6	7	8	9

750–800

N	0	1	2	3	4	5	6	7	8	9
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	564	570	576	581	587	593	599	604	610	616
752	622	628	633	639	645	651	656	662	668	674
753	679	685	691	697	703	708	714	720	726	731
754	737	743	749	754	760	766	772	777	783	789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	852	858	864	869	875	881	887	892	898	904
757	910	915	921	927	933	938	944	950	955	961
758	967	973	978	984	990	996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	058	064	070	076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	138	144	150	156	161	167	173	178	184	190
762	195	201	207	213	218	224	230	235	241	247
763	252	258	264	270	275	281	287	292	298	304
764	309	315	321	326	332	338	343	349	355	360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	423	429	434	440	446	451	457	463	468	474
767	480	485	491	497	502	508	513	519	525	530
768	536	542	547	553	559	564	570	576	581	587
769	593	598	604	610	615	621	627	632	638	643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	705	711	717	722	728	734	739	745	750	756
772	762	767	773	779	784	790	795	801	807	812
773	818	824	829	835	840	846	852	857	863	868
774	874	880	885	891	897	902	908	913	919	925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	986	992	997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	059	064	070	076	081	087	092
778	098	104	109	115	120	126	131	137	143	148
779	154	159	165	170	176	182	187	193	198	204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	265	271	276	282	287	293	298	304	310	315
782	321	326	332	337	343	348	354	360	365	371
783	376	382	387	393	398	404	409	415	421	426
784	432	437	443	448	454	459	465	470	476	481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	542	548	553	559	564	570	575	581	586	592
787	597	603	609	614	620	625	631	636	642	647
788	653	658	664	669	675	680	686	691	697	702
789	708	713	719	724	730	735	741	746	752	757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	818	823	829	834	840	845	851	856	862	867
792	873	878	883	889	894	900	905	911	916	922
793	927	933	938	944	949	955	960	966	971	977
794	982	988	993	998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	091	097	102	108	113	119	124	129	135	140
797	146	151	157	162	168	173	179	184	189	195
798	200	206	211	217	222	227	233	238	244	249
799	255	260	266	271	276	282	287	293	298	304
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
N	0	1	2	3	4	5	6	7	8	9

800–850

N	0	1	2	3	4	5	6	7	8	9
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	363	369	374	380	385	390	396	401	407	412
802	417	423	428	434	439	445	450	455	461	466
803	472	477	482	488	493	499	504	509	515	520
804	526	531	536	542	547	553	558	563	569	574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	634	639	644	650	655	660	666	671	677	682
807	687	693	698	703	709	714	720	725	730	736
808	741	747	752	757	763	768	773	779	784	789
809	795	800	806	811	816	822	827	832	838	843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	902	907	913	918	924	929	934	940	945	950
812	956	961	966	972	977	982	988	993	998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	057
814	062	068	073	078	084	089	094	100	105	110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	169	174	180	185	190	196	201	206	212	217
817	222	228	233	238	243	249	254	259	265	270
818	275	281	286	291	297	302	307	312	318	323
819	328	334	339	344	350	355	360	365	371	376
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	434	440	445	450	455	461	466	471	477	482
822	487	492	498	503	508	514	519	524	529	535
823	540	545	551	556	561	566	572	577	582	587
824	593	598	603	609	614	619	624	630	635	640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	698	703	709	714	719	724	730	735	740	745
827	751	756	761	766	772	777	782	787	793	798
828	803	808	814	819	824	829	834	840	845	850
829	855	861	866	871	876	882	887	892	897	903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	960	965	971	976	981	986	991	997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	054	059
833	065	070	075	080	085	091	096	101	106	111
834	117	122	127	132	137	143	148	153	158	163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	221	226	231	236	241	247	252	257	262	267
837	273	278	283	288	293	298	304	309	314	319
838	324	330	335	340	345	350	355	361	366	371
839	376	381	387	392	397	402	407	412	418	423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	480	485	490	495	500	505	511	516	521	526
842	531	536	542	547	552	557	562	567	572	578
843	583	588	593	598	603	609	614	619	624	629
844	634	639	645	650	655	660	665	670	675	681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	737	742	747	752	758	763	768	773	778	783
847	788	793	799	804	809	814	819	824	829	834
848	840	845	850	855	860	865	870	875	881	886
849	891	896	901	906	911	916	921	927	932	937
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
N	0	1	2	3	4	5	6	7	8	9

850–900

N	0	1	2	3	4	5	6	7	8	9
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	993	998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	054	059	064	069	075	080	085	090
853	095	100	105	110	115	120	125	131	136	141
854	146	151	156	161	166	171	176	181	186	192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	247	252	258	263	268	273	278	283	288	293
857	298	303	308	313	318	323	328	334	339	344
858	349	354	359	364	369	374	379	384	389	394
859	399	404	409	414	420	425	430	435	440	445
860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	500	505	510	515	520	526	531	536	541	546
862	551	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	682	687	692	697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	052	057	062	067	072	077	082	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	250	255	260	265	270	275	280	285	290	295
877	300	305	310	315	320	325	330	335	340	345
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	414	419	424	429	433	438	443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	498	503	507	512	517	522	527	532	537	542
882	547	552	557	562	567	571	576	581	586	591
883	596	601	606	611	616	621	626	630	635	640
884	645	650	655	660	665	670	675	680	685	689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	743	748	753	758	763	768	773	778	783	787
887	792	797	802	807	812	817	822	827	832	836
888	841	846	851	856	861	866	871	876	880	885
889	890	895	900	905	910	915	919	924	929	934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	988	993	998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	051	056	061	066	071	075	080
893	085	090	095	100	105	109	114	119	124	129
894	134	139	143	148	153	158	163	168	173	177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	231	236	240	245	250	255	260	265	270	274
897	279	284	289	294	299	303	308	313	318	323
898	328	332	337	342	347	352	357	361	366	371
899	376	381	386	390	395	400	405	410	415	419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
N	0	1	2	3	4	5	6	7	8	9

900–950

N	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	472	477	482	487	492	497	501	506	511	516
902	521	525	530	535	540	545	550	554	559	564
903	569	574	578	583	588	593	598	602	607	612
904	617	622	626	631	636	641	646	650	655	660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	713	718	722	727	732	737	742	746	751	756
907	761	766	770	775	780	785	789	794	799	804
908	809	813	818	823	828	832	837	842	847	852
909	856	861	866	871	875	880	885	890	895	899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	952	957	961	966	971	976	980	985	990	995
912	999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	052	057	061	066	071	076	080	085	090
914	095	099	104	109	114	118	123	128	133	137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	190	194	199	204	209	213	218	223	227	232
917	237	242	246	251	256	261	265	270	275	280
918	284	289	294	298	303	308	313	317	322	327
919	332	336	341	346	350	355	360	365	369	374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	426	431	435	440	445	450	454	459	464	468
922	473	478	483	487	492	497	501	506	511	515
923	520	525	530	534	539	544	548	553	558	562
924	567	572	577	581	586	591	595	600	605	609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	661	666	670	675	680	685	689	694	699	703
927	708	713	717	722	727	731	736	741	745	750
928	755	759	764	769	774	778	783	788	792	797
929	802	806	811	816	820	825	830	834	839	844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	895	900	904	909	914	918	923	928	932	937
932	942	946	951	956	960	965	970	974	979	984
933	988	993	997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	049	053	058	063	067	072	077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	128	132	137	142	146	151	155	160	165	169
937	174	179	183	188	192	197	202	206	211	216
938	220	225	230	234	239	243	248	253	257	262
939	267	271	276	280	285	290	294	299	304	308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	359	364	368	373	377	382	387	391	396	400
942	405	410	414	419	424	428	433	437	442	447
943	451	456	460	465	470	474	479	483	488	493
944	497	502	506	511	516	520	525	529	534	539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	589	594	598	603	607	612	617	621	626	630
947	635	640	644	649	653	658	663	667	672	676
948	681	685	690	695	699	704	708	713	717	722
949	727	731	736	740	745	749	754	759	763	768
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
N	0	1	2	3	4	5	6	7	8	9

950–1000

N	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	818	823	827	832	836	841	845	850	855	859
952	864	868	873	877	882	886	891	896	900	905
953	909	914	918	923	928	932	937	941	946	950
954	955	959	964	968	973	978	982	987	991	996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	046	050	055	059	064	068	073	078	082	087
957	091	096	100	105	109	114	118	123	127	132
958	137	141	146	150	155	159	164	168	173	177
959	182	186	191	195	200	204	209	214	218	223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	272	277	281	286	290	295	299	304	308	313
962	318	322	327	331	336	340	345	349	354	358
963	363	367	372	376	381	385	390	394	399	403
964	408	412	417	421	426	430	435	439	444	448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	498	502	507	511	516	520	525	529	534	538
967	543	547	552	556	561	565	570	574	579	583
968	588	592	597	601	605	610	614	619	623	628
969	632	637	641	646	650	655	659	664	668	673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	722	726	731	735	740	744	749	753	758	762
972	767	771	776	780	784	789	793	798	802	807
973	811	816	820	825	829	834	838	843	847	851
974	856	860	865	869	874	878	883	887	892	896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	945	949	954	958	963	967	972	976	981	985
977	989	994	998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	047	052	056	061	065	069	074
979	078	083	087	092	096	100	105	109	114	118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	167	171	176	180	185	189	193	198	202	207
982	211	216	220	224	229	233	238	242	247	251
983	255	260	264	269	273	277	282	286	291	295
984	300	304	308	313	317	322	326	330	335	339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	388	392	396	401	405	410	414	419	423	427
987	432	436	441	445	449	454	458	463	467	471
988	476	480	484	489	493	498	502	506	511	515
989	520	524	528	533	537	542	546	550	555	559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	607	612	616	621	625	629	634	638	642	647
992	651	656	660	664	669	673	677	682	686	691
993	695	699	704	708	712	717	721	726	730	734
994	739	743	747	752	756	760	765	769	774	778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	826	830	835	839	843	848	852	856	861	865
997	870	874	878	883	887	891	896	900	904	909
998	913	917	922	926	930	935	939	944	948	952
999	957	961	965	970	974	978	983	987	991	996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
N	0	1	2	3	4	5	6	7	8	9

TABLE II

Logarithms of Constants

Number	Log	Number	Log
Circle $= 360°$	2.55630	$\pi^2 = 9.86960$	0.99430
$= 21{,}600'$	4.33445	$\frac{1}{\pi^2} = 0.10132$	9.00570 − 10
$= 1{,}296{,}000''$	6.11261		
$\pi = 3.14159$	0.49715	$\sqrt{\pi} = 1.77245$	0.24857
$2\pi = 6.28319$	0.79818	$\frac{1}{\sqrt{\pi}} = 0.56419$	9.75143 − 10
$4\pi = 12.56637$	1.09921		
$\frac{4\pi}{3} = 4.18879$	0.62209	$\sqrt{\frac{4}{\pi}} = 1.12838$	0.05246
$\frac{\pi}{4} = 0.78540$	9.89509 − 10	$\sqrt[3]{\pi} = 1.46459$	0.16572
$\frac{\pi}{6} = 0.52360$	9.71900 − 10	$\frac{1}{\sqrt[3]{\pi}} = 0.68278$	9.83428 − 10
$\frac{1}{\pi} = 0.31831$	9.50285 − 10	$\sqrt[3]{\frac{3}{4\pi}} = 0.62035$	9.79264 − 10
$\frac{1}{2\pi} = 0.15915$	9.20182 − 10	$\sqrt[3]{\frac{\pi}{6}} = 0.80600$	9.90633 − 10
$\sqrt{2} = 1.41421$	0.15052	$\sqrt[3]{2} = 1.25992$	0.10034
$\sqrt{3} = 1.73205$	0.23856	$\sqrt[3]{3} = 1.44225$	0.15904
$\sqrt{5} = 2.23606$	0.34949	$\sqrt[3]{5} = 1.70997$	0.23299
$\sqrt{6} = 2.44948$	0.38908	$\sqrt[3]{6} = 1.81712$	0.25938
1 radian $= \frac{180°}{\pi}$		$1° = \frac{\pi}{180}$ radian	
$= 57.2958°$	1.75812	$1° = 0.01745$ radian	8.24188 − 10
$= 3437.75'$	3.53627	$1' = 0.00029$ radian	6.46373 − 10
$= 206{,}264.81''$	5.31443	$1'' = 0.000005$ radian	4.68557 − 10
Base of natural logs., e		$\log_{10} e = \log_{10} 2.71828$	0.43429
$e = 2.71828$	0.43429	$1 : \log_{10} e = 2.302585$	0.36222
1 m. $= 39.3708$ in.	1.59517	1 naut. mi. $= 6080.27$ ft.	3.78392
$= 1.0936$ yd.	0.03886	$= 1.1516$ mi.	0.06130
$= 3.2809$ ft.	0.51599	1 lb. Av. $= 7000$ gr.	3.84510
1 km. $= 0.6214$ mi.	9.79336 − 10	1 bu. $= 2150.42$ cu. in.	3.33252
1 mi. $= 1.6093$ km.	0.20664	1 U.S. gal. $= 231$ cu. in.	2.36361
1 mi. $= 5280$ ft.	3.72263	1 Brit. gal. $= 277.463$ cu. in.	2.44320
1 lb. Av. $= 453.5927$ g.	2.65666	Earth's radii	
1 acre $= 43{,}560$ sq. ft.	4.63909	$= 3963$ mi.	3.59802
$g = 32.17$ ft./sec.^{2}	1.50745	and 3950 mi.	3.59660
$g = 980.6$ cm./sec.^{2}	2.99149	1 ft.-lb. $= 0.1383$ kg.-m.	9.14082 − 10

TABLE III

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Table III gives the common logarithms of the functions sine, cosine, tangent, and cotangent of angles ranging from 0° to 90°. On pages 22–27 the values are given at intervals of 10″ from 0° to 2° and from 88° to 90°. On pages 28–49 the values are given at intervals of 1′ from 1° to 89°.

The mantissas only are printed in the main body of the table (pp. 28–49), the proper characteristic being shown at the top and bottom of the column. On pages 30 and 49 a heavy bar (—) placed between two consecutive entries in a column indicates that the characteristic at the top of the column is to be used with all logarithms above the bar, while the characteristic at the bottom is to be used with those below. *In every case — 10 is to be added to the printed characteristic.*

For all angles less than 45° the degrees are found at the *top* of the page, the minutes at the *left*, and the name of the function at the *top*. For all angles between 45° and 90° the degrees are found at the *bottom* of the page, the minutes at the *right*, and the name of the function at the *bottom*.

The values of log sec x and log csc x are found from the values of log cos x and log sin x, respectively, by the relations

$$\log \sec x = -\log \cos x,$$
$$\log \csc x = -\log \sin x.$$

0°

′ ″	log sin	log cos	log tan	′ ″
0 0	—	10.00000	—	**60** 0
10	5.68 557	10.00000	5.68 557	50
20	5.98 660	10.00000	5.98 660	40
30	6.16 270	10.00000	6.16 270	30
40	6.28 763	10.00000	6.28 763	20
50	6.38 454	10.00000	6.38 454	10
1 0	6.46 373	10.00000	6.46 373	**59** 0
10	6.53 067	10.00000	6.53 067	50
20	6.58 866	10.00000	6.58 866	40
30	6.63 982	10.00000	6.63 982	30
40	6.68 557	10.00000	6.68 557	20
50	6.72 697	10.00000	6.72 697	10
2 0	6.76 476	10.00000	6.76 476	**58** 0
10	6.79 952	10.00000	6.79 952	50
20	6.83 170	10.00000	6.83 170	40
30	6.86 167	10.00000	6.86 167	30
40	6.88 969	10.00000	6.88 969	20
50	6.91 602	10.00000	6.91 602	10
3 0	6.94 085	10.00000	6.94 085	**57** 0
10	6.96 433	10.00000	6.96 433	50
20	6.98 660	10.00000	6.98 661	40
30	7.00 779	10.00000	7.00 779	30
40	7.02 800	10.00000	7.02 800	20
50	7.04 730	10.00000	7.04 730	10
4 0	7.06 579	10.00000	7.06 579	**56** 0
10	7.08 351	10.00000	7.08 352	50
20	7.10 055	10.00000	7.10 055	40
30	7.11 694	10.00000	7.11 694	30
40	7.13 273	10.00000	7.13 273	20
50	7.14 797	10.00000	7.14 797	10
5 0	7.16 270	10.00000	7.16 270	**55** 0
10	7.17 694	10.00000	7.17 694	50
20	7.19 072	10.00000	7.19 073	40
30	7.20 409	10.00000	7.20 409	30
40	7.21 705	10.00000	7.21 705	20
50	7.22 964	10.00000	7.22 964	10
6 0	7.24 188	10.00000	7.24 188	**54** 0
10	7.25 378	10.00000	7.25 378	50
20	7.26 536	10.00000	7.26 536	40
30	7.27 664	10.00000	7.27 664	30
40	7.28 763	10.00000	7.28 764	20
50	7.29 836	10.00000	7.29 836	10
7 0	7.30 882	10.00000	7.30 882	**53** 0
10	7.31 904	10.00000	7.31 904	50
20	7.32 903	10.00000	7.32 903	40
30	7.33 879	10.00000	7.33 879	30
40	7.34 833	10.00000	7.34 833	20
50	7.35 767	10.00000	7.35 767	10
8 0	7.36 682	10.00000	7.36 682	**52** 0
10	7.37 577	10.00000	7.37 577	50
20	7.38 454	10.00000	7.38 455	40
30	7.39 314	10.00000	7.39 315	30
40	7.40 158	10.00000	7.40 158	20
50	7.40 985	10.00000	7.40 985	10
9 0	7.41 797	10.00000	7.41 797	**51** 0
10	7.42 594	10.00000	7.42 594	50
20	7.43 376	10.00000	7.43 376	40
30	7.44 145	10.00000	7.44 145	30
40	7.44 900	10.00000	7.44 900	20
50	7.45 643	10.00000	7.45 643	10
10 0	7.46 373	10.00000	7.46 373	**50** 0
′ ″	log cos	log sin	log cot	′ ″

′ ″	log sin	log cos	log tan	′ ″
10 0	7.46 373	10.00000	7.46 373	**50** 0
10	7.47 090	10.00000	7.47 091	50
20	7.47 797	10.00000	7.47 797	40
30	7.48 491	10.00000	7.48 492	30
40	7.49 175	10.00000	7.49 176	20
50	7.49 849	10.00000	7.49 849	10
11 0	7.50 512	10.00000	7.50 512	**49** 0
10	7.51 165	10.00000	7.51 165	50
20	7.51 808	10.00000	7.51 809	40
30	7.52 442	10.00000	7.52 443	30
40	7.53 067	10.00000	7.53 067	20
50	7.53 683	10.00000	7.53 683	10
12 0	7.54 291	10.00000	7.54 291	**48** 0
10	7.54 890	10.00000	7.54 890	50
20	7.55 481	10.00000	7.55 481	40
30	7.56 064	10.00000	7.56 064	30
40	7.56 639	10.00000	7.56 639	20
50	7.57 206	10.00000	7.57 207	10
13 0	7.57 767	10.00000	7.57 767	**47** 0
10	7.58 320	10.00000	7.58 320	50
20	7.58 866	10.00000	7.58 867	40
30	7.59 406	10.00000	7.59 406	30
40	7.59 939	10.00000	7.59 939	20
50	7.60 465	10.00000	7.60 466	10
14 0	7.60 985	10.00000	7.60 986	**46** 0
10	7.61 499	10.00000	7.61 500	50
20	7.62 007	10.00000	7.62 008	40
30	7.62 509	10.00000	7.62 510	30
40	7.63 006	10.00000	7.63 006	20
50	7.63 496	10.00000	7.63 497	10
15 0	7.63 982	10.00000	7.63 982	**45** 0
10	7.64 461	10.00000	7.64 462	50
20	7.64 936	10.00000	7.64 937	40
30	7.65 406	10.00000	7.65 406	30
40	7.65 870	10.00000	7.65 871	20
50	7.66 330	10.00000	7.66 330	10
16 0	7.66 784	10.00000	7.66 785	**44** 0
10	7.67 235	10.00000	7.67 235	50
20	7.67 680	10.00000	7.67 680	40
30	7.68 121	10.00000	7.68 121	30
40	7.68 557	9.99999	7.68 558	20
50	7.68 989	9.99999	7.68 990	10
17 0	7.69 417	9.99 999	7.69 418	**43** 0
10	7.69 841	9.99 999	7.69 842	50
20	7.70 261	9.99 999	7.70 261	40
30	7.70 676	9.99 999	7.70 677	30
40	7.71 088	9.99 999	7.71 088	20
50	7.71 496	9.99 999	7.71 496	10
18 0	7.71 900	9.99 999	7.71 900	**42** 0
10	7.72 300	9.99 999	7.72 301	50
20	7.72 697	9.99 999	7.72 697	40
30	7.73 090	9.99 999	7.73 090	30
40	7.73 479	9.99 999	7.73 480	20
50	7.73 865	9.99 999	7.73 866	10
19 0	7.74 248	9.99 999	7.74 248	**41** 0
10	7.74 627	9.99 999	7.74 628	50
20	7.75 003	9.99 999	7.75 004	40
30	7.75 376	9.99 999	7.75 377	30
40	7.75 745	9.99 999	7.75 746	20
50	7.76 112	9.99 999	7.76 113	10
20 0	7.76 475	9.99 999	7.76 476	**40** 0
′ ″	log cos	log sin	log cot	′ ″

89°

0°

′ ″	log sin	log cos	log tan	′ ″	′ ″	log sin	log cos	log tan	′ ″
20 0	7.76475	9.99999	7.76476	**40** 0	**30** 0	7.94084	9.99998	7.94086	**30** 0
10	7.76836	9.99999	7.76837	50	10	7.94325	9.99998	7.94326	50
20	7.77193	9.99999	7.77194	40	20	7.94564	9.99998	7.94566	40
30	7.77548	9.99999	7.77549	30	30	7.94802	9.99998	7.94804	30
40	7.77899	9.99999	7.77900	20	40	7.95039	9.99998	7.95040	20
50	7.78248	9.99999	7.78249	10	50	7.95274	9.99998	7.95276	10
21 0	7.78594	9.99999	7.78595	**39** 0	**31** 0	7.95508	9.99998	7.95510	**29** 0
10	7.78938	9.99999	7.78938	50	10	7.95741	9.99998	7.95743	50
20	7.79278	9.99999	7.79279	40	20	7.95973	9.99998	7.95974	40
30	7.79616	9.99999	7.79617	30	30	7.96203	9.99998	7.96205	30
40	7.79952	9.99999	7.79952	20	40	7.96432	9.99998	7.96434	20
50	7.80284	9.99999	7.80285	10	50	7.96660	9.99998	7.96662	10
22 0	7.80615	9.99999	7.80615	**38** 0	**32** 0	7.96887	9.99998	7.96889	**28** 0
10	7.80942	9.99999	7.80943	50	10	7.97113	9.99998	7.97114	50
20	7 81268	9.99999	7.81269	40	20	7.97337	9.99998	7.97339	40
30	7.81591	9.99999	7.81591	30	30	7.97560	9.99998	7.97562	30
40	7.81911	9.99999	7.81912	20	40	7.97782	9.99998	7.97784	20
50	7.82229	9.99999	7.82230	10	50	7.98003	9.99998	7.98005	10
23 0	7.82545	9.99999	7.82546	**37** 0	**33** 0	7.98223	9.99998	7.98225	**27** 0
10	7.82859	9.99999	7.82860	50	10	7.98442	9.99998	7.98444	50
20	7.83170	9.99999	7.83171	40	20	7.98660	9.99998	7.98662	40
30	7.83479	9.99999	7.83480	30	30	7.98876	9.99998	7.98878	30
40	7.83786	9.99999	7.83787	20	40	7.99092	9.99998	7.99094	20
50	7.84091	9.99999	7.84092	10	50	7.99306	9.99998	7.99308	10
24 0	7.84393	9.99999	7.84394	**36** 0	**34** 0	7.99520	9.99998	7.99522	**26** 0
10	7.84694	9.99999	7.84695	50	10	7.99732	9.99998	7.99734	50
20	7.84992	9.99999	7.84994	40	20	7.99943	9.99998	7.99946	40
30	7.85289	9.99999	7.85290	30	30	8.00154	9.99998	8.00156	30
40	7.85583	9.99999	7.85584	20	40	8.00363	9.99998	8.00365	20
50	7.85876	9.99999	7.85877	10	50	8.00571	9.99998	8.00574	10
25 0	7.86166	9.99999	7.86167	**35** 0	**35** 0	8.00779	9.99998	8.00781	**25** 0
10	7.86455	9.99999	7.86456	50	10	8.00985	9.99998	8.00987	50
20	7.86741	9.99999	7.86743	40	20	8.01190	9.99998	8.01193	40
30	7.87026	9.99999	7.87027	30	30	8.01395	9.99998	8.01397	30
40	7.87309	9.99999	7.87310	20	40	8.01598	9.99998	8.01600	20
50	7.87590	9.99999	7.87591	10	50	8.01801	9.99998	8.01803	10
26 0	7.87870	9.99999	7.87871	**34** 0	**36** 0	8.02002	9.99998	8.02004	**24** 0
10	7.88147	9.99999	7.88148	50	10	8.02203	9.99998	8.02205	50
20	7.88423	9.99999	7.88424	40	20	8.02402	9.99998	8.02405	40
30	7.88697	9.99999	7.88698	30	30	8.02601	9.99998	8.02604	30
40	7.88969	9.99999	7.88970	20	40	8.02799	9.99998	8.02801	20
50	7.89240	9.99999	7.89241	10	50	8.02996	9 99998	8.02998	10
27 0	7.89509	9.99999	7.89510	**33** 0	**37** 0	8.03192	9.99997	8.03194	**23** 0
10	7.89776	9.99999	7.89777	50	10	8.03387	9.99997	8.03390	50
20	7.90041	9.99999	7.90043	40	20	8.03581	9.99997	8.03584	40
30	7.90305	9.99999	7.90307	30	30	8.03775	9.99997	8.03777	30
40	7.90568	9.99999	7.90569	20	40	8.03967	9.99997	8.03970	20
50	7.90829	9.99999	7.90830	10	50	8.04159	9.99997	8.04162	10
28 0	7.91088	9.99999	7.91089	**32** 0	**38** 0	8.04350	9.99997	8.04353	**22** 0
10	7.91346	9.99999	7.91347	50	10	8.04540	9.99997	8.04543	50
20	7.91602	9.99999	7.91603	40	20	8.04729	9.99997	8.04732	40
30	7.91857	9.99999	7.91858	30	30	8.04918	9.99997	8.04921	30
40	7.92110	9.99998	7.92111	20	40	8.05105	9.99997	8.05108	20
50	7.92362	9.99998	7.92363	10	50	8.05292	9.99997	8.05295	10
29 0	7.92612	9.99998	7.92613	**31** 0	**39** 0	8.05478	9.99997	8.05481	**21** 0
10	7.92861	9.99998	7.92862	50	10	8.05663	9.99997	8.05666	50
20	7.93108	9.99998	7.93110	40	20	8.05848	9.99997	8.05851	40
30	7.93354	9.99998	7.93356	30	30	8.06031	9.99997	8.06034	30
40	7.93599	9.99998	7.93601	20	40	8.06214	9.99997	8.06217	20
50	7.93842	9.99998	7.93844	10	50	8.06396	9.99997	8.06399	10
30 0	7.94084	9.99998	7.94086	**30** 0	**40** 0	8.06578	9.99997	8.06581	**20** 0
′ ″	log cos	log sin	log cot	′ ″	′ ″	log cos	log sin	log cot	′ ″

89°

0°

′ ″	log sin	log cos	log tan	′ ″	′ ″	log sin	log cos	log tan	′ ″
40 0	8.06 578	9.99 997	8.06 581	**20** 0	**50** 0	8.16 268	9.99 995	8.16 273	**10** 0
10	8.06 758	9.99 997	8.06 761	50	10	8.16 413	9.99 995	8.16 417	50
20	8.06 938	9.99 997	8.06 941	40	20	8.16 557	9.99 995	8.16 561	40
30	8.07 117	9.99 997	8.07 120	30	30	8.16 700	9.99 995	8.16 705	30
40	8.07 295	9.99 997	8.07 299	20	40	8.16 843	9.99 995	8.16 848	20
50	8.07 473	9.99 997	8.07 476	10	50	8.16 986	9.99 995	8.16 991	10
41 0	8.07 650	9.99 997	8.07 653	**19** 0	**51** 0	8.17 128	9.99 995	8.17 133	**9** 0
10	8.07 826	9.99 997	8.07 829	50	10	8.17 270	9.99 995	8.17 275	50
20	8.08 002	9.99 997	8.08 005	40	20	8.17 411	9.99 995	8.17 416	40
30	8.08 176	9.99 997	8.08 180	30	30	8.17 552	9.99 995	8.17 557	30
40	8.08 350	9.99 997	8.08 354	20	40	8.17 692	9.99 995	8.17 697	20
50	8.08 524	9.99 997	8.08 527	10	50	8.17 832	9.99 995	8.17 837	10
42 0	8.08 696	9.99 997	8.08 700	**18** 0	**52** 0	8.17 971	9.99 995	8.17 976	**8** 0
10	8.08 868	9.99 997	8.08 872	50	10	8.18 110	9.99 995	8.18 115	50
20	8.09 040	9.99 997	8.09 043	40	20	8.18 249	9.99 995	8.18 254	40
30	8.09 210	9.99 997	8.09 214	30	30	8.18 387	9.99 995	8.18 392	30
40	8.09 380	9.99 997	8.09 384	20	40	8.18 524	9.99 995	8.18 530	20
50	8.09 550	9.99 997	8.09 553	10	50	8.18 662	9.99 995	8.18 667	10
43 0	8.09 718	9.99 997	8.09 722	**17** 0	**53** 0	8.18 798	9.99 995	8.18 804	**7** 0
10	8.09 886	9.99 997	8.09 890	50	10	8.18 935	9.99 995	8.18 940	50
20	8.10 054	9.99 997	8.10 057	40	20	8.19 071	9.99 995	8.19 076	40
30	8.10 220	9.99 997	8.10 224	30	30	8.19 206	9.99 995	8.19 212	30
40	8.10 386	9.99 997	8.10 390	20	40	8.19 341	9.99 995	8.19 347	20
50	8.10 552	9.99 996	8.10 555	10	50	8.19 476	9.99 995	8.19 481	10
44 0	8.10 717	9.99 996	8.10 720	**16** 0	**54** 0	8.19 610	9.99 995	8.19 616	**6** 0
10	8.10 881	9.99 996	8.10 884	50	10	8.19 744	9.99 995	8.19 749	50
20	8.11 044	9.99 996	8.11 048	40	20	8.19 877	9.99 995	8.19 883	40
30	8.11 207	9.99 996	8.11 211	30	30	8.20 010	9.99 995	8.20 016	30
40	8.11 370	9.99 996	8.11 373	20	40	8.20 143	9.99 995	8.20 149	20
50	8.11 531	9.99 996	8.11 535	10	50	8.20 275	9.99 994	8.20 281	10
45 0	8.11 693	9.99 996	8.11 696	**15** 0	**55** 0	8.20 407	9.99 994	8.20 413	**5** 0
10	8.11 853	9.99 996	8.11 857	50	10	8.20 538	9.99 994	8.20 544	50
20	8.12 013	9.99 996	8.12 017	40	20	8.20 669	9.99 994	8.20 675	40
30	8.12 172	9.99 996	8.12 176	30	30	8.20 800	9.99 994	8.20 806	30
40	8.12 331	9.99 996	8.12 335	20	40	8.20 930	9.99 994	8.20 936	20
50	8.12 489	9.99 996	8.12 493	10	50	8.21 060	9.99 994	8.21 066	10
46 0	8.12 647	9.99 996	8.12 651	**14** 0	**56** 0	8.21 189	9.99 994	8.21 195	**4** 0
10	8.12 804	9.99 996	8.12 808	50	10	8.21 319	9.99 994	8.21 324	50
20	8.12 961	9.99 996	8.12 965	40	20	8.21 447	9.99 994	8.21 453	40
30	8.13 117	9.99 996	8.13 121	30	30	8.21 576	9.99 994	8.21 581	30
40	8.13 272	9.99 996	8.13 276	20	40	8.21 703	9.99 994	8.21 709	20
50	8.13 427	9.99 996	8.13 431	10	50	8.21 831	9.99 994	8.21 837	10
47 0	8.13 581	9.99 996	8.13 585	**13** 0	**57** 0	8.21 958	9.99 994	8.21 964	**3** 0
10	8.13 735	9.99 996	8.13 739	50	10	8.22 085	9.99 994	8.22 091	50
20	8.13 888	9.99 996	8.13 892	40	20	8.22 211	9.99 994	8.22 217	40
30	8.14 041	9.99 996	8.14 045	30	30	8.22 337	9.99 994	8.22 343	30
40	8.14 193	9.99 996	8.14 197	20	40	8.22 463	9.99 994	8.22 469	20
50	8.14 344	9.99 996	8.14 348	10	50	8.22 588	9.99 994	8.22 595	10
48 0	8.14 495	9.99 996	8.14 500	**12** 0	**58** 0	8.22 713	9.99 994	8.22 720	**2** 0
10	8.14 646	9.99 996	8.14 650	50	10	8.22 838	9.99 994	8.22 844	50
20	8.14 796	9.99 996	8.14 800	40	20	8.22 962	9.99 994	8.22 968	40
30	8.14 945	9.99 996	8.14 950	30	30	8.23 086	9.99 994	8.23 092	30
40	8.15 094	9.99 996	8.15 099	20	40	8.23 210	9.99 994	8.23 216	20
50	8.15 243	9.99 996	8.15 247	10	50	8.23 333	9.99 994	8.23 339	10
49 0	8.15 391	9.99 996	8.15 395	**11** 0	**59** 0	8.23 456	9.99 994	8.23 462	**1** 0
10	8.15 538	9.99 996	8.15 543	50	10	8.23 578	9.99 994	8.23 585	50
20	8.15 685	9.99 996	8.15 690	40	20	8.23 700	9.99 994	8.23 707	40
30	8.15 832	9.99 996	8.15 836	30	30	8.23 822	9.99 993	8.23 829	30
40	8.15 978	9.99 995	8.15 982	20	40	8.23 944	9.99 993	8.23 950	20
50	8.16 123	9.99 995	8.16 128	10	50	8.24 065	9.99 993	8.24 071	10
50 0	8.16 268	9.99 995	8.16 273	**10** 0	**60** 0	8.24 186	9.99 993	8.24 192	**0** 0
′ ″	log cos	log sin	log cot	′ ″	′ ″	log cos	log sin	log cot	′ ″

89°

1°

′ ″	log sin	log cos	log tan	′ ″	′ ″	log sin	log cos	log tan	′ ″
0 0	8. 24 186	9. 99 993	8. 24 192	**60** 0	**10** 0	8. 30 879	9. 99 991	8. 30 888	**50** 0
10	8. 24 306	9. 99 993	8. 24 313	50	10	8. 30 983	9. 99 991	8. 30 992	50
20	8. 24 426	9. 99 993	8. 24 433	40	20	8. 31 086	9. 99 991	8. 31 095	40
30	8. 24 546	9. 99 993	8. 24 553	30	30	8. 31 188	9. 99 991	8. 31 198	30
40	8. 24 665	9. 99 993	8. 24 672	20	40	8. 31 291	9. 99 991	8. 31 300	20
50	8. 24 785	9. 99 993	8. 24 791	10	50	8. 31 393	9. 99 991	8. 31 403	10
1 0	8. 24 903	9. 99 993	8. 24 910	**59** 0	**11** 0	8. 31 495	9. 99 991	8. 31 505	**49** 0
10	8. 25 022	9. 99 993	8. 25 029	50	10	8. 31 597	9. 99 991	8. 31 606	50
20	8. 25 140	9. 99 993	8. 25 147	40	20	8. 31 699	9. 99 991	8. 31 708	40
30	8. 25 258	9. 99 993	8. 25 265	30	30	8. 31 800	9. 99 991	8. 31 809	30
40	8. 25 375	9. 99 993	8. 25 382	20	40	8. 31 901	9. 99 991	8. 31 911	20
50	8. 25 493	9. 99 993	8. 25 500	10	50	8. 32 002	9. 99 991	8. 32 012	10
2 0	8. 25 609	9. 99 993	8. 25 616	**58** 0	**12** 0	8. 32 103	9. 99 990	8. 32 112	**48** 0
10	8. 25 726	9. 99 993	8. 25 733	50	10	8. 32 203	9. 99 990	8. 32 213	50
20	8. 25 842	9. 99 993	8. 25 849	40	20	8. 32 303	9. 99 990	8. 32 313	40
30	8. 25 958	9. 99 993	8. 25 965	30	30	8. 32 403	9. 99 990	8. 32 413	30
40	8. 26 074	9. 99 993	8. 26 081	20	40	8. 32 503	9. 99 990	8. 32 513	20
50	8. 26 189	9. 99 993	8. 26 196	10	50	8. 32 602	9. 99 990	8. 32 612	10
3 0	8. 26 304	9. 99 993	8. 26 312	**57** 0	**13** 0	8. 32 702	9. 99 990	8. 32 711	**47** 0
10	8. 26 419	9. 99 993	8. 26 426	50	10	8. 32 801	9. 99 990	8. 32 811	50
20	8. 26 533	9. 99 993	8. 26 541	40	20	8. 32 899	9. 99 990	8. 32 909	40
30	8. 26 648	9. 99 993	8. 26 655	30	30	8. 32 998	9. 99 990	8. 33 008	30
40	8. 26 761	9. 99 993	8. 26 769	20	40	8. 33 096	9. 99 990	8. 33 106	20
50	8. 26 875	9. 99 993	8. 26 882	10	50	8. 33 195	9. 99 990	8. 33 205	10
4 0	8. 26 988	9. 99 992	8. 26 996	**56** 0	**14** 0	8. 33 292	9. 99 990	8. 33 302	**46** 0
10	8. 27 101	9. 99 992	8. 27 109	50	10	8. 33 390	9. 99 990	8. 33 400	50
20	8. 27 214	9. 99 992	8. 27 221	40	20	8. 33 488	9. 99 990	8. 33 498	40
30	8. 27 326	9. 99 992	8. 27 334	30	30	8. 33 585	9. 99 990	8. 33 595	30
40	8. 27 438	9. 99 992	8. 27 446	20	40	8. 33 682	9. 99 990	8. 33 692	20
50	8. 27 550	9. 99 992	8. 27 558	10	50	8. 33 779	9. 99 990	8. 33 789	10
5 0	8. 27 661	9. 99 992	8. 27 669	**55** 0	**15** 0	8. 33 875	9. 99 990	8. 33 886	**45** 0
10	8. 27 773	9. 99 992	8. 27 780	50	10	8. 33 972	9. 99 990	8. 33 982	50
20	8. 27 883	9. 99 992	8. 27 891	40	20	8. 34 068	9. 99 990	8. 34 078	40
30	8. 27 994	9. 99 992	8. 28 002	30	30	8. 34 164	9. 99 990	8. 34 174	30
40	8. 28 104	9. 99 992	8. 28 112	20	40	8. 34 260	9. 99 989	8. 34 270	20
50	8. 28 215	9. 99 992	8. 28 223	10	50	8. 34 355	9. 99 989	8. 34 366	10
6 0	8. 28 324	9. 99 992	8. 28 332	**54** 0	**16** 0	8. 34 450	9. 99 989	8. 34 461	**44** 0
10	8. 28 434	9. 99 992	8. 28 442	50	10	8. 34 546	9. 99 989	8. 34 556	50
20	8. 28 543	9. 99 992	8. 28 551	40	20	8. 34 640	9. 99 989	8. 34 651	40
30	8. 28 652	9. 99 992	8. 28 660	30	30	8. 34 735	9. 99 989	8. 34 746	30
40	8. 28 761	9. 99 992	8. 28 769	20	40	8. 34 830	9. 99 989	8. 34 840	20
50	8. 28 869	9. 99 992	8. 28 877	10	50	8. 34 924	9. 99 989	8. 34 935	10
7 0	8. 28 977	9. 99 992	8. 28 986	**53** 0	**17** 0	8. 35 018	9. 99 989	8. 35 029	**43** 0
10	8. 29 085	9. 99 992	8. 29 094	50	10	8. 35 112	9. 99 989	8. 35 123	50
20	8. 29 193	9. 99 992	8. 29 201	40	20	8. 35 206	9. 99 989	8. 35 217	40
30	8. 29 300	9. 99 992	8. 29 309	30	30	8. 35 299	9. 99 989	8. 35 310	30
40	8. 29 407	9. 99 992	8. 29 416	20	40	8. 35 392	9. 99 989	8. 35 403	20
50	8. 29 514	9. 99 992	8. 29 523	10	50	8. 35 485	9. 99 989	8. 35 497	10
8 0	8. 29 621	9. 99 992	8. 29 629	**52** 0	**18** 0	8. 35 578	9. 99 989	8. 35 590	**42** 0
10	8. 29 727	9. 99 991	8. 29 736	50	10	8. 35 671	9. 99 989	8. 35 682	50
20	8. 29 833	9. 99 991	8. 29 842	40	20	8. 35 764	9. 99 989	8. 35 775	40
30	8. 29 939	9. 99 991	8. 29 947	30	30	8. 35 856	9. 99 989	8. 35 867	30
40	8. 30 044	9. 99 991	8. 30 053	20	40	8. 35 948	9. 99 989	8. 35 959	20
50	8. 30 150	9. 99 991	8. 30 158	10	50	8. 36 040	9. 99 989	8. 36 051	10
9 0	8. 30 255	9. 99 991	8. 30 263	**51** 0	**19** 0	8. 36 131	9. 99 989	8. 36 143	**41** 0
10	8. 30 359	9. 99 991	8. 30 368	50	10	8. 36 223	9. 99 988	8. 36 235	50
20	8. 30 464	9. 99 991	8. 30 473	40	20	8 36 314	9. 99 988	8. 36 326	40
30	8. 30 568	9. 99 991	8. 30 577	30	30	8. 36 405	9. 99 988	8. 36 417	30
40	8. 30 672	9. 99 991	8. 30 681	20	40	8. 36 496	9. 99 988	8. 36 508	20
50	8. 30 776	9. 99 991	8. 30 785	10	50	8. 36 587	9. 99 988	8. 36 599	10
10 0	8. 30 879	9. 99 991	8. 30 888	**50** 0	**20** 0	8. 36 678	9. 99 988	8. 36 689	**40** 0
′ ″	log cos	log sin	log cot	′ ″	′ ″	log cos	log sin	log cot	′ ″

88°

1°

′ ″	log sin	log cos	log tan	′ ″	′ ″	log sin	log cos	log tan	′ ″
20 0	8. 36 678	9. 99 988	8. 36 689	**40** 0	**30** 0	8. 41 792	9. 99 985	8. 41 807	**30** 0
10	8. 36 768	9. 99 988	8. 36 780	50	10	8. 41 872	9. 99 985	8. 41 887	50
20	8. 36 858	9. 99 988	8. 36 870	40	20	8. 41 952	9. 99 985	8. 41 967	40
30	8. 36 948	9. 99 988	8. 36 960	30	30	8. 42 032	9. 99 985	8. 42 048	30
40	8. 37 038	9. 99 988	8. 37 050	20	40	8. 42 112	9. 99 985	8. 42 127	20
50	8. 37 128	9. 99 988	8. 37 140	10	50	8. 42 192	9. 99 985	8. 42 207	10
21 0	8. 37 217	9. 99 988	8. 37 229	**39** 0	**31** 0	8. 42 272	9. 99 985	8. 42 287	**29** 0
10	8. 37 306	9. 99 988	8. 37 318	50	10	8. 42 351	9. 99 985	8. 42 366	50
20	8. 37 395	9. 99 988	8. 37 408	40	20	8. 42 430	9. 99 985	8. 42 446	40
30	8. 37 484	9. 99 988	8. 37 497	30	30	8. 42 510	9. 99 985	8. 42 525	30
40	8. 37 573	9. 99 988	8. 37 585	20	40	8. 42 589	9. 99 985	8. 42 604	20
50	8. 37 662	9. 99 988	8. 37 674	10	50	8. 42 667	9. 99 985	8. 42 683	10
22 0	8. 37 750	9. 99 988	8. 37 762	**38** 0	**32** 0	8. 42 746	9. 99 984	8. 42 762	**28** 0
10	8. 37 838	9. 99 988	8. 37 850	50	10	8. 42 825	9. 99 984	8. 42 840	50
20	8. 37 926	9. 99 988	8. 37 938	40	20	8. 42 903	9. 99 984	8. 42 919	40
30	8. 38 014	9. 99 987	8. 38 026	30	30	8. 42 982	9. 99 984	8. 42 997	30
40	8. 38 101	9. 99 987	8. 38 114	20	40	8. 43 060	9. 99 984	8. 43 075	20
50	8. 38 189	9. 99 987	8. 38 202	10	50	8. 43 138	9. 99 984	8. 43 154	10
23 0	8. 38 276	9. 99 987	8. 38 289	**37** 0	**33** 0	8. 43 216	9. 99 984	8. 43 232	**27** 0
10	8. 38 363	9. 99 987	8. 38 376	50	10	8. 43 293	9. 99 984	8. 43 309	50
20	8. 38 450	9. 99 987	8. 38 463	40	20	8. 43 371	9. 99 984	8. 43 387	40
30	8. 38 537	9. 99 987	8. 38 550	30	30	8. 43 448	9. 99 984	8. 43 464	30
40	8. 38 624	9. 99 987	8. 38 636	20	40	8. 43 526	9. 99 984	8. 43 542	20
50	8. 38 710	9. 99 987	8. 38 723	10	50	8. 43 603	9. 99 984	8. 43 619	10
24 0	8. 38 796	9. 99 987	8. 38 809	**36** 0	**34** 0	8. 43 680	9. 99 984	8. 43 696	**26** 0
10	8. 38 882	9. 99 987	8. 38 895	50	10	8. 43 757	9. 99 984	8. 43 773	50
20	8. 38 968	9. 99 987	8. 38 981	40	20	8. 43 834	9. 99 984	8. 43 850	40
30	8. 39 054	9. 99 987	8. 39 067	30	30	8. 43 910	9. 99 984	8. 43 927	30
40	8. 39 139	9. 99 987	8. 39 153	20	40	8. 43 987	9. 99 984	8. 44 003	20
50	8. 39 225	9. 99 987	8. 39 238	10	50	8. 44 063	9. 99 983	8. 44 080	10
25 0	8. 39 310	9. 99 987	8. 39 323	**35** 0	**35** 0	8. 44 139	9. 99 983	8. 44 156	**25** 0
10	8. 39 395	9. 99 987	8. 39 408	50	10	8. 44 216	9. 99 983	8. 44 232	50
20	8. 39 480	9. 99 987	8. 39 493	40	20	8. 44 292	9. 99 983	8. 44 308	40
30	8. 39 565	9. 99 987	8. 39 578	30	30	8. 44 367	9. 99 983	8. 44 384	30
40	8. 39 649	9. 99 987	8. 39 663	20	40	8. 44 443	9. 99 983	8. 44 460	20
50	8. 39 734	9. 99 986	8. 39 747	10	50	8. 44 519	9. 99 983	8. 44 536	10
26 0	8. 39 818	9. 99 986	8. 39 832	**34** 0	**36** 0	8. 44 594	9. 99 983	8. 44 611	**24** 0
10	8. 39 902	9. 99 986	8. 39 916	50	10	8. 44 669	9. 99 983	8. 44 686	50
20	8. 39 986	9. 99 986	8. 40 000	40	20	8. 44 745	9. 99 983	8. 44 762	40
30	8. 40 070	9. 99 986	8. 40 083	30	30	8. 44 820	9. 99 983	8. 44 837	30
40	8. 40 153	9. 99 986	8. 40 167	20	40	8. 44 895	9. 99 983	8. 44 912	20
50	8. 40 237	9. 99 986	8. 40 251	10	50	8. 44 969	9. 99 983	8. 44 987	10
27 0	8. 40 320	9. 99 986	8. 40 334	**33** 0	**37** 0	8. 45 044	9. 99 983	8. 45 061	**23** 0
10	8. 40 403	9. 99 986	8. 40 417	50	10	8. 45 119	9. 99 983	8. 45 136	50
20	8. 40 486	9. 99 986	8. 40 500	40	20	8. 45 193	9. 99 983	8. 45 210	40
30	8. 40 569	9. 99 986	8. 40 583	30	30	8. 45 267	9. 99 983	8. 45 285	30
40	8. 40 651	9. 99 986	8. 40 665	20	40	8. 45 341	9. 99 982	8. 45 359	20
50	8. 40 734	9. 99 986	8. 40 748	10	50	8. 45 415	9. 99 982	8. 45 433	10
28 0	8. 40 816	9. 99 986	8. 40 830	**32** 0	**38** 0	8. 45 489	9. 99 982	8. 45 507	**22** 0
10	8. 40 898	9. 99 986	8. 40 913	50	10	8. 45 563	9. 99 982	8. 45 581	50
20	8. 40 980	9. 99 986	8. 40 995	40	20	8. 45 637	9. 99 982	8. 45 655	40
30	8. 41 062	9. 99 986	8. 41 077	30	30	8. 45 710	9. 99 982	8. 45 728	30
40	8. 41 144	9. 99 986	8. 41 158	20	40	8. 45 784	9. 99 982	8. 45 802	20
50	8. 41 225	9. 99 986	8. 41 240	10	50	8. 45 857	9. 99 982	8. 45 875	10
29 0	8. 41 307	9. 99 985	8. 41 321	**31** 0	**39** 0	8. 45 930	9. 99 982	8. 45 948	**21** 0
10	8. 41 388	9. 99 985	8. 41 403	50	10	8. 46 003	9. 99 982	8. 46 021	50
20	8. 41 469	9. 99 985	8. 41 484	40	20	8. 46 076	9. 99 982	8. 46 094	40
30	8. 41 550	9. 99 985	8. 41 565	30	30	8. 46 149	9. 99 982	8. 46 167	30
40	8. 41 631	9. 99 985	8. 41 646	20	40	8. 46 222	9. 99 982	8. 46 240	20
50	8. 41 711	9. 99 985	8. 41 726	10	50	8. 46 294	9. 99 982	8. 46 312	10
30 0	8. 41 792	9. 99 985	8. 41 807	**30** 0	**40** 0	8. 46 366	9. 99 982	8. 46 385	**20** 0
′ ″	log cos	log sin	log cot	′ ″	′ ″	log cos	log sin	log cot	′ ″

88°

1°

′ ″	log sin	log cos	log tan	′ ″
40 0	8.46 366	9.99 982	8.46 385	**20** 0
10	8.46 439	9.99 982	8.46 457	50
20	8.46 511	9.99 982	8.46 529	40
30	8.46 583	9.99 981	8.46 602	30
40	8.46 655	9.99 981	8.46 674	20
50	8.46 727	9.99 981	8.46 745	10
41 0	8.46 799	9.99 981	8.46 817	**19** 0
10	8.46 870	9.99 981	8.46 889	50
20	8.46 942	9.99 981	8.46 960	40
30	8.47 013	9.99 981	8.47 032	30
40	8.47 084	9.99 981	8.47 103	20
50	8.47 155	9.99 981	8.47 174	10
42 0	8.47 226	9.99 981	8.47 245	**18** 0
10	8.47 297	9.99 981	8.47 316	50
20	8.47 368	9.99 981	8.47 387	40
30	8.47 439	9.99 981	8.47 458	30
40	8.47 509	9.99 981	8.47 528	20
50	8.47 580	9.99 981	8.47 599	10
43 0	8.47 650	9.99 981	8.47 669	**17** 0
10	8.47 720	9.99 980	8.47 740	50
20	8.47 790	9.99 980	8.47 810	40
30	8.47 860	9.99 980	8.47 880	30
40	8.47 930	9.99 980	8.47 950	20
50	8.48 000	9.99 980	8.48 020	10
44 0	8.48 069	9.99 980	8.48 089	**16** 0
10	8.48 139	9 99 980	8.48 159	50
20	8.48 208	9.99 980	8.48 228	40
30	8.48 278	9.99 980	8.48 298	30
40	8.48 347	9.99 980	8.48 367	20
50	8.48 416	9.99 980	8.48 436	10
45 0	8.48 485	9.99 980	8.48 505	**15** 0
10	8.48 554	9.99 980	8.48 574	50
20	8.48 622	9.99 980	8.48 643	40
30	8.48 691	9.99 980	8.48 711	30
40	8.48 760	9.99 979	8.48 780	20
50	8.48 828	9.99 979	8.48 849	10
46 0	8.48 896	9.99 979	8.48 917	**14** 0
10	8.48 965	9.99 979	8.48 985	50
20	8.49 033	9.99 979	8.49 053	40
30	8.49 101	9.99 979	8.49 121	30
40	8.49 169	9.99 979	8.49 189	20
50	8.49 236	9.99 979	8.49 257	10
47 0	8.49 304	9.99 979	8.49 325	**13** 0
10	8.49 372	9.99 979	8.49 393	50
20	8.49 439	9.99 979	8.49 460	40
30	8.49 506	9.99 979	8.49 528	30
40	8.49 574	9.99 979	8.49 595	20
50	8.49 641	9.99 979	8.49 662	10
48 0	8.49 708	9.99 979	8.49 729	**12** 0
10	8.49 775	9.99 979	8.49 796	50
20	8.49 842	9.99 978	8.49 863	40
30	8.49 908	9.99 978	8.49 930	30
40	8.49 975	9.99 978	8.49 997	20
50	8.50 042	9.99 978	8.50 063	10
49 0	8.50 108	9.99 978	8.50 130	**11** 0
10	8.50 174	9.99 978	8.50 196	50
20	8.50 241	9.99 978	8.50 263	40
30	8.50 307	9.99 978	8.50 329	30
40	8.50 373	9.99 978	8.50 395	20
50	8.50 439	9.99 978	8.50 461	10
50 0	8.50 504	9.99 978	8.50 527	**10** 0
′ ″	log cos	log sin	log cot	′ ″

′ ″	log sin	log cos	log tan	′ ″
50 0	8.50 504	9.99 978	8.50 527	**10** 0
10	8.50 570	9.99 978	8.50 593	50
20	8.50 636	9.99 978	8.50 658	40
30	8.50 701	9.99 978	8.50 724	30
40	8.50 767	9.99 977	8.50 789	20
50	8.50 832	9.99 977	8.50 855	10
51 0	8.50 897	9.99 977	8.50 920	**9** 0
10	8.50 963	9.99 977	8.50 985	50
20	8.51 028	9.99 977	8.51 050	40
30	8.51 092	9.99 977	8.51 115	30
40	8.51 157	9.99 977	8.51 180	20
50	8.51 222	9.99 977	8.51 245	10
52 0	8.51 287	9.99 977	8.51 310	**8** 0
10	8.51 351	9.99 977	8.51 374	50
20	8.51 416	9.99 977	8.51 439	40
30	8.51 480	9.99 977	8.51 503	30
40	8.51 544	9.99 977	8.51 568	20
50	8.51 609	9.99 977	8.51 632	10
53 0	8.51 673	9.99 977	8.51 696	**7** 0
10	8.51 737	9.99 976	8.51 760	50
20	8.51 801	9.99 976	8.51 824	40
30	8.51 864	9.99 976	8.51 888	30
40	8.51 928	9.99 976	8.51 952	20
50	8.51 992	9.99 976	8.52 015	10
54 0	8.52 055	9.99 976	8.52 079	**6** 0
10	8.52 119	9.99 976	8.52 143	50
20	8.52 182	9.99 976	8.52 206	40
30	8.52 245	9.99 976	8.52 269	30
40	8.52 308	9.99 976	8.52 332	20
50	8.52 371	9.99 976	8.52 396	10
55 0	8.52 434	9.99 976	8.52 459	**5** 0
10	8.52 497	9.99 976	8.52 522	50
20	8.52 560	9.99 976	8.52 584	40
30	8.52 623	9.99 975	8.52 647	30
40	8.52 685	9.99 975	8.52 710	20
50	8.52 748	9.99 975	8.52 772	10
56 0	8.52 810	9.99 975	8.52 835	**4** 0
10	8.52 872	9.99 975	8.52 897	50
20	8.52 935	9.99 975	8.52 960	40
30	8.52 997	9.99 975	8.53 022	30
40	8.53 059	9.99 975	8.53 084	20
50	8.53 121	9.99 975	8.53 146	10
57 0	8.53 183	9.99 975	8.53 208	**3** 0
10	8.53 245	9.99 975	8.53 270	50
20	8.53 306	9.99 975	8.53 332	40
30	8.53 368	9.99 975	8.53 393	30
40	8.53 429	9.99 975	8.53 455	20
50	8.53 491	9.99 974	8.53 516	10
58 0	8.53 552	9.99 974	8.53 578	**2** 0
10	8.53 614	9.99 974	8.53 639	50
20	8.53 675	9.99 974	8.53 700	40
30	8.53 736	9.99 974	8.53 762	30
40	8.53 797	9.99 974	8.53 823	20
50	8.53 858	9.99 974	8.53 884	10
59 0	8.53 919	9.99 974	8.53 945	**1** 0
10	8.53 979	9.99 974	8.54 005	50
20	8.54 040	9.99 974	8.54 066	40
30	8.54 101	9.99 974	8.54 127	30
40	8.54 161	9.99 974	8.54 187	20
50	8.54 222	9.99 974	8.54 248	10
60 0	8.54 282	9.99 974	8.54 308	**0** 0
′ ″	log cos	log sin	log cot	′ ″

88°

1°

′	log sin	log cos	log tan	log cot	′
	8	9	8	11	
0	24 186	99 993	24 192	75 808	**60**
1	24 903	99 993	24 910	75 090	59
2	25 609	99 993	25 616	74 384	58
3	26 304	99 993	26 312	73 688	57
4	26 988	99 992	26 996	73 004	56
5	27 661	99 992	27 669	72 331	**55**
6	28 324	99 992	28 332	71 668	54
7	28 977	99 992	28 986	71 014	53
8	29 621	99 992	29 629	70 371	52
9	30 255	99 991	30 263	69 737	51
10	30 879	99 991	30 888	69 112	**50**
11	31 495	99 991	31 505	68 495	49
12	32 103	99 990	32 112	67 888	48
13	32 702	99 990	32 711	67 289	47
14	33 292	99 990	33 302	66 698	46
15	33 875	99 990	33 886	66 114	**45**
16	34 450	99 989	34 461	65 539	44
17	35 018	99 989	35 029	64 971	43
18	35 578	99 989	35 590	64 410	42
19	36 131	99 989	36 143	63 857	41
20	36 678	99 988	36 689	63 311	**40**
21	37 217	99 988	37 229	62 771	39
22	37 750	99 988	37 762	62 238	38
23	38 276	99 987	38 289	61 711	37
24	38 796	99 987	38 809	61 191	36
25	39 310	99 987	39 323	60 677	**35**
26	39 818	99 986	39 832	60 168	34
27	40 320	99 986	40 334	59 666	33
28	40 816	99 986	40 830	59 170	32
29	41 307	99 985	41 321	58 679	31
30	41 792	99 985	41 807	58 193	**30**
31	42 272	99 985	42 287	57 713	29
32	42 746	99 984	42 762	57 238	28
33	43 216	99 984	43 232	56 768	27
34	43 680	99 984	43 696	56 304	26
35	44 139	99 983	44 156	55 844	**25**
36	44 594	99 983	44 611	55 389	24
37	45 044	99 983	45 061	54 939	23
38	45 489	99 982	45 507	54 493	22
39	45 930	99 982	45 948	54 052	21
40	46 366	99 982	46 385	53 615	**20**
41	46 799	99 981	46 817	53 183	19
42	47 226	99 981	47 245	52 755	18
43	47 650	99 981	47 669	52 331	17
44	48 069	99 980	48 089	51 911	16
45	48 485	99 980	48 505	51 495	**15**
46	48 896	99 979	48 917	51 083	14
47	49 304	99 979	49 325	50 675	13
48	49 708	99 979	49 729	50 271	12
49	50 108	99 978	50 130	49 870	11
50	50 504	99 978	50 527	49 473	**10**
51	50 897	99 977	50 920	49 080	9
52	51 287	99 977	51 310	48 690	8
53	51 673	99 977	51 696	48 304	7
54	52 055	99 976	52 079	47 921	6
55	52 434	99 976	52 459	47 541	**5**
56	52 810	99 975	52 835	47 165	4
57	53 183	99 975	53 208	46 792	3
58	53 552	99 974	53 578	46 422	2
59	53 919	99 974	53 945	46 055	1
60	54 282	99 974	54 308	45 692	**0**
′	8 log cos	9 log sin	8 log cot	11 log tan	′

88°

2°

′	log sin	log cos	log tan	log cot	′
	8	9	8	11	
0	54 282	99 974	54 308	45 692	**60**
1	54 642	99 973	54 669	45 331	59
2	54 999	99 973	55 027	44 973	58
3	55 354	99 972	55 382	44 618	57
4	55 705	99 972	55 734	44 266	56
5	56 054	99 971	56 083	43 917	**55**
6	56 400	99 971	56 429	43 571	54
7	56 743	99 970	56 773	43 227	53
8	57 084	99 970	57 114	42 886	52
9	57 421	99 969	57 452	42 548	51
10	57 757	99 969	57 788	42 212	**50**
11	58 089	99 968	58 121	41 879	49
12	58 419	99 968	58 451	41 549	48
13	58 747	99 967	58 779	41 221	47
14	59 072	99 967	59 105	40 895	46
15	59 395	99 967	59 428	40 572	**45**
16	59 715	99 966	59 749	40 251	44
17	60 033	99 966	60 068	39 932	43
18	60 349	99 965	60 384	39 616	42
19	60 662	99 964	60 698	39 302	41
20	60 973	99 964	61 009	38 991	**40**
21	61 282	99 963	61 319	38 681	39
22	61 589	99 963	61 626	38 374	38
23	61 894	99 962	61 931	38 069	37
24	62 196	99 962	62 234	37 766	36
25	62 497	99 961	62 535	37 465	**35**
26	62 795	99 961	62 834	37 166	34
27	63 091	99 960	63 131	36 869	33
28	63 385	99 960	63 426	36 574	32
29	63 678	99 959	63 718	36 282	31
30	63 968	99 959	64 009	35 991	**30**
31	64 256	99 958	64 298	35 702	29
32	64 543	99 958	64 585	35 415	28
33	64 827	99 957	64 870	35 130	27
34	65 110	99 956	65 154	34 846	26
35	65 391	99 956	65 435	34 565	**25**
36	65 670	99 955	65 715	34 285	24
37	65 947	99 955	65 993	34 007	23
38	66 223	99 954	66 269	33 731	22
39	66 497	99 954	66 543	33 457	21
40	66 769	99 953	66 816	33 184	**20**
41	67 039	99 952	67 087	32 913	19
42	67 308	99 952	67 356	32 644	18
43	67 575	99 951	67 624	32 376	17
44	67 841	99 951	67 890	32 110	16
45	68 104	99 950	68 154	31 846	**15**
46	68 367	99 949	68 417	31 583	14
47	68 627	99 949	68 678	31 322	13
48	68 886	99 948	68 938	31 062	12
49	69 144	99 948	69 196	30 804	11
50	69 400	99 947	69 453	30 547	**10**
51	69 654	99 946	69 708	30 292	9
52	69 907	99 946	69 962	30 038	8
53	70 159	99 945	70 214	29 786	7
54	70 409	99 944	70 465	29 535	6
55	70 658	99 944	70 714	29 286	**5**
56	70 905	99 943	70 962	29 038	4
57	71 151	99 942	71 208	28 792	3
58	71 395	99 942	71 453	28 547	2
59	71 638	99 941	71 697	28 303	1
60	71 880	99 940	71 940	28 060	**0**
′	8 log cos	9 log sin	8 log cot	11 log tan	′

87°

3°

′	log sin 8	log cos 9	log tan 8	log cot 11	′
0	71 880	99 940	71 940	28 060	**60**
1	72 120	940	72 181	27 819	59
2	359	939	420	580	58
3	597	938	659	341	57
4	72 834	938	72 896	27 104	56
5	73 069	99 937	73 132	26 868	**55**
6	303	936	366	634	54
7	535	936	600	400	53
8	767	935	73 832	26 168	52
9	73 997	934	74 063	25 937	51
10	74 226	99 934	74 292	25 708	**50**
11	454	933	521	479	49
12	680	932	748	252	48
13	74 906	932	74 974	25 026	47
14	75 130	931	75 199	24 801	46
15	75 353	99 930	75 423	24 577	**45**
16	575	929	645	355	44
17	75 795	929	75 867	24 133	43
18	76 015	928	76 087	23 913	42
19	234	927	306	694	41
20	76 451	99 926	76 525	23 475	**40**
21	667	926	742	258	39
22	76 883	925	76 958	23 042	38
23	77 097	924	77 173	22 827	37
24	310	923	387	613	36
25	77 522	99 923	77 600	22 400	**35**
26	733	922	77 811	22 189	34
27	77 943	921	78 022	21 978	33
28	78 152	920	232	768	32
29	360	920	441	559	31
30	78 568	99 919	78 649	21 351	**30**
31	774	918	78 855	21 145	29
32	78 979	917	79 061	20 939	28
33	79 183	917	266	734	27
34	386	916	470	530	26
35	79 588	99 915	79 673	20 327	**25**
36	789	914	79 875	20 125	24
37	79 990	913	80 076	19 924	23
38	80 189	913	277	723	22
39	388	912	476	524	21
40	80 585	99 911	80 674	19 326	**20**
41	782	910	80 872	19 128	19
42	80 978	909	81 068	18 932	18
43	81 173	909	264	736	17
44	367	908	459	541	16
45	81 560	99 907	81 653	18 347	**15**
46	752	906	81 846	18 154	14
47	81 944	905	82 038	17 962	13
48	82 134	904	230	770	12
49	324	904	420	580	11
50	82 513	99 903	82 610	17 390	**10**
51	701	902	799	201	9
52	82 888	901	82 987	17 013	8
53	83 075	900	83 175	16 825	7
54	261	899	361	639	6
55	83 446	99 898	83 547	16 453	**5**
56	630	898	732	268	4
57	813	897	83 916	16 084	3
58	83 996	896	84 100	15 900	2
59	84 177	895	282	718	1
60	84 358	99 894	84 464	15 536	**0**
′	**8** log cos	**9** log sin	**8** log cot	**11** log tan	′

86°

4°

′	log sin 8	log cos 9	log tan 8	log cot 11	′
0	84 358	99 894	84 464	15 536	**60**
1	539	893	646	354	59
2	718	892	84 826	15 174	58
3	84 897	891	85 006	14 994	57
4	85 075	891	185	815	56
5	85 252	99 890	85 363	14 637	**55**
6	429	889	540	460	54
7	605	888	717	283	53
8	780	887	85 893	14 107	52
9	85 955	886	86 069	13 931	51
10	86 128	99 885	86 243	13 757	**50**
11	301	884	417	583	49
12	474	883	591	409	48
13	645	882	763	237	47
14	816	881	86 935	13 065	46
15	86 987	99 880	87 106	12 894	**45**
16	87 156	879	277	723	44
17	325	879	447	553	43
18	494	878	616	384	42
19	661	877	785	215	41
20	87 829	99 876	87 953	12 047	**40**
21	87 995	875	88 120	11 880	39
22	88 161	874	287	713	38
23	326	873	453	547	37
24	490	872	618	382	36
25	88 654	99 871	88 783	11 217	**35**
26	817	870	88 948	11 052	34
27	88 980	869	89 111	10 889	33
28	89 142	868	274	726	32
29	304	867	437	563	31
30	89 464	99 866	89 598	10 402	**30**
31	625	865	760	240	29
32	784	864	89 920	10 080	28
33	89 943	863	90 080	09 920	27
34	90 102	862	240	760	26
35	90 260	99 861	90 399	09 601	**25**
36	417	860	557	443	24
37	574	859	715	285	23
38	730	858	90 872	09 128	22
39	90 885	857	91 029	08 971	21
40	91 040	99 856	91 185	08 815	**20**
41	195	855	340	660	19
42	349	854	495	505	18
43	502	853	650	350	17
44	655	852	803	197	16
45	91 807	99 851	91 957	08 043	**15**
46	91 959	850	92 110	07 890	14
47	92 110	848	262	738	13
48	261	847	414	586	12
49	411	846	565	435	11
50	92 561	99 845	92 716	07 284	**10**
51	710	844	92 866	07 134	9
52	92 859	843	93 016	06 984	8
53	93 007	842	165	835	7
54	154	841	313	687	6
55	93 301	99 840	93 462	06 538	**5**
56	448	839	609	391	4
57	594	838	756	244	3
58	740	837	93 903	06 097	2
59	93 885	836	94 049	05 951	1
60	94 030	99 834	94 195	05 805	**0**
′	**8** log cos	**9** log sin	**8** log cot	**11** log tan	′

85°

5°

′	log sin	log cos	log tan	log cot	′
	8	9	8	11	
0	94 030	99 834	94 195	05 805	60
1	174	833	340	660	59
2	317	832	485	515	58
3	461	831	630	370	57
4	603	830	773	227	56
5	94 746	99 829	94 917	05 083	55
6	94 887	828	95 060	04 940	54
7	95 029	827	202	798	53
8	170	825	344	656	52
9	310	824	486	514	51
10	95 450	99 823	95 627	04 373	50
11	589	822	767	233	49
12	728	821	95 908	04 092	48
13	95 867	820	96 047	03 953	47
14	96 005	819	187	813	46
15	96 143	99 817	96 325	03 675	45
16	280	816	464	536	44
17	417	815	602	398	43
18	553	814	739	261	42
19	689	813	96 877	03 123	41
20	96 825	99 812	97 013	02 987	40
21	96 960	810	150	850	39
22	97 095	809	285	715	38
23	229	808	421	579	37
24	363	807	556	444	36
25	97 496	99 806	97 691	02 309	35
26	629	804	825	175	34
27	762	803	97 959	02 041	33
28	97 894	802	98 092	01 908	32
29	98 026	801	225	775	31
30	98 157	99 800	98 358	01 642	30
31	288	798	490	510	29
32	419	797	622	378	28
33	549	796	753	247	27
34	679	795	98 884	01 116	26
35	98 808	99 793	99 015	00 985	25
36	98 937	792	145	855	24
37	99 066	791	275	725	23
38	194	790	405	595	22
39	322	788	534	466	21
40	99 450	99 787	99 662	00 338	20
41	577	786	791	209	19
42	704	785	99 919	00 081	18
43	830	783	00 046	99 954	17
44	99 956	782	174	826	16
45	00 082	99 781	00 301	99 699	15
46	207	780	427	573	14
47	332	778	553	447	13
48	456	777	679	321	12
49	581	776	805	195	11
50	00 704	99 775	00 930	99 070	10
51	828	773	01 055	98 945	9
52	00 951	772	179	821	8
53	01 074	771	303	697	7
54	196	769	427	573	6
55	01 318	99 768	01 550	98 450	5
56	440	767	673	327	4
57	561	765	796	204	3
58	682	764	01 918	98 082	2
59	803	763	02 040	97 960	1
60	01 923	99 761	02 162	97 838	0
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

84°

6°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	01 923	99 761	02 162	97 838	60
1	02 043	760	283	717	59
2	163	759	404	596	58
3	283	757	525	475	57
4	402	756	645	355	56
5	02 520	99 755	02 766	97 234	55
6	639	753	02 885	97 115	54
7	757	752	03 005	96 995	53
8	874	751	124	876	52
9	02 992	749	242	758	51
10	03 109	99 748	03 361	96 639	50
11	226	747	479	521	49
12	342	745	597	403	48
13	458	744	714	286	47
14	574	742	832	168	46
15	03 690	99 741	03 948	96 052	45
16	805	740	04 065	95 935	44
17	03 920	738	181	819	43
18	04 034	737	297	703	42
19	149	736	413	587	41
20	04 262	99 734	04 528	95 472	40
21	376	733	643	357	39
22	490	731	758	242	38
23	603	730	873	127	37
24	715	728	04 987	95 013	36
25	04 828	99 727	05 101	94 899	35
26	04 940	726	214	786	34
27	05 052	724	328	672	33
28	164	723	441	559	32
29	275	721	553	447	31
30	05 386	99 720	05 666	94 334	30
31	497	718	778	222	29
32	607	717	05 890	94 110	28
33	717	716	06 002	93 998	27
34	827	714	113	887	26
35	05 937	99 713	06 224	93 776	25
36	06 046	711	335	665	24
37	155	710	445	555	23
38	264	708	556	444	22
39	372	707	666	334	21
40	06 481	99 705	06 775	93 225	20
41	589	704	885	115	19
42	696	702	06 994	93 006	18
43	804	701	07 103	92 897	17
44	06 911	699	211	789	16
45	07 018	99 698	07 320	92 680	15
46	124	696	428	572	14
47	231	695	536	464	13
48	337	693	643	357	12
49	442	692	751	249	11
50	07 548	99 690	07 858	92 142	10
51	653	689	07 964	92 036	9
52	758	687	08 071	91 929	8
53	863	686	177	823	7
54	07 968	684	283	717	6
55	08 072	99 683	08 389	91 611	5
56	176	681	495	505	4
57	280	680	600	400	3
58	383	678	705	295	2
59	486	677	810	190	1
60	08 589	99 675	08 914	91 086	0
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

83°

7°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	08 589	99 675	08 914	91 086	60
1	692	674	09 019	90 981	59
2	795	672	123	877	58
3	897	670	227	773	57
4	08 999	669	330	670	56
5	09 101	99 667	09 434	90 566	55
6	202	666	537	463	54
7	304	664	640	360	53
8	405	663	742	258	52
9	506	661	845	155	51
10	09 606	99 659	09 947	90 053	50
11	707	658	10 049	89 951	49
12	807	656	150	850	48
13	09 907	655	252	748	47
14	10 006	653	353	647	46
15	10 106	99 651	10 454	89 546	45
16	205	650	555	445	44
17	304	648	656	344	43
18	402	647	756	244	42
19	501	645	856	144	41
20	10 599	99 643	10 956	89 044	40
21	697	642	11 056	88 944	39
22	795	640	155	845	38
23	893	638	254	746	37
24	10 990	637	353	647	36
25	11 087	99 635	11 452	88 548	35
26	184	633	551	449	34
27	281	632	649	351	33
28	377	630	747	253	32
29	474	629	845	155	31
30	11 570	99 627	11 943	88 057	30
31	666	625	12 040	87 960	29
32	761	624	138	862	28
33	857	622	235	765	27
34	11 952	620	332	668	26
35	12 047	99 618	12 428	87 572	25
36	142	617	525	475	24
37	236	615	621	379	23
38	331	613	717	283	22
39	425	612	813	187	21
40	12 519	99 610	12 909	87 091	20
41	612	608	13 004	86 996	19
42	706	607	099	901	18
43	799	605	194	806	17
44	892	603	289	711	16
45	12 985	99 601	13 384	86 616	15
46	13 078	600	478	522	14
47	171	598	573	427	13
48	263	596	667	333	12
49	355	595	761	239	11
50	13 447	99 593	13 854	86 146	10
51	539	591	13 948	86 052	9
52	630	589	14 041	85 959	8
53	722	588	134	866	7
54	813	586	227	773	6
55	13 904	99 584	14 320	85 680	5
56	13 994	582	412	588	4
57	14 085	581	504	496	3
58	175	579	597	403	2
59	266	577	688	312	1
60	14 356	99 575	14 780	85 220	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

82°

8°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	14 356	99 575	14 780	85 220	60
1	445	574	872	128	59
2	535	572	14 963	85 037	58
3	624	570	15 054	84 946	57
4	714	568	145	855	56
5	14 803	99 566	15 236	84 764	55
6	891	565	327	673	54
7	14 980	563	417	583	53
8	15 069	561	508	492	52
9	157	559	598	402	51
10	15 245	99 557	15 688	84 312	50
11	333	556	777	223	49
12	421	554	867	133	48
13	508	552	15 956	84 044	47
14	596	550	16 046	83 954	46
15	15 683	99 548	16 135	83 865	45
16	770	546	224	776	44
17	857	545	312	688	43
18	15 944	543	401	599	42
19	16 030	541	489	511	41
20	16 116	99 539	16 577	83 423	40
21	203	537	665	335	39
22	289	535	753	247	38
23	374	533	841	159	37
24	460	532	16 928	83 072	36
25	16 545	99 530	17 016	82 984	35
26	631	528	103	897	34
27	716	526	190	810	33
28	801	524	277	723	32
29	886	522	363	637	31
30	16 970	99 520	17 450	82 550	30
31	17 055	518	536	464	29
32	139	517	622	378	28
33	223	515	708	292	27
34	307	513	794	206	26
35	17 391	99 511	17 880	82 120	25
36	474	509	17 965	82 035	24
37	558	507	18 051	81 949	23
38	641	505	136	864	22
39	724	503	221	779	21
40	17 807	99 501	18 306	81 694	20
41	890	499	391	609	19
42	17 973	497	475	525	18
43	18 055	495	560	440	17
44	137	494	644	356	16
45	18 220	99 492	18 728	81 272	15
46	302	490	812	188	14
47	383	488	896	104	13
48	465	486	18 979	81 021	12
49	547	484	19 063	80 937	11
50	18 628	99 482	19 146	80 854	10
51	709	480	229	771	9
52	790	478	312	688	8
53	871	476	395	605	7
54	18 952	474	478	522	6
55	19 033	99 472	19 561	80 439	5
56	113	470	643	357	4
57	193	468	725	275	3
58	273	466	807	193	2
59	353	464	889	111	1
60	19 433	99 462	19 971	80 029	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

81°

9°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	19 433	99 462	19 971	80 029	60
1	513	460	20 053	79 947	59
2	592	458	134	866	58
3	672	456	216	784	57
4	751	454	297	703	56
5	19 830	99 452	20 378	79 622	55
6	909	450	459	541	54
7	19 988	448	540	460	53
8	20 067	446	621	379	52
9	145	444	701	299	51
10	20 223	99 442	20 782	79 218	50
11	302	440	862	138	49
12	380	438	20 942	79 058	48
13	458	436	21 022	78 978	47
14	535	434	102	898	46
15	20 613	99 432	21 182	78 818	45
16	691	429	261	739	44
17	768	427	341	659	43
18	845	425	420	580	42
19	922	423	499	501	41
20	20 999	99 421	21 578	78 422	40
21	21 076	419	657	343	39
22	153	417	736	264	38
23	229	415	814	186	37
24	306	413	893	107	36
25	21 382	99 411	21 971	78 029	35
26	458	409	22 049	77 951	34
27	534	407	127	873	33
28	610	404	205	795	32
29	685	402	283	717	31
30	21 761	99 400	22 361	77 639	30
31	836	398	438	562	29
32	912	396	516	484	28
33	21 987	394	593	407	27
34	22 062	392	670	330	26
35	22 137	99 390	22 747	77 253	25
36	211	388	824	176	24
37	286	385	901	099	23
38	361	383	22 977	77 023	22
39	435	381	23 054	76 946	21
40	22 509	99 379	23 130	76 870	20
41	583	377	206	794	19
42	657	375	283	717	18
43	731	372	359	641	17
44	805	370	435	565	16
45	22 878	99 368	23 510	76 490	15
46	22 952	366	586	414	14
47	23 025	364	661	339	13
48	098	362	737	263	12
49	171	359	812	188	11
50	23 244	99 357	23 887	76 113	10
51	317	355	23 962	76 038	9
52	390	353	24 037	75 963	8
53	462	351	112	888	7
54	535	348	186	814	6
55	23 607	99 346	24 261	75 739	5
56	679	344	335	665	4
57	752	342	410	590	3
58	823	340	484	516	2
59	895	337	558	442	1
60	23 967	99 335	24 632	75 368	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

80°

10°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	23 967	99 335	24 632	75 368	60
1	24 039	333	706	294	59
2	110	331	779	221	58
3	181	328	853	147	57
4	253	326	24 926	074	56
5	24 324	99 324	25 000	75 000	55
6	395	322	073	74 927	54
7	466	319	146	854	53
8	536	317	219	781	52
9	607	315	292	708	51
10	24 677	99 313	25 365	74 635	50
11	748	310	437	563	49
12	818	308	510	490	48
13	888	306	582	418	47
14	24 958	304	655	345	46
15	25 028	99 301	25 727	74 273	45
16	098	299	799	201	44
17	168	297	871	129	43
18	237	294	25 943	74 057	42
19	307	292	26 015	73 985	41
20	25 376	99 290	26 086	73 914	40
21	445	288	158	842	39
22	514	285	229	771	38
23	583	283	301	699	37
24	652	281	372	628	36
25	25 721	99 278	26 443	73 557	35
26	790	276	514	486	34
27	858	274	585	415	33
28	927	271	655	345	32
29	25 995	269	726	274	31
30	26 063	99 267	26 797	73 203	30
31	131	264	867	133	29
32	199	262	26 937	73 063	28
33	267	260	27 008	72 992	27
34	335	257	078	922	26
35	26 403	99 255	27 148	72 852	25
36	470	252	218	782	24
37	538	250	288	712	23
38	605	248	357	643	22
39	672	245	427	573	21
40	26 739	99 243	27 496	72 504	20
41	806	241	566	434	19
42	873	238	635	365	18
43	26 940	236	704	296	17
44	27 007	233	773	227	16
45	27 073	99 231	27 842	72 158	15
46	140	229	911	089	14
47	206	226	27 980	72 020	13
48	273	224	28 049	71 951	12
49	339	221	117	883	11
50	27 405	99 219	28 186	71 814	10
51	471	217	254	746	9
52	537	214	323	677	8
53	602	212	391	609	7
54	668	209	459	541	6
55	27 734	99 207	28 527	71 473	5
56	799	204	595	405	4
57	864	202	662	338	3
58	930	200	730	270	2
59	27 995	197	798	202	1
60	28 060	99 195	28 865	71 135	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

79°

11°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	28 060	99 195	28 865	71 135	60
1	125	192	28 933	067	59
2	190	190	29 000	71 000	58
3	254	187	067	70 933	57
4	319	185	134	866	56
5	28 384	99 182	29 201	70 799	55
6	448	180	268	732	54
7	512	177	335	665	53
8	577	175	402	598	52
9	641	172	468	532	51
10	28 705	99 170	29 535	70 465	50
11	769	167	601	399	49
12	833	165	668	332	48
13	896	162	734	266	47
14	28 960	160	800	200	46
15	29 024	99 157	29 866	70 134	45
16	087	155	932	068	44
17	150	152	29 998	70 002	43
18	214	150	30 064	69 936	42
19	277	147	130	870	41
20	29 340	99 145	30 195	69 805	40
21	403	142	261	739	39
22	466	140	326	674	38
23	529	137	391	609	37
24	591	135	457	543	36
25	29 654	99 132	30 522	69 478	35
26	716	130	587	413	34
27	779	127	652	348	33
28	841	124	717	283	32
29	903	122	782	218	31
30	29 966	99 119	30 846	69 154	30
31	30 028	117	911	089	29
32	090	114	30 975	69 025	28
33	151	112	31 040	68 960	27
34	213	109	104	896	26
35	30 275	99 106	31 168	68 832	25
36	336	104	233	767	24
37	398	101	297	703	23
38	459	099	361	639	22
39	521	096	425	575	21
40	30 582	99 093	31 489	68 511	20
41	643	091	552	448	19
42	704	088	616	384	18
43	765	086	679	321	17
44	826	083	743	257	16
45	30 887	99 080	31 806	68 194	15
46	30 947	078	870	130	14
47	31 008	075	933	067	13
48	068	072	31 996	68 004	12
49	129	070	32 059	67 941	11
50	31 189	99 067	32 122	67 878	10
51	250	064	185	815	9
52	310	062	248	752	8
53	370	059	311	689	7
54	430	056	373	627	6
55	31 490	99 054	32 436	67 564	5
56	549	051	498	502	4
57	609	048	561	439	3
58	669	046	623	377	2
59	728	043	685	315	1
60	31 788	99 040	32 747	67 253	0
′	9	9	9	10	′
	log cos	log sin	log cot	log tan	

78°

12°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	31 788	99 040	32 747	67 253	60
1	847	038	810	190	59
2	907	035	872	128	58
3	31 966	032	933	067	57
4	32 025	030	32 995	67 005	56
5	32 084	99 027	33 057	66 943	55
6	143	024	119	881	54
7	202	022	180	820	53
8	261	019	242	758	52
9	319	016	303	697	51
10	32 378	99 013	33 365	66 635	50
11	437	011	426	574	49
12	495	008	487	513	48
13	553	005	548	452	47
14	612	002	609	391	46
15	32 670	99 000	33 670	66 330	45
16	728	98 997	731	269	44
17	786	994	792	208	43
18	844	991	853	147	42
19	902	989	913	087	41
20	32 960	98 986	33 974	66 026	40
21	33 018	983	34 034	65 966	39
22	075	980	095	905	38
23	133	978	155	845	37
24	190	975	215	785	36
25	33 248	98 972	34 276	65 724	35
26	305	969	336	664	34
27	362	967	396	604	33
28	420	964	456	544	32
29	477	961	516	484	31
30	33 534	98 958	34 576	65 424	30
31	591	955	635	365	29
32	647	953	695	305	28
33	704	950	755	245	27
34	761	947	814	186	26
35	33 818	98 944	34 874	65 126	25
36	874	941	933	067	24
37	931	938	34 992	65 008	23
38	33 987	936	35 051	64 949	22
39	34 043	933	111	889	21
40	34 100	98 930	35 170	64 830	20
41	156	927	229	771	19
42	212	924	288	712	18
43	268	921	347	653	17
44	324	919	405	595	16
45	34 380	98 916	35 464	64 536	15
46	436	913	523	477	14
47	491	910	581	419	13
48	547	907	640	360	12
49	602	904	698	302	11
50	34 658	98 901	35 757	64 243	10
51	713	898	815	185	9
52	769	896	873	127	8
53	824	893	931	069	7
54	879	890	35 989	64 011	6
55	34 934	98 887	36 047	63 953	5
56	34 989	884	105	895	4
57	35 044	881	163	837	3
58	099	878	221	779	2
59	154	875	279	721	1
60	35 209	98 872	36 336	63 664	0
′	9	9	9	10	′
	log cos	log sin	log cot	log tan	

77°

13°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	35 209	98 872	36 336	63 664	**60**
1	263	869	394	606	59
2	318	867	452	548	58
3	373	864	509	491	57
4	427	861	566	434	56
5	35 481	98 858	36 624	63 376	**55**
6	536	855	681	319	54
7	590	852	738	262	53
8	644	849	795	205	52
9	698	846	852	148	51
10	35 752	98 843	36 909	63 091	**50**
11	806	840	36 966	63 034	49
12	860	837	37 023	62 977	48
13	914	834	080	920	47
14	35 968	831	137	863	46
15	36 022	98 828	37 193	62 807	**45**
16	075	825	250	750	44
17	129	822	306	694	43
18	182	819	363	637	42
19	236	816	419	581	41
20	36 289	98 813	37 476	62 524	**40**
21	342	810	532	468	39
22	395	807	588	412	38
23	449	804	644	356	37
24	502	801	700	300	36
25	36 555	98 798	37 756	62 244	**35**
26	608	795	812	188	34
27	660	792	868	132	33
28	713	789	924	076	32
29	766	786	37 980	62 020	31
30	36 819	98 783	38 035	61 965	**30**
31	871	780	091	909	29
32	924	777	147	853	28
33	36 976	774	202	798	27
34	37 028	771	257	743	26
35	37 081	98 768	38 313	61 687	**25**
36	133	765	368	632	24
37	185	762	423	577	23
38	237	759	479	521	22
39	289	756	534	466	21
40	37 341	98 753	38 589	61 411	**20**
41	393	750	644	356	19
42	445	746	699	301	18
43	497	743	754	246	17
44	549	740	808	192	16
45	37 600	98 737	38 863	61 137	**15**
46	652	734	918	082	14
47	703	731	38 972	61 028	13
48	755	728	39 027	60 973	12
49	806	725	082	918	11
50	37 858	98 722	39 136	60 864	**10**
51	909	719	190	810	9
52	37 960	715	245	755	8
53	38 011	712	299	701	7
54	062	709	353	647	6
55	38 113	98 706	39 407	60 593	**5**
56	164	703	461	539	4
57	215	700	515	485	3
58	266	697	569	431	2
59	317	694	623	377	1
60	38 368	98 690	39 677	60 323	**0**
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

76°

14°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	38 368	98 690	39 677	60 323	**60**
1	418	687	731	269	59
2	469	684	785	215	58
3	519	681	838	162	57
4	570	678	892	108	56
5	38 620	98 675	39 945	60 055	**55**
6	670	671	39 999	60 001	54
7	721	668	40 052	59 948	53
8	771	665	106	894	52
9	821	662	159	841	51
10	38 871	98 659	40 212	59 788	**50**
11	921	656	266	734	49
12	38 971	652	319	681	48
13	39 021	649	372	628	47
14	071	646	425	575	46
15	39 121	98 643	40 478	59 522	**45**
16	170	640	531	469	44
17	220	636	584	416	43
18	270	633	636	364	42
19	319	630	689	311	41
20	39 369	98 627	40 742	59 258	**40**
21	418	623	795	205	39
22	467	620	847	153	38
23	517	617	900	100	37
24	566	614	40 952	59 048	36
25	39 615	98 610	41 005	58 995	**35**
26	664	607	057	943	34
27	713	604	109	891	33
28	762	601	161	839	32
29	811	597	214	786	31
30	39 860	98 594	41 266	58 734	**30**
31	909	591	318	682	29
32	39 958	588	370	630	28
33	40 006	584	422	578	27
34	055	581	474	526	26
35	40 103	98 578	41 526	58 474	**25**
36	152	574	578	422	24
37	200	571	629	371	23
38	249	568	681	319	22
39	297	565	733	267	21
40	40 346	98 561	41 784	58 216	**20**
41	394	558	836	164	19
42	442	555	887	113	18
43	490	551	939	061	17
44	538	548	41 990	58 010	16
45	40 586	98 545	42 041	57 959	**15**
46	634	541	093	907	14
47	682	538	144	856	13
48	730	535	195	805	12
49	778	531	246	754	11
50	40 825	98 528	42 297	57 703	**10**
51	873	525	348	652	9
52	921	521	399	601	8
53	40 968	518	450	550	7
54	41 016	515	501	499	6
55	41 063	98 511	42 552	57 448	**5**
56	111	508	603	397	4
57	158	505	653	347	3
58	205	501	704	296	2
59	252	498	755	245	1
60	41 300	98 494	42 805	57 195	**0**
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

75°

15°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	41 300	98 494	42 805	57 195	60
1	347	491	856	144	59
2	394	488	906	094	58
3	441	484	42 957	57 043	57
4	488	481	43 007	56 993	56
5	41 535	98 477	43 057	56 943	55
6	582	474	108	892	54
7	628	471	158	842	53
8	675	467	208	792	52
9	722	464	258	742	51
10	41 768	98 460	43 308	56 692	50
11	815	457	358	642	49
12	861	453	408	592	48
13	908	450	458	542	47
14	41 954	447	508	492	46
15	42 001	98 443	43 558	56 442	45
16	047	440	607	393	44
17	093	436	657	343	43
18	140	433	707	293	42
19	186	429	756	244	41
20	42 232	98 426	43 806	56 194	40
21	278	422	855	145	39
22	324	419	905	095	38
23	370	415	43 954	56 046	37
24	416	412	44 004	55 996	36
25	42 461	98 409	44 053	55 947	35
26	507	405	102	898	34
27	553	402	151	849	33
28	599	398	201	799	32
29	644	395	250	750	31
30	42 690	98 391	44 299	55 701	30
31	735	388	348	652	29
32	781	384	397	603	28
33	826	381	446	554	27
34	872	377	495	505	26
35	42 917	98 373	44 544	55 456	25
36	42 962	370	592	408	24
37	43 008	366	641	359	23
38	053	363	690	310	22
39	098	359	738	262	21
40	43 143	98 356	44 787	55 213	20
41	188	352	836	164	19
42	233	349	884	116	18
43	278	345	933	067	17
44	323	342	44 981	55 019	16
45	43 367	98 338	45 029	54 971	15
46	412	334	078	922	14
47	457	331	126	874	13
48	502	327	174	826	12
49	546	324	222	778	11
50	43 591	98 320	45 271	54 729	10
51	635	317	319	681	9
52	680	313	367	633	8
53	724	309	415	585	7
54	769	306	463	537	6
55	43 813	98 302	45 511	54 489	5
56	857	299	559	441	4
57	901	295	606	394	3
58	946	291	654	346	2
59	43 990	288	702	298	1
60	44 034	98 284	45 750	54 250	0
′	log cos 9	log sin 9	log cot 9	log tan 10	′

74°

16°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	44 034	98 284	45 750	54 250	60
1	078	281	797	203	59
2	122	277	845	155	58
3	166	273	892	108	57
4	210	270	940	060	56
5	44 253	98 266	45 987	54 013	55
6	297	262	46 035	53 965	54
7	341	259	082	918	53
8	385	255	130	870	52
9	428	251	177	823	51
10	44 472	98 248	46 224	53 776	50
11	516	244	271	729	49
12	559	240	319	681	48
13	602	237	366	634	47
14	646	233	413	587	46
15	44 689	98 229	46 460	53 540	45
16	733	226	507	493	44
17	776	222	554	446	43
18	819	218	601	399	42
19	862	215	648	352	41
20	44 905	98 211	46 694	53 306	40
21	948	207	741	259	39
22	44 992	204	788	212	38
23	45 035	200	835	165	37
24	077	196	881	119	36
25	45 120	98 192	46 928	53 072	35
26	163	189	46 975	53 025	34
27	206	185	47 021	52 979	33
28	249	181	068	932	32
29	292	177	114	886	31
30	45 334	98 174	47 160	52 840	30
31	377	170	207	793	29
32	419	166	253	747	28
33	462	162	299	701	27
34	504	159	346	654	26
35	45 547	98 155	47 392	52 608	25
36	589	151	438	562	24
37	632	147	484	516	23
38	674	144	530	470	22
39	716	140	576	424	21
40	45 758	98 136	47 622	52 378	20
41	801	132	668	332	19
42	843	129	714	286	18
43	885	125	760	240	17
44	927	121	806	194	16
45	45 969	98 117	47 852	52 148	15
46	46 011	113	897	103	14
47	053	110	943	057	13
48	095	106	47 989	52 011	12
49	136	102	48 035	51 965	11
50	46 178	98 098	48 080	51 920	10
51	220	094	126	874	9
52	262	090	171	829	8
53	303	087	217	783	7
54	345	083	262	738	6
55	46 386	98 079	48 307	51 693	5
56	428	075	353	647	4
57	469	071	398	602	3
58	511	067	443	557	2
59	552	063	489	511	1
60	46 594	98 060	48 534	51 466	0
′	log cos 9	log sin 9	log cot 9	log tan 10	′

73°

17°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	46 594	98 060	48 534	51 466	**60**
1	635	056	579	421	59
2	676	052	624	376	58
3	717	048	669	331	57
4	758	044	714	286	56
5	46 800	98 040	48 759	51 241	**55**
6	841	036	804	196	54
7	882	032	849	151	53
8	923	029	894	106	52
9	46 964	025	939	061	51
10	47 005	98 021	48 984	51 016	**50**
11	045	017	49 029	50 971	49
12	086	013	073	927	48
13	127	009	118	882	47
14	168	005	163	837	46
15	47 209	98 001	49 207	50 793	**45**
16	249	97 997	252	748	44
17	290	993	296	704	43
18	330	989	341	659	42
19	371	986	385	615	41
20	47 411	97 982	49 430	50 570	**40**
21	452	978	474	526	39
22	492	974	519	481	38
23	533	970	563	437	37
24	573	966	607	393	36
25	47 613	97 962	49 652	50 348	**35**
26	654	958	696	304	34
27	694	954	740	260	33
28	734	950	784	216	32
29	774	946	828	172	31
30	47 814	97 942	49 872	50 128	**30**
31	854	938	916	084	29
32	894	934	49 960	50 040	28
33	934	930	50 004	49 996	27
34	47 974	926	048	952	26
35	48 014	97 922	50 092	49 908	**25**
36	054	918	136	864	24
37	094	914	180	820	23
38	133	910	223	777	22
39	173	906	267	733	21
40	48 213	97 902	50 311	49 689	**20**
41	252	898	355	645	19
42	292	894	398	602	18
43	332	890	442	558	17
44	371	886	485	515	16
45	48 411	97 882	50 529	49 471	**15**
46	450	878	572	428	14
47	490	874	616	384	13
48	529	870	659	341	12
49	568	866	703	297	11
50	48 607	97 861	50 746	49 254	**10**
51	647	857	789	211	9
52	686	853	833	167	8
53	725	849	876	124	7
54	764	845	919	081	6
55	48 803	97 841	50 962	49 038	**5**
56	842	837	51 005	48 995	4
57	881	833	048	952	3
58	920	829	092	908	2
59	959	825	135	865	1
60	48 998	97 821	51 178	48 822	**0**
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

72°

18°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	48 998	97 821	51 178	48 822	**60**
1	49 037	817	221	779	59
2	076	812	264	736	58
3	115	808	306	694	57
4	153	804	349	651	56
5	49 192	97 800	51 392	48 608	**55**
6	231	796	435	565	54
7	269	792	478	522	53
8	308	788	520	480	52
9	347	784	563	437	51
10	49 385	97 779	51 606	48 394	**50**
11	424	775	648	352	49
12	462	771	691	309	48
13	500	767	734	266	47
14	539	763	776	224	46
15	49 577	97 759	51 819	48 181	**45**
16	615	754	861	139	44
17	654	750	903	097	43
18	692	746	946	054	42
19	730	742	51 988	48 012	41
20	49 768	97 738	52 031	47 969	**40**
21	806	734	073	927	39
22	844	729	115	885	38
23	882	725	157	843	37
24	920	721	200	800	36
25	49 958	97 717	52 242	47 758	**35**
26	49 996	713	284	716	34
27	50 034	708	326	674	33
28	072	704	368	632	32
29	110	700	410	590	31
30	50 148	97 696	52 452	47 548	**30**
31	185	691	494	506	29
32	223	687	536	464	28
33	261	683	578	422	27
34	298	679	620	380	26
35	50 336	97 674	52 661	47 339	**25**
36	374	670	703	297	24
37	411	666	745	255	23
38	449	662	787	213	22
39	486	657	829	171	21
40	50 523	97 653	52 870	47 130	**20**
41	561	649	912	088	19
42	598	645	953	047	18
43	635	640	52 995	47 005	17
44	673	636	53 037	46 963	16
45	50 710	97 632	53 078	46 922	**15**
46	747	628	120	880	14
47	784	623	161	839	13
48	821	619	202	798	12
49	858	615	244	756	11
50	50 896	97 610	53 285	46 715	**10**
51	933	606	327	673	9
52	50 970	602	368	632	8
53	51 007	597	409	591	7
54	043	593	450	550	6
55	51 080	97 589	53 492	46 508	**5**
56	117	584	533	467	4
57	154	580	574	426	3
58	191	576	615	385	2
59	227	571	656	344	1
60	51 264	97 567	53 697	46 303	**0**
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

71°

19°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	51 264	97 567	53 697	46 303	60
1	301	563	738	262	59
2	338	558	779	221	58
3	374	554	820	180	57
4	411	550	861	139	56
5	51 447	97 545	53 902	46 098	55
6	484	541	943	057	54
7	520	536	53 984	46 016	53
8	557	532	54 025	45 975	52
9	593	528	065	935	51
10	51 629	97 523	54 106	45 894	50
11	666	519	147	853	49
12	702	515	187	813	48
13	738	510	228	772	47
14	774	506	269	731	46
15	51 811	97 501	54 309	45 691	45
16	847	497	350	650	44
17	883	492	390	610	43
18	919	488	431	569	42
19	955	484	471	529	41
20	51 991	97 479	54 512	45 488	40
21	52 027	475	552	448	39
22	063	470	593	407	38
23	099	466	633	367	37
24	135	461	673	327	36
25	52 171	97 457	54 714	45 286	35
26	207	453	754	246	34
27	242	448	794	206	33
28	278	444	835	165	32
29	314	439	875	125	31
30	52 350	97 435	54 915	45 085	30
31	385	430	955	045	29
32	421	426	54 995	45 005	28
33	456	421	55 035	44 965	27
34	492	417	075	925	26
35	52 527	97 412	55 115	44 885	25
36	563	408	155	845	24
37	598	403	195	805	23
38	634	399	235	765	22
39	669	394	275	725	21
40	52 705	97 390	55 315	44 685	20
41	740	385	355	645	19
42	775	381	395	605	18
43	811	376	434	566	17
44	846	372	474	526	16
45	52 881	97 367	55 514	44 486	15
46	916	363	554	446	14
47	951	358	593	407	13
48	52 986	353	633	367	12
49	53 021	349	673	327	11
50	53 056	97 344	55 712	44 288	10
51	092	340	752	248	9
52	126	335	791	209	8
53	161	331	831	169	7
54	196	326	870	130	6
55	53 231	97 322	55 910	44 090	5
56	266	317	949	051	4
57	301	312	55 989	44 011	3
58	336	308	56 028	43 972	2
59	370	303	067	933	1
60	53 405	97 299	56 107	43 893	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

70°

20°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	53 405	97 299	56 107	43 893	60
1	440	294	146	854	59
2	475	289	185	815	58
3	509	285	224	776	57
4	544	280	264	736	56
5	53 578	97 276	56 303	43 697	55
6	613	271	342	658	54
7	647	266	381	619	53
8	682	262	420	580	52
9	716	257	459	541	51
10	53 751	97 252	56 498	43 502	50
11	785	248	537	463	49
12	819	243	576	424	48
13	854	238	615	385	47
14	888	234	654	346	46
15	53 922	97 229	56 693	43 307	45
16	957	224	732	268	44
17	53 991	220	771	229	43
18	54 025	215	810	190	42
19	059	210	849	151	41
20	54 093	97 206	56 887	43 113	40
21	127	201	926	074	39
22	161	196	56 965	43 035	38
23	195	192	57 004	42 996	37
24	229	187	042	958	36
25	54 263	97 182	57 081	42 919	35
26	297	178	120	880	34
27	331	173	158	842	33
28	365	168	197	803	32
29	399	163	235	765	31
30	54 433	97 159	57 274	42 726	30
31	466	154	312	688	29
32	500	149	351	649	28
33	534	145	389	611	27
34	567	140	428	572	26
35	54 601	97 135	57 466	42 534	25
36	635	130	504	496	24
37	668	126	543	457	23
38	702	121	581	419	22
39	735	116	619	381	21
40	54 769	97 111	57 658	42 342	20
41	802	107	696	304	19
42	836	102	734	266	18
43	869	097	772	228	17
44	903	092	810	190	16
45	54 936	97 087	57 849	42 151	15
46	54 969	083	887	113	14
47	55 003	078	925	075	13
48	036	073	57 963	42 037	12
49	069	068	58 001	41 999	11
50	55 102	97 063	58 039	41 961	10
51	136	059	077	923	9
52	169	054	115	885	8
53	202	049	153	847	7
54	235	044	191	809	6
55	55 268	97 039	58 229	41 771	5
56	301	035	267	733	4
57	334	030	304	696	3
58	367	025	342	658	2
59	400	020	380	620	1
60	55 433	97 015	58 418	41 582	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

69°

21°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	55 433	97 015	58 418	41 582	**60**
1	466	010	455	545	59
2	499	005	493	507	58
3	532	97 001	531	469	57
4	564	96 996	569	431	56
5	55 597	96 991	58 606	41 394	**55**
6	630	986	644	356	54
7	663	981	681	319	53
8	695	976	719	281	52
9	728	971	757	243	51
10	55 761	96 966	58 794	41 206	**50**
11	793	962	832	168	49
12	826	957	869	131	48
13	858	952	907	093	47
14	891	947	944	056	46
15	55 923	96 942	58 981	41 019	**45**
16	956	937	59 019	40 981	44
17	55 988	932	056	944	43
18	56 021	927	094	906	42
19	053	922	131	869	41
20	56 085	96 917	59 168	40 832	**40**
21	118	912	205	795	39
22	150	907	243	757	38
23	182	903	280	720	37
24	215	898	317	683	36
25	56 247	96 893	59 354	40 646	**35**
26	279	888	391	609	34
27	311	883	429	571	33
28	343	878	466	534	32
29	375	873	503	497	31
30	56 408	96 868	59 540	40 460	**30**
31	440	863	577	423	29
32	472	858	614	386	28
33	504	853	651	349	27
34	536	848	688	312	26
35	56 568	96 843	59 725	40 275	**25**
36	599	838	762	238	24
37	631	833	799	201	23
38	663	828	835	165	22
39	695	823	872	128	21
40	56 727	96 818	59 909	40 091	**20**
41	759	813	946	054	19
42	790	808	59 983	40 017	18
43	822	803	60 019	39 981	17
44	854	798	056	944	16
45	56 886	96 793	60 093	39 907	**15**
46	917	788	130	870	14
47	949	783	166	834	13
48	56 980	778	203	797	12
49	57 012	772	240	760	11
50	57 044	96 767	60 276	39 724	**10**
51	075	762	313	687	9
52	107	757	349	651	8
53	138	752	386	614	7
54	169	747	422	578	6
55	57 201	96 742	60 459	39 541	**5**
56	232	737	495	505	4
57	264	732	532	468	3
58	295	727	568	432	2
59	326	722	605	395	1
60	57 358	96 717	60 641	39 359	**0**
′	9 log cos	9 log sin	9 log cot	10 log tan	′

22°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	57 358	96 717	60 641	39 359	**60**
1	389	711	677	323	59
2	420	706	714	286	58
3	451	701	750	250	57
4	482	696	786	214	56
5	57 514	96 691	60 823	39 177	**55**
6	545	686	859	141	54
7	576	681	895	105	53
8	607	676	931	069	52
9	638	670	60 967	39 033	51
10	57 669	96 665	61 004	38 996	**50**
11	700	660	040	960	49
12	731	655	076	924	48
13	762	650	112	888	47
14	793	645	148	852	46
15	57 824	96 640	61 184	38 816	**45**
16	855	634	220	780	44
17	885	629	256	744	43
18	916	624	292	708	42
19	947	619	328	672	41
20	57 978	96 614	61 364	38 636	**40**
21	58 008	608	400	600	39
22	039	603	436	564	38
23	070	598	472	528	37
24	101	593	508	492	36
25	58 131	96 588	61 544	38 456	**35**
26	162	582	579	421	34
27	192	577	615	385	33
28	223	572	651	349	32
29	253	567	687	313	31
30	58 284	96 562	61 722	38 278	**30**
31	314	556	758	242	29
32	345	551	794	206	28
33	375	546	830	170	27
34	406	541	865	135	26
35	58 436	96 535	61 901	38 099	**25**
36	467	530	936	064	24
37	497	525	61 972	38 028	23
38	527	520	62 008	37 992	22
39	557	514	043	957	21
40	58 588	96 509	62 079	37 921	**20**
41	618	504	114	886	19
42	648	498	150	850	18
43	678	493	185	815	17
44	709	488	221	779	16
45	58 739	96 483	62 256	37 744	**15**
46	769	477	292	708	14
47	799	472	327	673	13
48	829	467	362	638	12
49	859	461	398	602	11
50	58 889	96 456	62 433	37 567	**10**
51	919	451	468	532	9
52	949	445	504	496	8
53	58 979	440	539	461	7
54	59 009	435	574	426	6
55	59 039	96 429	62 609	37 391	**5**
56	069	424	645	355	4
57	098	419	680	320	3
58	128	413	715	285	2
59	158	408	750	250	1
60	59 188	96 403	62 785	37 215	**0**
′	9 log cos	9 log sin	9 log cot	10 log tan	′

68° 67°

23°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	59 188	96 403	62 785	37 215	60
1	218	397	820	180	59
2	247	392	855	145	58
3	277	387	890	110	57
4	307	381	926	074	56
5	59 336	96 376	62 961	37 039	55
6	366	370	62 996	37 004	54
7	396	365	63 031	36 969	53
8	425	360	066	934	52
9	455	354	101	899	51
10	59 484	96 349	63 135	36 865	50
11	514	343	170	830	49
12	543	338	205	795	48
13	573	333	240	760	47
14	602	327	275	725	46
15	59 632	96 322	63 310	36 690	45
16	661	316	345	655	44
17	690	311	379	621	43
18	720	305	414	586	42
19	749	300	449	551	41
20	59 778	96 294	63 484	36 516	40
21	808	289	519	481	39
22	837	284	553	447	38
23	866	278	588	412	37
24	895	273	623	377	36
25	59 924	96 267	63 657	36 343	35
26	954	262	692	308	34
27	59 983	256	726	274	33
28	60 012	251	761	239	32
29	041	245	796	204	31
30	60 070	96 240	63 830	36 170	30
31	099	234	865	135	29
32	128	229	899	101	28
33	157	223	934	066	27
34	186	218	63 968	36 032	26
35	60 215	96 212	64 003	35 997	25
36	244	207	037	963	24
37	273	201	072	928	23
38	302	196	106	894	22
39	331	190	140	860	21
40	60 359	96 185	64 175	35 825	20
41	388	179	209	791	19
42	417	174	243	757	18
43	446	168	278	722	17
44	474	162	312	688	16
45	60 503	96 157	64 346	35 654	15
46	532	151	381	619	14
47	561	146	415	585	13
48	589	140	449	551	12
49	618	135	483	517	11
50	60 646	96 129	64 517	35 483	10
51	675	123	552	448	9
52	704	118	586	414	8
53	732	112	620	380	7
54	761	107	654	346	6
55	60 789	96 101	64 688	35 312	5
56	818	095	722	278	4
57	846	090	756	244	3
58	875	084	790	210	2
59	903	079	824	176	1
60	60 931	96 073	64 858	35 142	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

66°

24°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	60 931	96 073	64 858	35 142	60
1	960	067	892	108	59
2	60 988	062	926	074	58
3	61 016	056	960	040	57
4	045	050	64 994	35 006	56
5	61 073	96 045	65 028	34 972	55
6	101	039	062	938	54
7	129	034	096	904	53
8	158	028	130	870	52
9	186	022	164	836	51
10	61 214	96 017	65 197	34 803	50
11	242	011	231	769	49
12	270	005	265	735	48
13	298	96 000	299	701	47
14	326	95 994	333	667	46
15	61 354	95 988	65 366	34 634	45
16	382	982	400	600	44
17	411	977	434	566	43
18	438	971	467	533	42
19	466	965	501	499	41
20	61 494	95 960	65 535	34 465	40
21	522	954	568	432	39
22	550	948	602	398	38
23	578	942	636	364	37
24	606	937	669	331	36
25	61 634	95 931	65 703	34 297	35
26	662	925	736	264	34
27	689	920	770	230	33
28	717	914	803	197	32
29	745	908	837	163	31
30	61 773	95 902	65 870	34 130	30
31	800	897	904	096	29
32	828	891	937	063	28
33	856	885	65 971	34 029	27
34	883	879	66 004	33 996	26
35	61 911	95 873	66 038	33 962	25
36	939	868	071	929	24
37	966	862	104	896	23
38	61 994	856	138	862	22
39	62 021	850	171	829	21
40	62 049	95 844	66 204	33 796	20
41	076	839	238	762	19
42	104	833	271	729	18
43	131	827	304	696	17
44	159	821	337	663	16
45	62 186	95 815	66 371	33 629	15
46	214	810	404	596	14
47	241	804	437	563	13
48	268	798	470	530	12
49	296	792	503	497	11
50	62 323	95 786	66 537	33 463	10
51	350	780	570	430	9
52	377	775	603	397	8
53	405	769	636	364	7
54	432	763	669	331	6
55	62 459	95 757	66 702	33 298	5
56	486	751	735	265	4
57	513	745	768	232	3
58	541	739	801	199	2
59	568	733	834	166	1
60	62 595	95 728	66 867	33 133	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

65°

25°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	62 595	95 728	66 867	33 133	**60**
1	622	722	900	100	59
2	649	716	933	067	58
3	676	710	966	034	57
4	703	704	66 999	33 001	56
5	62 730	95 698	67 032	32 968	**55**
6	757	692	065	935	54
7	784	686	098	902	53
8	811	680	131	869	52
9	838	674	163	837	51
10	62 865	95 668	67 196	32 804	**50**
11	892	663	229	771	49
12	918	657	262	738	48
13	945	651	295	705	47
14	972	645	327	673	46
15	62 999	95 639	67 360	32 640	**45**
16	63 026	633	393	607	44
17	052	627	426	574	43
18	079	621	458	542	42
19	106	615	491	509	41
20	63 133	95 609	67 524	32 476	**40**
21	159	603	556	444	39
22	186	597	589	411	38
23	213	591	622	378	37
24	239	585	654	346	36
25	63 266	95 579	67 687	32 313	**35**
26	292	573	719	281	34
27	319	567	752	248	33
28	345	561	785	215	32
29	372	555	817	183	31
30	63 398	95 549	67 850	32 150	**30**
31	425	543	882	118	29
32	451	537	915	085	28
33	478	531	947	053	27
34	504	525	67 980	32 020	26
35	63 531	95 519	68 012	31 988	**25**
36	557	513	044	956	24
37	583	507	077	923	23
38	610	500	109	891	22
39	636	494	142	858	21
40	63 662	95 488	68 174	31 826	**20**
41	689	482	206	794	19
42	715	476	239	761	18
43	741	470	271	729	17
44	767	464	303	697	16
45	63 794	95 458	68 336	31 664	**15**
46	820	452	368	632	14
47	846	446	400	600	13
48	872	440	432	568	12
49	898	434	465	535	11
50	63 924	95 427	68 497	31 503	**10**
51	950	421	529	471	9
52	63 976	415	561	439	8
53	64 002	409	593	407	7
54	028	403	626	374	6
55	64 054	95 397	68 658	31 342	**5**
56	080	391	690	310	4
57	106	384	722	278	3
58	132	378	754	246	2
59	158	372	786	214	1
60	64 184	95 366	68 818	31 182	**0**
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

26°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	64 184	95 366	68 818	31 182	**60**
1	210	360	850	150	59
2	236	354	882	118	58
3	262	348	914	086	57
4	288	341	946	054	56
5	64 313	95 335	68 978	31 022	**55**
6	339	329	69 010	30 990	54
7	365	323	042	958	53
8	391	317	074	926	52
9	417	310	106	894	51
10	64 442	95 304	69 138	30 862	**50**
11	468	298	170	830	49
12	494	292	202	798	48
13	519	286	234	766	47
14	545	279	266	734	46
15	64 571	95 273	69 298	30 702	**45**
16	596	267	329	671	44
17	622	261	361	639	43
18	647	254	393	607	42
19	673	248	425	575	41
20	64 698	95 242	69 457	30 543	**40**
21	724	236	488	512	39
22	749	229	520	480	38
23	775	223	552	448	37
24	800	217	584	416	36
25	64 826	95 211	69 615	30 385	**35**
26	851	204	647	353	34
27	877	198	679	321	33
28	902	192	710	290	32
29	927	185	742	258	31
30	64 953	95 179	69 774	30 226	**30**
31	64 978	173	805	195	29
32	65 003	167	837	163	28
33	029	160	868	132	27
34	054	154	900	100	26
35	65 079	95 148	69 932	30 068	**25**
36	104	141	963	037	24
37	130	135	69 995	30 005	23
38	155	129	70 026	29 974	22
39	180	122	058	942	21
40	65 205	95 116	70 089	29 911	**20**
41	230	110	121	879	19
42	255	103	152	848	18
43	281	097	184	816	17
44	306	090	215	785	16
45	65 331	95 084	70 247	29 753	**15**
46	356	078	278	722	14
47	381	071	309	691	13
48	406	065	341	659	12
49	431	059	372	628	11
50	65 456	95 052	70 404	29 596	**10**
51	481	046	435	565	9
52	506	039	466	534	8
53	531	033	498	502	7
54	556	027	529	471	6
55	65 580	95 020	70 560	29 440	**5**
56	605	014	592	408	4
57	630	007	623	377	3
58	655	95 001	654	346	2
59	680	94 995	685	315	1
60	65 705	94 988	70 717	29 283	**0**
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

64° 63°

27°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	65 705	94 988	70 717	29 283	**60**
1	729	982	748	252	59
2	754	975	779	221	58
3	779	969	810	190	57
4	804	962	841	159	56
5	65 828	94 956	70 873	29 127	**55**
6	853	949	904	096	54
7	878	943	935	065	53
8	902	936	966	034	52
9	927	930	70 997	29 003	51
10	65 952	94 923	71 028	28 972	**50**
11	65 976	917	059	941	49
12	66 001	911	090	910	48
13	025	904	121	879	47
14	050	898	153	847	46
15	66 075	94 891	71 184	28 816	**45**
16	099	885	215	785	44
17	124	878	246	754	43
18	148	871	277	723	42
19	173	865	308	692	41
20	66 197	94 858	71 339	28 661	**40**
21	221	852	370	630	39
22	246	845	401	599	38
23	270	839	431	569	37
24	295	832	462	538	36
25	66 319	94 826	71 493	28 507	**35**
26	343	819	524	476	34
27	368	813	555	445	33
28	392	806	586	414	32
29	416	799	617	383	31
30	66 441	94 793	71 648	28 352	**30**
31	465	786	679	321	29
32	489	780	709	291	28
33	513	773	740	260	27
34	537	767	771	229	26
35	66 562	94 760	71 802	28 198	**25**
36	586	753	833	167	24
37	610	747	863	137	23
38	634	740	894	106	22
39	658	734	925	075	21
40	66 682	94 727	71 955	28 045	**20**
41	706	720	71 986	28 014	19
42	731	714	72 017	27 983	18
43	755	707	048	952	17
44	779	700	078	922	16
45	66 803	94 694	72 109	27 891	**15**
46	827	687	140	860	14
47	851	680	170	830	13
48	875	674	201	799	12
49	899	667	231	769	11
50	66 922	94 660	72 262	27 738	**10**
51	946	654	293	707	9
52	970	647	323	677	8
53	66 994	640	354	646	7
54	67 018	634	384	616	6
55	67 042	94 627	72 415	27 585	**5**
56	066	620	445	555	4
57	090	614	476	524	3
58	113	607	506	494	2
59	137	600	537	463	1
60	67 161	94 593	72 567	27 433	**0**
′	9 log cos	9 log sin	9 log cot	10 log tan	′

62°

28°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	67 161	94 593	72 567	27 433	**60**
1	185	587	598	402	59
2	208	580	628	372	58
3	232	573	659	341	57
4	256	567	689	311	56
5	67 280	94 560	72 720	27 280	**55**
6	303	553	750	250	54
7	327	546	780	220	53
8	350	540	811	189	52
9	374	533	841	159	51
10	67 398	94 526	72 872	27 128	**50**
11	421	519	902	098	49
12	445	513	932	068	48
13	468	506	963	037	47
14	492	499	72 993	27 007	46
15	67 515	94 492	73 023	26 977	**45**
16	539	485	054	946	44
17	562	479	084	916	43
18	586	472	114	886	42
19	609	465	144	856	41
20	67 633	94 458	73 175	26 825	**40**
21	656	451	205	795	39
22	680	445	235	765	38
23	703	438	265	735	37
24	726	431	295	705	36
25	67 750	94 424	73 326	26 674	**35**
26	773	417	356	644	34
27	796	410	386	614	33
28	820	404	416	584	32
29	843	397	446	554	31
30	67 866	94 390	73 476	26 524	**30**
31	890	383	507	493	29
32	913	376	537	463	28
33	936	369	567	433	27
34	959	362	597	403	26
35	67 982	94 355	73 627	26 373	**25**
36	68 006	349	657	343	24
37	029	342	687	313	23
38	052	335	717	283	22
39	075	328	747	253	21
40	68 098	94 321	73 777	26 223	**20**
41	121	314	807	193	19
42	144	307	837	163	18
43	167	300	867	133	17
44	190	293	897	103	16
45	68 213	94 286	73 927	26 073	**15**
46	237	279	957	043	14
47	260	273	73 987	26 013	13
48	283	266	74 017	25 983	12
49	305	259	047	953	11
50	68 328	94 252	74 077	25 923	**10**
51	351	245	107	893	9
52	374	238	137	863	8
53	397	231	166	834	7
54	420	224	196	804	6
55	68 443	94 217	74 226	25 774	**5**
56	466	210	256	744	4
57	489	203	286	714	3
58	512	196	316	684	2
59	534	189	345	655	1
60	68 557	94 182	74 375	25 625	**0**
′	9 log cos	9 log sin	9 log cot	10 log tan	′

61°

29°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	68 557	94 182	74 375	25 625	60
1	580	175	405	595	59
2	603	168	435	565	58
3	625	161	465	535	57
4	648	154	494	506	56
5	68 671	94 147	74 524	25 476	55
6	694	140	554	446	54
7	716	133	583	417	53
8	739	126	613	387	52
9	762	119	643	357	51
10	68 784	94 112	74 673	25 327	50
11	807	105	702	298	49
12	829	098	732	268	48
13	852	090	762	238	47
14	875	083	791	209	46
15	68 897	94 076	74 821	25 179	45
16	920	069	851	149	44
17	942	062	880	120	43
18	965	055	910	090	42
19	68 987	048	939	061	41
20	69 010	94 041	74 969	25 031	40
21	032	034	74 998	25 002	39
22	055	027	75 028	24 972	38
23	077	020	058	942	37
24	100	012	087	913	36
25	69 122	94 005	75 117	24 883	35
26	144	93 998	146	854	34
27	167	991	176	824	33
28	189	984	205	795	32
29	212	977	235	765	31
30	69 234	93 970	75 264	24 736	30
31	256	963	294	706	29
32	279	955	323	677	28
33	301	948	353	647	27
34	323	941	382	618	26
35	69 345	93 934	75 411	24 589	25
36	368	927	441	559	24
37	390	920	470	530	23
38	412	912	500	500	22
39	434	905	529	471	21
40	69 456	93 898	75 558	24 442	20
41	479	891	588	412	19
42	501	884	617	383	18
43	523	876	647	353	17
44	545	869	676	324	16
45	69 567	93 862	75 705	24 295	15
46	589	855	735	265	14
47	611	847	764	236	13
48	633	840	793	207	12
49	655	833	822	178	11
50	69 677	93 826	75 852	24 148	10
51	699	819	881	119	9
52	721	811	910	090	8
53	743	804	939	061	7
54	765	797	969	031	6
55	69 787	93 789	75 998	24 002	5
56	809	782	76 027	23 973	4
57	831	775	056	944	3
58	853	768	086	914	2
59	875	760	115	885	1
60	69 897	93 753	76 144	23 856	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

30°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	69 897	93 753	76 144	23 856	60
1	919	746	173	827	59
2	941	738	202	798	58
3	963	731	231	769	57
4	69 984	724	261	739	56
5	70 006	93 717	76 290	23 710	55
6	028	709	319	681	54
7	050	702	348	652	53
8	072	695	377	623	52
9	093	687	406	594	51
10	70 115	93 680	76 435	23 565	50
11	137	673	464	536	49
12	159	665	493	507	48
13	180	658	522	478	47
14	202	650	551	449	46
15	70 224	93 643	76 580	23 420	45
16	245	636	609	391	44
17	267	628	639	361	43
18	288	621	668	332	42
19	310	614	697	303	41
20	70 332	93 606	76 725	23 275	40
21	353	599	754	246	39
22	375	591	783	217	38
23	396	584	812	188	37
24	418	577	841	159	36
25	70 439	93 569	76 870	23 130	35
26	461	562	899	101	34
27	482	554	928	072	33
28	504	547	957	043	32
29	525	539	76 986	23 014	31
30	70 547	93 532	77 015	22 985	30
31	568	525	044	956	29
32	590	517	073	927	28
33	611	510	101	899	27
34	633	502	130	870	26
35	70 654	93 495	77 159	22 841	25
36	675	487	188	812	24
37	697	480	217	783	23
38	718	472	246	754	22
39	739	465	274	726	21
40	70 761	93 457	77 303	22 697	20
41	782	450	332	668	19
42	803	442	361	639	18
43	824	435	390	610	17
44	846	427	418	582	16
45	70 867	93 420	77 447	22 553	15
46	888	412	476	524	14
47	909	405	505	495	13
48	931	397	533	467	12
49	952	390	562	438	11
50	70 973	93 382	77 591	22 409	10
51	70 994	375	619	381	9
52	71 015	367	648	352	8
53	036	360	677	323	7
54	058	352	706	294	6
55	71 079	93 344	77 734	22 266	5
56	100	337	763	237	4
57	121	329	791	209	3
58	142	322	820	180	2
59	163	314	849	151	1
60	71 184	93 307	77 877	22 123	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

60° 59°

31°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	71 184	93 307	77 877	22 123	**60**
1	205	299	906	094	59
2	226	291	935	065	58
3	247	284	963	037	57
4	268	276	77 992	22 008	56
5	71 289	93 269	78 020	21 980	**55**
6	310	261	049	951	54
7	331	253	077	923	53
8	352	246	106	894	52
9	373	238	135	865	51
10	71 393	93 230	78 163	21 837	**50**
11	414	223	192	808	49
12	435	215	220	780	48
13	456	207	249	751	47
14	477	200	277	723	46
15	71 498	93 192	78 306	21 694	**45**
16	519	184	334	666	44
17	539	177	363	637	43
18	560	169	391	609	42
19	581	161	419	581	41
20	71 602	93 154	78 448	21 552	**40**
21	622	146	476	524	39
22	643	138	505	495	38
23	664	131	533	467	37
24	685	123	562	438	36
25	71 705	93 115	78 590	21 410	**35**
26	726	108	618	382	34
27	747	100	647	353	33
28	767	092	675	325	32
29	788	084	704	296	31
30	71 809	93 077	78 732	21 268	**30**
31	829	069	760	240	29
32	850	061	789	211	28
33	870	053	817	183	27
34	891	046	845	155	26
35	71 911	93 038	78 874	21 126	**25**
36	932	030	902	098	24
37	952	022	930	070	23
38	973	014	959	.041	22
39	71 994	93 007	78 987	21 013	21
40	72 014	92 999	79 015	20 985	**20**
41	034	991	043	957	19
42	055	983	072	928	18
43	075	976	100	900	17
44	096	968	128	872	16
45	72 116	92 960	79 156	20 844	**15**
46	137	952	185	815	14
47	157	944	213	787	13
48	177	936	241	759	12
49	198	929	269	731	11
50	72 218	92 921	79 297	20 703	**10**
51	238	913	326	674	9
52	259	905	354	646	8
53	279	897	382	618	7
54	299	889	410	590	6
55	72 320	92 881	79 438	20 562	**5**
56	340	874	466	534	4
57	360	866	495	505	3
58	381	858	523	477	2
59	401	850	551	449	1
60	72 421	92 842	79 579	20 421	**0**
′	9 log cos	9 log sin	9 log cot	10 log tan	′

32°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	72 421	92 842	79 579	20 421	**60**
1	441	834	607	393	59
2	461	826	635	365	58
3	482	818	663	337	57
4	502	810	691	309	56
5	72 522	92 803	79 719	20 281	**55**
6	542	795	747	253	54
7	562	787	776	224	53
8	582	779	804	196	52
9	602	771	832	168	51
10	72 622	92 763	79 860	20 140	**50**
11	643	755	888	112	49
12	663	747	916	084	48
13	683	739	944	056	47
14	703	731	79 972	20 028	46
15	72 723	92 723	80 000	20 000	**45**
16	743	715	028	19 972	44
17	763	707	056	944	43
18	783	699	084	916	42
19	803	691	112	888	41
20	72 823	92 683	80 140	19 860	**40**
21	843	675	168	832	39
22	863	667	195	805	38
23	883	659	223	777	37
24	902	651	251	749	36
25	72 922	92 643	80 279	19 721	**35**
26	942	635	307	693	34
27	962	627	335	665	33
28	72 982	619	363	637	32
29	73 002	611	391	609	31
30	73 022	92 603	80 419	19 581	**30**
31	041	595	447	553	29
32	061	587	474	526	28
33	081	579	502	498	27
34	101	571	530	470	26
35	73 121	92 563	80 558	19 442	**25**
36	140	555	586	414	24
37	160	546	614	386	23
38	180	538	642	358	22
39	200	530	669	331	21
40	73 219	92 522	80 697	19 303	**20**
41	239	514	725	275	19
42	259	506	753	247	18
43	278	498	781	219	17
44	298	490	808	192	16
45	73 318	92 482	80 836	19 164	**15**
46	337	473	864	136	14
47	357	465	892	108	13
48	377	457	919	081	12
49	396	449	947	053	11
50	73 416	92 441	80 975	19 025	**10**
51	435	433	81 003	18 997	9
52	455	425	030	970	8
53	474	416	058	942	7
54	494	408	086	914	6
55	73 513	92 400	81 113	18 887	**5**
56	533	392	141	859	4
57	552	384	169	831	3
58	572	376	196	804	2
59	591	367	224	776	1
60	73 611	92 359	81 252	18 748	**0**
′	9 log cos	9 log sin	9 log cot	10 log tan	′

58° 57°

33°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	73 611	92 359	81 252	18 748	60
1	630	351	279	721	59
2	650	343	307	693	58
3	669	335	335	665	57
4	689	326	362	638	56
5	73 708	92 318	81 390	18 610	55
6	727	310	418	582	54
7	747	302	445	555	53
8	766	293	473	527	52
9	785	285	500	500	51
10	73 805	92 277	81 528	18 472	50
11	824	269	556	444	49
12	843	260	583	417	48
13	863	252	611	389	47
14	882	244	638	362	46
15	73 901	92 235	81 666	18 334	45
16	921	227	693	307	44
17	940	219	721	279	43
18	959	211	748	252	42
19	978	202	776	224	41
20	73 997	92 194	81 803	18 197	40
21	74 017	186	831	169	39
22	036	177	858	142	38
23	055	169	886	114	37
24	074	161	913	087	36
25	74 093	92 152	81 941	18 059	35
26	113	144	968	032	34
27	132	136	81 996	18 004	33
28	151	127	82 023	17 977	32
29	170	119	051	949	31
30	74 189	92 111	82 078	17 922	30
31	208	102	106	894	29
32	227	094	133	867	28
33	246	086	161	839	27
34	265	077	188	812	26
35	74 284	92 069	82 215	17 785	25
36	303	060	243	757	24
37	322	052	270	730	23
38	341	044	298	702	22
39	360	035	325	675	21
40	74 379	92 027	82 352	17 648	20
41	398	018	380	620	19
42	417	010	407	593	18
43	436	92 002	435	565	17
44	455	91 993	462	538	16
45	74 474	91 985	82 489	17 511	15
46	493	976	517	483	14
47	512	968	544	456	13
48	531	959	571	429	12
49	549	951	599	401	11
50	74 568	91 942	82 626	17 374	10
51	587	934	653	347	9
52	606	925	681	319	8
53	625	917	708	292	7
54	644	908	735	265	6
55	74 662	91 900	82 762	17 238	5
56	681	891	790	210	4
57	700	883	817	183	3
58	719	874	844	156	2
59	737	866	871	129	1
60	74 756	91 857	82 899	17 101	0
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

56°

34°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	74 756	91 857	82 899	17 101	60
1	775	849	926	074	59
2	794	840	953	047	58
3	812	832	82 980	17 020	57
4	831	823	83 008	16 992	56
5	74 850	91 815	83 035	16 965	55
6	868	806	062	938	54
7	887	798	089	911	53
8	906	789	117	883	52
9	924	781	144	856	51
10	74 943	91 772	83 171	16 829	50
11	961	763	198	802	49
12	980	755	225	775	48
13	74 999	746	252	748	47
14	75 017	738	280	720	46
15	75 036	91 729	83 307	16 693	45
16	054	720	334	666	44
17	073	712	361	639	43
18	091	703	388	612	42
19	110	695	415	585	41
20	75 128	91 686	83 442	16 558	40
21	147	677	470	530	39
22	165	669	497	503	38
23	184	660	524	476	37
24	202	651	551	449	36
25	75 221	91 643	83 578	16 422	35
26	239	634	605	395	34
27	258	625	632	368	33
28	276	617	659	341	32
29	294	608	686	314	31
30	75 313	91 599	83 713	16 287	30
31	331	591	740	260	29
32	350	582	768	232	28
33	368	573	795	205	27
34	386	565	822	178	26
35	75 405	91 556	83 849	16 151	25
36	423	547	876	124	24
37	441	538	903	097	23
38	459	530	930	070	22
39	478	521	957	043	21
40	75 496	91 512	83 984	16 016	20
41	514	504	84 011	15 989	19
42	533	495	038	962	18
43	551	486	065	935	17
44	569	477	092	908	16
45	75 587	91 469	84 119	15 881	15
46	605	460	146	854	14
47	624	451	173	827	13
48	642	442	200	800	12
49	660	433	227	773	11
50	75 678	91 425	84 254	15 746	10
51	696	416	280	720	9
52	714	407	307	693	8
53	733	398	334	666	7
54	751	389	361	639	6
55	75 769	91 381	84 388	15 612	5
56	787	372	415	585	4
57	805	363	442	558	3
58	823	354	469	531	2
59	841	345	496	504	1
60	75 859	91 336	84 523	15 477	0
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

55°

35°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	75 859	91 336	84 523	15 477	**60**
1	877	328	550	450	59
2	895	319	576	424	58
3	913	310	603	397	57
4	931	301	630	370	56
5	75 949	91 292	84 657	15 343	**55**
6	967	283	684	316	54
7	75 985	274	711	289	53
8	76 003	266	738	262	52
9	021	257	764	236	51
10	76 039	91 248	84 791	15 209	**50**
11	057	239	818	182	49
12	075	230	845	155	48
13	093	221	872	128	47
14	111	212	899	101	46
15	76 129	91 203	84 925	15 075	**45**
16	146	194	952	048	44
17	164	185	84 979	15 021	43
18	182	176	85 006	14 994	42
19	200	167	033	967	41
20	76 218	91 158	85 059	14 941	**40**
21	236	149	086	914	39
22	253	141	113	887	38
23	271	132	140	860	37
24	289	123	166	834	36
25	76 307	91 114	85 193	14 807	**35**
26	324	105	220	780	34
27	342	096	247	753	33
28	360	087	273	727	32
29	378	078	300	700	31
30	76 395	91 069	85 327	14 673	**30**
31	413	060	354	646	29
32	431	051	380	620	28
33	448	042	407	593	27
34	466	033	434	566	26
35	76 484	91 023	85 460	14 540	**25**
36	501	014	487	513	24
37	519	91 005	514	486	23
38	537	90 996	540	460	22
39	554	987	567	433	21
40	76 572	90 978	85 594	14 406	**20**
41	590	969	620	380	19
42	607	960	647	353	18
43	625	951	674	326	17
44	642	942	700	300	16
45	76 660	90 933	85 727	14 273	**15**
46	677	924	754	246	14
47	695	915	780	220	13
48	712	906	807	193	12
49	730	896	834	166	11
50	76 747	90 887	85 860	14 140	**10**
51	765	878	887	113	9
52	782	869	913	087	8
53	800	860	940	060	7
54	817	851	967	033	6
55	76 835	90 842	85 993	14 007	**5**
56	852	832	86 020	13 980	4
57	870	823	046	954	3
58	887	814	073	927	2
59	904	805	100	900	1
60	76 922	90 796	86 126	13 874	**0**
′	9 log cos	9 log sin	9 log cot	10 log tan	′

54°

36°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	76 922	90 796	86 126	13 874	**60**
1	939	787	153	847	59
2	957	777	179	821	58
3	974	768	206	794	57
4	76 991	759	232	768	56
5	77 009	90 750	86 259	13 741	**55**
6	026	741	285	715	54
7	043	731	312	688	53
8	061	722	338	662	52
9	078	713	365	635	51
10	77 095	90 704	86 392	13 608	**50**
11	112	694	418	582	49
12	130	685	445	555	48
13	147	676	471	529	47
14	164	667	498	502	46
15	77 181	90 657	86 524	13 476	**45**
16	199	648	551	449	44
17	216	639	577	423	43
18	233	630	603	397	42
19	250	620	630	370	41
20	77 268	90 611	86 656	13 344	**40**
21	285	602	683	317	39
22	302	592	709	291	38
23	319	583	736	264	37
24	336	574	762	238	36
25	77 353	90 565	86 789	13 211	**35**
26	370	555	815	185	34
27	387	546	842	158	33
28	405	537	868	132	32
29	422	527	894	106	31
30	77 439	90 518	86 921	13 079	**30**
31	456	509	947	053	29
32	473	499	86 974	026	28
33	490	490	87 000	13 000	27
34	507	480	027	12 973	26
35	77 524	90 471	87 053	12 947	**25**
36	541	462	079	921	24
37	558	452	106	894	23
38	575	443	132	868	22
39	592	434	158	842	21
40	77 609	90 424	87 185	12 815	**20**
41	626	415	211	789	19
42	643	405	238	762	18
43	660	396	264	736	17
44	677	386	290	710	16
45	77 694	90 377	87 317	12 683	**15**
46	711	368	343	657	14
47	728	358	369	631	13
48	744	349	396	604	12
49	761	339	422	578	11
50	77 778	90 330	87 448	12 552	**10**
51	795	320	475	525	9
52	812	311	501	499	8
53	829	301	527	473	7
54	846	292	554	446	6
55	77 862	90 282	87 580	12 420	**5**
56	879	273	606	394	4
57	896	263	633	367	3
58	913	254	659	341	2
59	930	244	685	315	1
60	77 946	90 235	87 711	12 289	**0**
′	9 log cos	9 log sin	9 log cot	10 log tan	′

53°

37°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	77 946	90 235	87 711	12 289	60
1	963	225	738	262	59
2	980	216	764	236	58
3	77 997	206	790	210	57
4	78 013	197	817	183	56
5	78 030	90 187	87 843	12 157	55
6	047	178	869	131	54
7	063	168	895	105	53
8	080	159	922	078	52
9	097	149	948	052	51
10	78 113	90 139	87 974	12 026	50
11	130	130	88 000	12 000	49
12	147	120	027	11 973	48
13	163	111	053	947	47
14	180	101	079	921	46
15	78 197	90 091	88 105	11 895	45
16	213	082	131	869	44
17	230	072	158	842	43
18	246	063	184	816	42
19	263	053	210	790	41
20	78 280	90 043	88 236	11 764	40
21	296	034	262	738	39
22	313	024	289	711	38
23	329	014	315	685	37
24	346	90 005	341	659	36
25	78 362	89 995	88 367	11 633	35
26	379	985	393	607	34
27	395	976	420	580	33
28	412	966	446	554	32
29	428	956	472	528	31
30	78 445	89 947	88 498	11 502	30
31	461	937	524	476	29
32	478	927	550	450	28
33	494	918	577	423	27
34	510	908	603	397	26
35	78 527	89 898	88 629	11 371	25
36	543	888	655	345	24
37	560	879	681	319	23
38	576	869	707	293	22
39	592	859	733	267	21
40	78 609	89 849	88 759	11 241	20
41	625	840	786	214	19
42	642	830	812	188	18
43	658	820	838	162	17
44	674	810	864	136	16
45	78 691	89 801	88 890	11 110	15
46	707	791	916	084	14
47	723	781	942	058	13
48	739	771	968	032	12
49	756	761	88 994	11 006	11
50	78 772	89 752	89 020	10 980	10
51	788	742	046	954	9
52	805	732	073	927	8
53	821	722	099	901	7
54	837	712	125	875	6
55	78 853	89 702	89 151	10 849	5
56	869	693	177	823	4
57	886	683	203	797	3
58	902	673	229	771	2
59	918	663	255	745	1
60	78 934	89 653	89 281	10 719	0
′	log cos 9	log sin 9	log cot 9	log tan 10	′

52°

38°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	78 934	89 653	89 281	10 719	60
1	950	643	307	693	59
2	967	633	333	667	58
3	983	624	359	641	57
4	78 999	614	385	615	56
5	79 015	89 604	89 411	10 589	55
6	031	594	437	563	54
7	047	584	463	537	53
8	063	574	489	511	52
9	079	564	515	485	51
10	79 095	89 554	89 541	10 459	50
11	111	544	567	433	49
12	128	534	593	407	48
13	144	524	619	381	47
14	160	514	645	355	46
15	79 176	89 504	89 671	10 329	45
16	192	495	697	303	44
17	208	485	723	277	43
18	224	475	749	251	42
19	240	465	775	225	41
20	79 256	89 455	89 801	10 199	40
21	272	445	827	173	39
22	288	435	853	147	38
23	304	425	879	121	37
24	319	415	905	095	36
25	79 335	89 405	89 931	10 069	35
26	351	395	957	043	34
27	367	385	89 983	10 017	33
28	383	375	90 009	09 991	32
29	399	364	035	965	31
30	79 415	89 354	90 061	09 939	30
31	431	344	086	914	29
32	447	334	112	888	28
33	463	324	138	862	27
34	478	314	164	836	26
35	79 494	89 304	90 190	09 810	25
36	510	294	216	784	24
37	526	284	242	758	23
38	542	274	268	732	22
39	558	264	294	706	21
40	79 573	89 254	90 320	09 680	20
41	589	244	346	654	19
42	605	233	371	629	18
43	621	223	397	603	17
44	636	213	423	577	16
45	79 652	89 203	90 449	09 551	15
46	668	193	475	525	14
47	684	183	501	499	13
48	699	173	527	473	12
49	715	162	553	447	11
50	79 731	89 152	90 578	09 422	10
51	746	142	604	396	9
52	762	132	630	370	8
53	778	122	656	344	7
54	793	112	682	318	6
55	79 809	89 101	90 708	09 292	5
56	825	091	734	266	4
57	840	081	759	241	3
58	856	071	785	215	2
59	872	060	811	189	1
60	79 887	89 050	90 837	09 163	0
′	log cos 9	log sin 9	log cot 9	log tan 10	′

51°

39°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	79 887	89 050	90 837	09 163	60
1	903	040	863	137	59
2	918	030	889	111	58
3	934	020	914	086	57
4	950	89 009	940	060	56
5	79 965	88 999	90 966	09 034	55
6	981	989	90 992	09 008	54
7	79 996	978	91 018	08 982	53
8	80 012	968	043	957	52
9	027	958	069	931	51
10	80 043	88 948	91 095	08 905	50
11	058	937	121	879	49
12	074	927	147	853	48
13	089	917	172	828	47
14	105	906	198	802	46
15	80 120	88 896	91 224	08 776	45
16	136	886	250	750	44
17	151	875	276	724	43
18	166	865	301	699	42
19	182	855	327	673	41
20	80 197	88 844	91 353	08 647	40
21	213	834	379	621	39
22	228	824	404	596	38
23	244	813	430	570	37
24	259	803	456	544	36
25	80 274	88 793	91 482	08 518	35
26	290	782	507	493	34
27	305	772	533	467	33
28	320	761	559	441	32
29	336	751	585	415	31
30	80 351	88 741	91 610	08 390	30
31	366	730	636	364	29
32	382	720	662	338	28
33	397	709	688	312	27
34	412	699	713	287	26
35	80 428	88 688	91 739	08 261	25
36	443	678	765	235	24
37	458	668	791	209	23
38	473	657	816	184	22
39	489	647	842	158	21
40	80 504	88 636	91 868	08 132	20
41	519	626	893	107	19
42	534	615	919	081	18
43	550	605	945	055	17
44	565	594	971	029	16
45	80 580	88 584	91 996	08 004	15
46	595	573	92 022	07 978	14
47	610	563	048	952	13
48	625	552	073	927	12
49	641	542	099	901	11
50	80 656	88 531	92 125	07 875	10
51	671	521	150	850	9
52	686	510	176	824	8
53	701	499	202	798	7
54	716	489	227	773	6
55	80 731	88 478	92 253	07 747	5
56	746	468	279	721	4
57	762	457	304	696	3
58	777	447	330	670	2
59	792	436	356	644	1
60	80 807	88 425	92 381	07 619	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

40°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	80 807	88 425	92 381	07 619	60
1	822	415	407	593	59
2	837	404	433	567	58
3	852	394	458	542	57
4	867	383	484	516	56
5	80 882	88 372	92 510	07 490	55
6	897	362	535	465	54
7	912	351	561	439	53
8	927	340	587	413	52
9	942	330	612	388	51
10	80 957	88 319	92 638	07 362	50
11	972	308	663	337	49
12	80 987	298	689	311	48
13	81 002	287	715	285	47
14	017	276	740	260	46
15	81 032	88 266	92 766	07 234	45
16	047	255	792	208	44
17	061	244	817	183	43
18	076	234	843	157	42
19	091	223	868	132	41
20	81 106	88 212	92 894	07 106	40
21	121	201	920	080	39
22	136	191	945	055	38
23	151	180	971	029	37
24	166	169	92 996	07 004	36
25	81 180	88 158	93 022	06 978	35
26	195	148	048	952	34
27	210	137	073	927	33
28	225	126	099	901	32
29	240	115	124	876	31
30	81 254	88 105	93 150	06 850	30
31	269	094	175	825	29
32	284	083	201	799	28
33	299	072	227	773	27
34	314	061	252	748	26
35	81 328	88 051	93 278	06 722	25
36	343	040	303	697	24
37	358	029	329	671	23
38	372	018	354	646	22
39	387	88 007	380	620	21
40	81 402	87 996	93 406	06 594	20
41	417	985	431	569	19
42	431	975	457	543	18
43	446	964	482	518	17
44	461	953	508	492	16
45	81 475	87 942	93 533	06 467	15
46	490	931	559	441	14
47	505	920	584	416	13
48	519	909	610	390	12
49	534	898	636	364	11
50	81 549	87 887	93 661	06 339	10
51	563	877	687	313	9
52	578	866	712	288	8
53	592	855	738	262	7
54	607	844	763	237	6
55	81 622	87 833	93 789	06 211	5
56	636	822	814	186	4
57	651	811	840	160	3
58	665	800	865	135	2
59	680	789	891	109	1
60	81 694	87 778	93 916	06 084	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

50°

49°

41°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	81 694	87 778	93 916	06 084	60
1	709	767	942	058	59
2	723	756	967	033	58
3	738	745	93 993	06 007	57
4	752	734	94 018	05 982	56
5	81 767	87 723	94 044	05 956	55
6	781	712	069	931	54
7	796	701	095	905	53
8	810	690	120	880	52
9	825	679	146	854	51
10	81 839	87 668	94 171	05 829	50
11	854	657	197	803	49
12	868	646	222	778	48
13	882	635	248	752	47
14	897	624	273	727	46
15	81 911	87 613	94 299	05 701	45
16	926	601	324	676	44
17	940	590	350	650	43
18	955	579	375	625	42
19	969	568	401	599	41
20	81 983	87 557	94 426	05 574	40
21	81 998	546	452	548	39
22	82 012	535	477	523	38
23	026	524	503	497	37
24	041	513	528	472	36
25	82 055	87 501	94 554	05 446	35
26	069	490	579	421	34
27	084	479	604	396	33
28	098	468	630	370	32
29	112	457	655	345	31
30	82 126	87 446	94 681	05 319	30
31	141	434	706	294	29
32	155	423	732	268	28
33	169	412	757	243	27
34	184	401	783	217	26
35	82 198	87 390	94 808	05 192	25
36	212	378	834	166	24
37	226	367	859	141	23
38	240	356	884	116	22
39	255	345	910	090	21
40	82 269	87 334	94 935	05 065	20
41	283	322	961	039	19
42	297	311	94 986	05 014	18
43	311	300	95 012	04 988	17
44	326	288	037	963	16
45	82 340	87 277	95 062	04 938	15
46	354	266	088	912	14
47	368	255	113	887	13
48	382	243	139	861	12
49	396	232	164	836	11
50	82 410	87 221	95 190	04 810	10
51	424	209	215	785	9
52	439	198	240	760	8
53	453	187	266	734	7
54	467	175	291	709	6
55	82 481	87 164	95 317	04 683	5
56	495	153	342	658	4
57	509	141	368	632	3
58	523	130	393	607	2
59	537	119	418	582	1
60	82 551	87 107	95 444	04 556	0
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

42°

′	log sin	log cos	log tan	log cot	′
	9	9	9	10	
0	82 551	87 107	95 444	04 556	60
1	565	096	469	531	59
2	579	085	495	505	58
3	593	073	520	480	57
4	607	062	545	455	56
5	82 621	87 050	95 571	04 429	55
6	635	039	596	404	54
7	649	028	622	378	53
8	663	016	647	353	52
9	677	87 005	672	328	51
10	82 691	86 993	95 698	04 302	50
11	705	982	723	277	49
12	719	970	748	252	48
13	733	959	774	226	47
14	747	947	799	201	46
15	82 761	86 936	95 825	04 175	45
16	775	924	850	150	44
17	788	913	875	125	43
18	802	902	901	099	42
19	816	890	926	074	41
20	82 830	86 879	95 952	04 048	40
21	844	867	95 977	04 023	39
22	858	855	96 002	03 998	38
23	872	844	028	972	37
24	885	832	053	947	36
25	82 899	86 821	96 078	03 922	35
26	913	809	104	896	34
27	927	798	129	871	33
28	941	786	155	845	32
29	955	775	180	820	31
30	82 968	86 763	96 205	03 795	30
31	982	752	231	769	29
32	82 996	740	256	744	28
33	83 010	728	281	719	27
34	023	717	307	693	26
35	83 037	86 705	96 332	03 668	25
36	051	694	357	643	24
37	065	682	383	617	23
38	078	670	408	592	22
39	092	659	433	567	21
40	83 106	86 647	96 459	03 541	20
41	120	635	484	516	19
42	133	624	510	490	18
43	147	612	535	465	17
44	161	600	560	440	16
45	83 174	86 589	96 586	03 414	15
46	188	577	611	389	14
47	202	565	636	364	13
48	215	554	662	338	12
49	229	542	687	313	11
50	83 242	86 530	96 712	03 288	10
51	256	518	738	262	9
52	270	507	763	237	8
53	283	495	788	212	7
54	297	483	814	186	6
55	83 310	86 472	96 839	03 161	5
56	324	460	864	136	4
57	338	448	890	110	3
58	351	436	915	085	2
59	365	425	940	060	1
60	83 378	86 413	96 966	03 034	0
	9	9	9	10	
′	log cos	log sin	log cot	log tan	′

48° 47°

43°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	83 378	86 413	96 966	03 034	60
1	392	401	96 991	03 009	59
2	405	389	97 016	02 984	58
3	419	377	042	958	57
4	432	366	067	933	56
5	83 446	86 354	97 092	02 908	55
6	459	342	118	882	54
7	473	330	143	857	53
8	486	318	168	832	52
9	500	306	193	807	51
10	83 513	86 295	97 219	02 781	50
11	527	283	244	756	49
12	540	271	269	731	48
13	554	259	295	705	47
14	567	247	320	680	46
15	83 581	86 235	97 345	02 655	45
16	594	223	371	629	44
17	608	211	396	604	43
18	621	200	421	579	42
19	634	188	447	553	41
20	83 648	86 176	97 472	02 528	40
21	661	164	497	503	39
22	674	152	523	477	38
23	688	140	548	452	37
24	701	128	573	427	36
25	83 715	86 116	97 598	02 402	35
26	728	104	624	376	34
27	741	092	649	351	33
28	755	080	674	326	32
29	768	068	700	300	31
30	83 781	86 056	97 725	02 275	30
31	795	044	750	250	29
32	808	032	776	224	28
33	821	020	801	199	27
34	834	86 008	826	174	26
35	83 848	85 996	97 851	02 149	25
36	861	984	877	123	24
37	874	972	902	098	23
38	887	960	927	073	22
39	901	948	953	047	21
40	83 914	85 936	97 978	02 022	20
41	927	924	98 003	01 997	19
42	940	912	029	971	18
43	954	900	054	946	17
44	967	888	079	921	16
45	83 980	85 876	98 104	01 896	15
46	83 993	864	130	870	14
47	84 006	851	155	845	13
48	020	839	180	820	12
49	033	827	206	794	11
50	84 046	85 815	98 231	01 769	10
51	059	803	256	744	9
52	072	791	281	719	8
53	085	779	307	693	7
54	098	766	332	668	6
55	84 112	85 754	98 357	01 643	5
56	125	742	383	617	4
57	138	730	408	592	3
58	151	718	433	567	2
59	164	706	458	542	1
60	84 177	85 693	98 484	01 516	0
′	9 log cos	9 log sin	9 log cot	10 log tan	′

44°

′	log sin 9	log cos 9	log tan 9	log cot 10	′
0	84 177	85 693	98 484	01 516	60
1	190	681	509	491	59
2	203	669	534	466	58
3	216	657	560	440	57
4	229	645	585	415	56
5	84 242	85 632	98 610	01 390	55
6	255	620	635	365	54
7	269	608	661	339	53
8	282	596	686	314	52
9	295	583	711	289	51
10	84 308	85 571	98 737	01 263	50
11	321	559	762	238	49
12	334	547	787	213	48
13	347	534	812	188	47
14	360	522	838	162	46
15	84 373	85 510	98 863	01 137	45
16	385	497	888	112	44
17	398	485	913	087	43
18	411	473	939	061	42
19	424	460	964	036	41
20	84 437	85 448	98 989	01 011	40
21	450	436	99 015	00 985	39
22	463	423	040	960	38
23	476	411	065	935	37
24	489	399	090	910	36
25	84 502	85 386	99 116	00 884	35
26	515	374	141	859	34
27	528	361	166	834	33
28	540	349	191	809	32
29	553	337	217	783	31
30	84 566	85 324	99 242	00 758	30
31	579	312	267	733	29
32	592	299	293	707	28
33	605	287	318	682	27
34	618	274	343	657	26
35	84 630	85 262	99 368	00 632	25
36	643	250	394	606	24
37	656	237	419	581	23
38	669	225	444	556	22
39	682	212	469	531	21
40	84 694	85 200	99 495	00 505	20
41	707	187	520	480	19
42	720	175	545	455	18
43	733	162	570	430	17
44	745	150	596	404	16
45	84 758	85 137	99 621	00 379	15
46	771	125	646	354	14
47	784	112	672	328	13
48	796	100	697	303	12
49	809	087	722	278	11
50	84 822	85 074	99 747	00 253	10
51	835	062	773	227	9
52	847	049	798	202	8
53	860	037	823	177	7
54	873	024	848	152	6
55	84 885	85 012	99 874	00 126	5
56	898	84 999	899	101	4
57	911	986	924	076	3
58	923	974	949	051	2
59	936	961	975	025	1
60	84 949	84 949	00 000	00 000	0
′	9 log cos	9 log sin	10 log cot	10 log tan	′

46°

45°

TABLE IV

NATURAL TRIGONOMETRIC FUNCTIONS

Table IV gives the actual values of the functions sine, cosine, tangent, and cotangent of angles ranging from 0° to 90°. The values are given at intervals of 1′.

For all angles less than 45° the degrees are found at the *top* of the page, the minutes at the *left*, and the name of the function at the *top*. For all angles between 45° and 90° the degrees are found at the *bottom* of the page, the minutes at the *right*, and the name of the function at the *bottom*.

Interpolation for seconds is just like ordinary interpolation except that sixtieths instead of tenths must be used.

EXAMPLE 1. Find tan 32° 24′ 48″.

$$\tan 32° 24' 48'' = 0.63462 + \tfrac{48}{60} \cdot 41 = 0.63495.$$

Note that sine and tangent are *increasing* functions, so that the proportional part is to be *added* in interpolation. On the other hand cosine and cotangent are *decreasing* functions, so that proportional parts are to be *subtracted*.

EXAMPLE 2. Find cos 43° 51′ 24″.

$$\cos 43° 51' 24'' = 0.72116 - \tfrac{24}{60} \cdot 21 = 0.72108.$$

Interpolation is not accurate for values of the tangent near 90° nor for values of the cotangent near 0°.

0°

/	sin	cos	tan	cot	/
0	.00000	1.0000	.00000	—	**60**
1	029	000	029	3437.7	59
2	058	000	058	1718.9	58
3	087	000	087	1145.9	57
4	116	000	116	859.44	56
5	.00145	1.0000	.00145	687.55	**55**
6	175	000	175	572.96	54
7	204	000	204	491.11	53
8	233	000	233	429.72	52
9	262	000	262	381.97	51
10	.00291	1.0000	.00291	343.77	**50**
11	320	.99999	320	312.52	49
12	349	999	349	286.48	48
13	378	999	378	264.44	47
14	407	999	407	245.55	46
15	.00436	.99999	.00436	229.18	**45**
16	465	999	465	214.86	44
17	495	999	495	202.22	43
18	524	999	524	190.98	42
19	553	998	553	180.93	41
20	.00582	.99998	.00582	171.89	**40**
21	611	998	611	163.70	39
22	640	998	640	156.26	38
23	669	998	669	149.47	37
24	698	998	698	143.24	36
25	.00727	.99997	.00727	137.51	**35**
26	756	997	756	132.22	34
27	785	997	785	127.32	33
28	814	997	815	122.77	32
29	844	996	844	118.54	31
30	.00873	.99996	.00873	114.59	**30**
31	902	996	902	110.89	29
32	931	996	931	107.43	28
33	960	995	960	104.17	27
34	.00989	995	.00989	101.11	26
35	.01018	.99995	.01018	98.218	**25**
36	047	995	047	95.489	24
37	076	994	076	92.908	23
38	105	994	105	90.463	22
39	134	994	135	88.144	21
40	.01164	.99993	.01164	85.940	**20**
41	193	993	193	83.844	19
42	222	993	222	81.847	18
43	251	992	251	79.943	17
44	280	992	280	78.126	16
45	.01309	.99991	.01309	76.390	**15**
46	338	991	338	74.729	14
47	367	991	367	73.139	13
48	396	990	396	71.615	12
49	425	990	425	70.153	11
50	.01454	.99989	.01455	68.750	**10**
51	483	989	484	67.402	9
52	513	989	513	66.105	8
53	542	988	542	64.858	7
54	571	988	571	63.657	6
55	.01600	.99987	.01600	62.499	**5**
56	629	987	629	61.383	4
57	658	986	658	60.306	3
58	687	986	687	59.266	2
59	716	985	716	58.261	1
60	.01745	99985	.01746	57.290	**0**
/	cos	sin	cot	tan	/

89°

1°

/	sin	cos	tan	cot	/
0	.01745	.99985	.01746	57.290	**60**
1	774	984	775	56.351	59
2	803	984	804	55.442	58
3	832	983	833	54.561	57
4	862	983	862	53.709	56
5	.01891	.99982	.01891	52.882	**55**
6	920	982	920	52.081	54
7	949	981	949	51.303	53
8	.01978	980	.01978	50.549	52
9	.02007	980	02007	49.816	51
10	.02036	.99979	.02036	49.104	**50**
11	065	979	066	48.412	49
12	094	978	095	47.740	48
13	123	977	124	47.085	47
14	152	977	153	46.449	46
15	.02181	.99976	.02182	45.829	**45**
16	211	976	211	45.226	44
17	240	975	240	44.639	43
18	269	974	269	44.066	42
19	298	974	298	43.508	41
20	.02327	.99973	.02328	42.964	**40**
21	356	972	357	42.433	39
22	385	972	386	41.916	38
23	414	971	415	41.411	37
24	443	970	444	40.917	36
25	.02472	.99969	.02473	40.436	**35**
26	501	969	502	39.965	34
27	530	968	531	39.506	33
28	560	967	560	39.057	32
29	589	966	589	38.618	31
30	.02618	.99966	.02619	38.188	**30**
31	647	965	648	37.769	29
32	676	964	677	37.358	28
33	705	963	706	36.956	27
34	734	963	735	36.563	26
35	.02763	.99962	.02764	36.178	**25**
36	792	961	793	35.801	24
37	821	960	822	35.431	23
38	850	959	851	35.070	22
39	879	959	881	34.715	21
40	.02908	.99958	.02910	34.368	**20**
41	938	957	939	34.027	19
42	967	956	968	33.694	18
43	.02996	955	.02997	33.366	17
44	.03025	954	.03026	33.045	16
45	.03054	.99953	.03055	32.730	**15**
46	083	952	084	32.421	14
47	112	952	114	32.118	13
48	141	951	143	31.821	12
49	170	950	172	31.528	11
50	.03199	.99949	.03201	31.242	**10**
51	228	948	230	30.960	9
52	257	947	259	30.683	8
53	286	946	288	30.412	7
54	316	945	317	30.145	6
55	.03345	.99944	.03346	29.882	**5**
56	374	943	376	29.624	4
57	403	942	405	29.371	3
58	432	941	434	29.122	2
59	461	940	463	28.877	1
60	.03490	.99939	.03492	28.636	**0**
/	cos	sin	cot	tan	/

88°

2°

′	sin	cos	tan	cot	′
0	.03490	.99939	.03492	28.636	**60**
1	519	938	521	.399	59
2	548	937	550	28.166	58
3	577	936	579	27.937	57
4	606	935	609	.712	56
5	.03635	.99934	.03638	27.490	**55**
6	664	933	667	.271	54
7	693	932	696	27.057	53
8	723	931	725	26.845	52
9	752	930	754	.637	51
10	.03781	.99929	.03783	26.432	**50**
11	810	927	812	.230	49
12	839	926	842	26.031	48
13	868	925	871	25.835	47
14	897	924	900	.642	46
15	.03926	.99923	.03929	25.452	**45**
16	955	922	958	.264	44
17	.03984	921	.03987	25.080	43
18	.04013	919	.04016	24.898	42
19	042	918	046	.719	41
20	.04071	.99917	.04075	24.542	**40**
21	100	916	104	.368	39
22	129	915	133	.196	38
23	159	913	162	24.026	37
24	188	912	191	23.859	36
25	.04217	.99911	.04220	23.695	**35**
26	246	910	250	.532	34
27	275	909	279	.372	33
28	304	907	308	.214	32
29	333	906	337	23.058	31
30	.04362	.99905	.04366	22.904	**30**
31	391	904	395	.752	29
32	420	902	424	.602	28
33	449	901	454	.454	27
34	478	900	483	.308	26
35	.04507	.99898	.04512	22.164	**25**
36	536	897	541	22.022	24
37	565	896	570	21.881	23
38	594	894	599	.743	22
39	623	893	628	.606	21
40	.04653	.99892	.04658	21.470	**20**
41	682	890	687	.337	19
42	711	889	716	.205	18
43	740	888	745	21.075	17
44	769	886	774	20.946	16
45	.04798	.99885	.04803	20.819	**15**
46	827	883	833	.693	14
47	856	882	862	.569	13
48	885	881	891	.446	12
49	914	879	920	.325	11
50	.04943	.99878	.04949	20.206	**10**
51	.04972	876	.04978	20.087	9
52	.05001	875	.05007	19.970	8
53	030	873	037	.855	7
54	059	872	066	.740	6
55	.05088	.99870	.05095	19.627	**5**
56	117	869	124	.516	4
57	146	867	153	.405	3
58	175	866	182	.296	2
59	205	864	212	.188	1
60	.05234	.99863	.05241	19.081	**0**
′	cos	sin	cot	tan	′

87°

3°

′	sin	cos	tan	cot	′
0	.05234	.99863	.05241	19.081	**60**
1	263	861	270	18.976	59
2	292	860	299	.871	58
3	321	858	328	.768	57
4	350	857	357	.666	56
5	.05379	.99855	.05387	18.564	**55**
6	408	854	416	.464	54
7	437	852	445	.366	53
8	466	851	474	.268	52
9	495	849	503	.171	51
10	.05524	.99847	.05533	18.075	**50**
11	553	846	562	17.980	49
12	582	844	591	.886	48
13	611	842	620	.793	47
14	640	841	649	.702	46
15	.05669	.99839	.05678	17.611	**45**
16	698	838	708	.521	44
17	727	836	737	.431	43
18	756	834	766	.343	42
19	785	833	795	.256	41
20	.05814	.99831	.05824	17.169	**40**
21	844	829	854	17.084	39
22	873	827	883	16.999	38
23	902	826	912	.915	37
24	931	824	941	.832	36
25	.05960	.99822	.05970	16.750	**35**
26	.05989	821	.05999	.668	34
27	.06018	819	.06029	.587	33
28	047	817	058	.507	32
29	076	815	087	.428	31
30	.06105	.99813	.06116	16.350	**30**
31	134	812	145	.272	29
32	163	810	175	.195	28
33	192	808	204	.119	27
34	221	806	233	16.043	26
35	.06250	.99804	.06262	15.969	**25**
36	279	803	291	.895	24
37	308	801	321	.821	23
38	337	799	350	.748	22
39	366	797	379	.676	21
40	.06395	.99795	.06408	15.605	**20**
41	424	793	438	.534	19
42	453	792	467	.464	18
43	482	790	496	.394	17
44	511	788	525	.325	16
45	.06540	.99786	.06554	15.257	**15**
46	569	784	584	.189	14
47	598	782	613	.122	13
48	627	780	642	15.056	12
49	656	778	671	14.990	11
50	.06685	.99776	.06700	14.924	**10**
51	714	774	730	.860	9
52	743	772	759	.795	8
53	773	770	788	.732	7
54	802	768	817	.669	6
55	.06831	.99766	.06847	14.606	**5**
56	860	764	876	.544	4
57	889	762	905	.482	3
58	918	760	934	.421	2
59	947	758	963	.361	1
60	.06976	.99756	.06993	14.301	**0**
′	cos	sin	cot	tan	′

86°

4°

/	sin	cos	tan	cot	/
0	.06976	.99756	.06993	14.301	**60**
1	.07005	754	.07022	.241	59
2	034	752	051	.182	58
3	063	750	080	.124	57
4	092	748	110	.065	56
5	.07121	.99746	.07139	14.008	**55**
6	150	744	168	13.951	54
7	179	742	197	.894	53
8	208	740	227	.838	52
9	237	738	256	.782	51
10	.07266	.99736	.07285	13.727	**50**
11	295	734	314	.672	49
12	324	731	344	.617	48
13	353	729	373	.563	47
14	382	727	402	.510	46
15	.07411	.99725	.07431	13.457	**45**
16	440	723	461	.404	44
17	469	721	490	.352	43
18	498	719	519	.300	42
19	527	716	548	.248	41
20	.07556	.99714	.07578	13.197	**40**
21	585	712	607	.146	39
22	614	710	636	.096	38
23	643	708	665	13.046	37
24	672	705	695	12.996	36
25	.07701	.99703	.07724	12.947	**35**
26	730	701	753	.898	34
27	759	699	782	.850	33
28	788	696	812	.801	32
29	817	694	841	.754	31
30	.07846	.99692	.07870	12.706	**30**
31	875	689	899	.659	29
32	904	687	929	.612	28
33	933	685	958	.566	27
34	962	683	.07987	.520	26
35	.07991	.99680	.08017	12.474	**25**
36	.08020	678	046	.429	24
37	049	676	075	.384	23
38	078	673	104	.339	22
39	107	671	134	.295	21
40	.08136	.99668	.08163	12.251	**20**
41	165	666	192	.207	19
42	194	664	221	.163	18
43	223	661	251	.120	17
44	252	659	280	.077	16
45	.08281	.99657	.08309	12.035	**15**
46	310	654	339	11.992	14
47	339	652	368	.950	13
48	368	649	397	.909	12
49	397	647	427	.867	11
50	.08426	.99644	.08456	11.826	**10**
51	455	642	485	.785	9
52	484	639	514	.745	8
53	513	637	544	.705	7
54	542	635	573	.664	6
55	.08571	.99632	.08602	11.625	**5**
56	600	630	632	.585	4
57	629	627	661	.546	3
58	658	625	690	.507	2
59	687	622	720	.468	1
60	.08716	.99619	.08749	11.430	**0**
/	cos	sin	cot	tan	/

85°

5°

/	sin	cos	tan	cot	/
0	.08716	.99619	.08749	11.430	**60**
1	745	617	778	.392	59
2	774	614	807	.354	58
3	803	612	837	.316	57
4	831	609	866	.279	56
5	.08860	.99607	.08895	11.242	**55**
6	889	604	925	.205	54
7	918	602	954	.168	53
8	947	599	.08983	.132	52
9	.08976	596	.09013	.095	51
10	.09005	.99594	.09042	11.059	**50**
11	034	591	071	11.024	49
12	063	588	101	10.988	48
13	092	586	130	.953	47
14	121	583	159	.918	46
15	.09150	.99580	.09189	10.883	**45**
16	179	578	218	.848	44
17	208	575	247	.814	43
18	237	572	277	.780	42
19	266	570	306	.746	41
20	.09295	.99567	.09335	10.712	**40**
21	324	564	365	.678	39
22	353	562	394	.645	38
23	382	559	423	.612	37
24	411	556	453	.579	36
25	.09440	.99553	.09482	10.546	**35**
26	469	551	511	.514	34
27	498	548	541	.481	33
28	527	545	570	.449	32
29	556	542	600	.417	31
30	.09585	.99540	.09629	10.385	**30**
31	614	537	658	.354	29
32	642	534	688	.322	28
33	671	531	717	.291	27
34	700	528	746	.260	26
35	.09729	.99526	.09776	10.229	**25**
36	758	523	805	.199	24
37	787	520	834	.168	23
38	816	517	864	.138	22
39	845	514	893	.108	21
40	.09874	.99511	.09923	10.078	**20**
41	903	508	952	.048	19
42	932	506	.09981	10.019	18
43	961	503	.10011	9.9893	17
44	.09990	500	040	.9601	16
45	.10019	.99497	.10069	9.9310	**15**
46	048	494	099	.9021	14
47	077	491	128	.8734	13
48	106	488	158	.8448	12
49	135	485	187	.8164	11
50	.10164	.99482	.10216	9.7882	**10**
51	192	479	246	.7601	9
52	221	476	275	.7322	8
53	250	473	305	.7044	7
54	279	470	334	.6768	6
55	.10308	.99467	.10363	9.6493	**5**
56	337	464	393	.6220	4
57	366	461	422	.5949	3
58	395	458	452	.5679	2
59	424	455	481	.5411	1
60	.10453	.99452	.10510	9.5144	**0**
/	cos	sin	cot	tan	/

84°

6°

/	sin	cos	tan	cot	/
0	.10453	.99452	.10510	9.5144	**60**
1	482	449	540	.4878	59
2	511	446	569	.4614	58
3	540	443	599	.4352	57
4	569	440	628	.4090	56
5	.10597	.99437	.10657	9.3831	**55**
6	626	434	687	.3572	54
7	655	431	716	.3315	53
8	684	428	746	.3060	52
9	713	424	775	.2806	51
10	.10742	.99421	.10805	9.2553	**50**
11	771	418	834	.2302	49
12	800	415	863	.2052	48
13	829	412	893	.1803	47
14	858	409	922	.1555	46
15	.10887	.99406	.10952	9.1309	**45**
16	916	402	.10981	.1065	44
17	945	399	.11011	.0821	43
18	.10973	396	040	.0579	42
19	.11002	393	070	.0338	41
20	.11031	.99390	.11099	9.0098	**40**
21	060	386	128	8.9860	39
22	089	383	158	.9623	38
23	118	380	187	.9387	37
24	147	377	217	.9152	36
25	.11176	.99374	.11246	8.8919	**35**
26	205	370	276	.8686	34
27	234	367	305	.8455	33
28	263	364	335	.8225	32
29	291	360	364	.7996	31
30	.11320	.99357	.11394	8.7769	**30**
31	349	354	423	.7542	29
32	378	351	452	.7317	28
33	407	347	482	.7093	27
34	436	344	511	.6870	26
35	.11465	.99341	.11541	8.6648	**25**
36	494	337	570	.6427	24
37	523	334	600	.6208	23
38	552	331	629	.5989	22
39	580	327	659	.5772	21
40	.11609	.99324	.11688	8.5555	**20**
41	638	320	718	.5340	19
42	667	317	747	.5126	18
43	696	314	777	.4913	17
44	725	310	806	.4701	16
45	.11754	.99307	.11836	8.4490	**15**
46	783	303	865	.4280	14
47	812	300	895	.4071	13
48	840	297	924	.3863	12
49	869	293	954	.3656	11
50	.11898	.99290	.11983	8.3450	**10**
51	927	286	.12013	.3245	9
52	956	283	042	.3041	8
53	.11985	279	072	.2838	7
54	.12014	276	101	.2636	6
55	.12043	.99272	.12131	8.2434	**5**
56	071	269	160	.2234	4
57	100	265	190	.2035	3
58	129	262	219	.1837	2
59	158	258	249	.1640	1
60	.12187	.99255	.12278	8.1443	**0**
/	cos	sin	cot	tan	/

83°

7°

/	sin	cos	tan	cot	/
0	.12187	.99255	.12278	8.1443	**60**
1	216	251	308	.1248	59
2	245	248	338	.1054	58
3	274	244	367	.0860	57
4	302	240	397	.0667	56
5	.12331	.99237	.12426	8.0476	**55**
6	360	233	456	.0285	54
7	389	230	485	8.0095	53
8	418	226	515	7.9906	52
9	447	222	544	.9718	51
10	.12476	.99219	.12574	7.9530	**50**
11	504	215	603	.9344	49
12	533	211	633	.9158	48
13	562	208	662	.8973	47
14	591	204	692	.8789	46
15	.12620	.99200	.12722	7.8606	**45**
16	649	197	751	.8424	44
17	678	193	781	.8243	43
18	706	189	810	.8062	42
19	735	186	840	.7882	41
20	.12764	.99182	.12869	7.7704	**40**
21	793	178	899	.7525	39
22	822	175	929	.7348	38
23	851	171	958	.7171	37
24	880	167	.12988	.6996	36
25	.12908	.99163	.13017	7.6821	**35**
26	937	160	047	.6647	34
27	966	156	076	.6473	33
28	.12995	152	106	.6301	32
29	.13024	148	136	.6129	31
30	.13053	.99144	.13165	7.5958	**30**
31	081	141	195	.5787	29
32	110	137	224	.5618	28
33	139	133	254	.5449	27
34	168	129	284	.5281	26
35	.13197	.99125	.13313	7.5113	**25**
36	226	122	343	.4947	24
37	254	118	372	.4781	23
38	283	114	402	.4615	22
39	312	110	432	.4451	21
40	.13341	.99106	.13461	7.4287	**20**
41	370	102	491	.4124	19
42	399	098	521	.3962	18
43	427	094	550	.3800	17
44	456	091	580	.3639	16
45	.13485	.99087	.13609	7.3479	**15**
46	514	083	639	.3319	14
47	543	079	669	.3160	13
48	572	075	698	.3002	12
49	600	071	728	.2844	11
50	.13629	.99067	.13758	7.2687	**10**
51	658	063	787	.2531	9
52	687	059	817	.2375	8
53	716	055	846	.2220	7
54	744	051	876	.2066	6
55	.13773	.99047	.13906	7.1912	**5**
56	802	043	935	.1759	4
57	831	039	965	.1607	3
58	860	035	.13995	.1455	2
59	889	031	.14024	.1304	1
60	.13917	.99027	.14054	7.1154	**0**
/	cos	sin	cot	tan	/

82°

8°

/	sin	cos	tan	cot	/
0	.13917	.99027	.14054	7.1154	**60**
1	946	023	084	.1004	59
2	.13975	019	113	.0855	58
3	.14004	015	143	.0706	57
4	033	011	173	.0558	56
5	.14061	.99006	.14202	7.0410	**55**
6	090	.99002	232	.0264	54
7	119	.98998	262	7.0117	53
8	148	994	291	6.9972	52
9	177	990	321	.9827	51
10	.14205	.98986	.14351	6.9682	**50**
11	234	982	381	.9538	49
12	263	978	410	.9395	48
13	292	973	440	.9252	47
14	320	969	470	.9110	46
15	.14349	.98965	.14499	6.8969	**45**
16	378	961	529	.8828	44
17	407	957	559	.8687	43
18	436	953	588	.8548	42
19	464	948	618	.8408	41
20	.14493	.98944	.14648	6.8269	**40**
21	522	940	678	.8131	39
22	551	936	707	.7994	38
23	580	931	737	.7856	37
24	608	927	767	.7720	36
25	.14637	.98923	.14796	6.7584	**35**
26	666	919	826	.7448	34
27	695	914	856	.7313	33
28	723	910	886	.7179	32
29	752	906	915	.7045	31
30	.14781	.98902	.14945	6.6912	**30**
31	810	897	.14975	.6779	29
32	838	893	.15005	.6646	28
33	867	889	034	.6514	27
34	896	884	064	.6383	26
35	.14925	.98880	.15094	6.6252	**25**
36	954	876	124	.6122	24
37	.14982	871	153	.5992	23
38	.15011	867	183	.5863	22
39	040	863	213	.5734	21
40	.15069	.98858	.15243	6.5606	**20**
41	097	854	272	.5478	19
42	126	849	302	.5350	18
43	155	845	332	.5223	17
44	184	841	362	.5097	16
45	.15212	.98836	.15391	6.4971	**15**
46	241	832	421	.4846	14
47	270	827	451	.4721	13
48	299	823	481	.4596	12
49	327	818	511	.4472	11
50	.15356	.98814	.15540	6.4348	**10**
51	385	809	570	.4225	9
52	414	805	600	.4103	8
53	442	800	630	.3980	7
54	471	796	660	.3859	6
55	.15500	.98791	.15689	6.3737	**5**
56	529	787	719	.3617	4
57	557	782	749	.3496	3
58	586	778	779	.3376	2
59	615	773	809	.3257	1
60	.15643	.98769	.15838	6.3138	**0**
/	cos	sin	cot	tan	/

81°

9°

/	sin	cos	tan	cot	/
0	.15643	.98769	.15838	6.3138	**60**
1	672	764	868	.3019	59
2	701	760	898	.2901	58
3	730	755	928	.2783	57
4	758	751	958	.2666	56
5	.15787	.98746	.15988	6.2549	**55**
6	816	741	.16017	.2432	54
7	845	737	047	.2316	53
8	873	732	077	.2200	52
9	902	728	107	.2085	51
10	.15931	.98723	.16137	6.1970	**50**
11	959	718	167	.1856	49
12	.15988	714	196	.1742	48
13	.16017	709	226	.1628	47
14	046	704	256	.1515	46
15	.16074	.98700	.16286	6.1402	**45**
16	103	695	316	.1290	44
17	132	690	346	.1178	43
18	160	686	376	.1066	42
19	189	681	405	.0955	41
20	.16218	.98676	.16435	6.0844	**40**
21	246	671	465	.0734	39
22	275	667	495	.0624	38
23	304	662	525	.0514	37
24	333	657	555	.0405	36
25	.16361	.98652	.16585	6.0296	**35**
26	390	648	615	.0188	34
27	419	643	645	6.0080	33
28	447	638	674	5.9972	32
29	476	633	704	.9865	31
30	.16505	.98629	.16734	5.9758	**30**
31	533	624	764	.9651	29
32	562	619	794	.9545	28
33	591	614	824	.9439	27
34	620	609	854	.9333	26
35	.16648	.98604	.16884	5.9228	**25**
36	677	600	914	.9124	24
37	706	595	944	.9019	23
38	734	590	.16974	.8915	22
39	763	585	.17004	.8811	21
40	.16792	.98580	.17033	5.8708	**20**
41	820	575	063	.8605	19
42	849	570	093	.8502	18
43	878	565	123	.8400	17
44	906	561	153	.8298	16
45	.16935	.98556	.17183	5.8197	**15**
46	964	551	213	.8095	14
47	.16992	546	243	.7994	13
48	.17021	541	273	.7894	12
49	050	536	303	.7794	11
50	.17078	.98531	.17333	5.7694	**10**
51	107	526	363	.7594	9
52	136	521	393	.7495	8
53	164	516	423	.7396	7
54	193	511	453	.7297	6
55	.17222	.98506	.17483	5.7199	**5**
56	250	501	513	.7101	4
57	279	496	543	.7004	3
58	308	491	573	.6906	2
59	336	486	603	.6809	1
60	.17365	.98481	.17633	5.6713	**0**
/	cos	sin	cot	tan	/

80°

10°

′	sin	cos	tan	cot	′
0	.17365	.98481	.17633	5.6713	**60**
1	393	476	663	.6617	59
2	422	471	693	.6521	58
3	451	466	723	.6425	57
4	479	461	753	.6329	56
5	.17508	.98455	.17783	5.6234	**55**
6	537	450	813	.6140	54
7	565	445	843	.6045	53
8	594	440	873	.5951	52
9	623	435	903	.5857	51
10	.17651	.98430	.17933	5.5764	**50**
11	680	425	963	.5671	49
12	708	420	.17993	.5578	48
13	737	414	.18023	.5485	47
14	766	409	053	.5393	46
15	.17794	.98404	.18083	5.5301	**45**
16	823	399	113	.5209	44
17	852	394	143	.5118	43
18	880	389	173	.5026	42
19	909	383	203	.4936	41
20	.17937	.98378	.18233	5.4845	**40**
21	966	373	263	.4755	39
22	.17995	368	293	.4665	38
23	.18023	362	323	.4575	37
24	052	357	353	.4486	36
25	.18081	.98352	.18384	5.4397	**35**
26	109	347	414	.4308	34
27	138	341	444	.4219	33
28	166	336	474	.4131	32
29	195	331	504	.4043	31
30	.18224	.98325	.18534	5.3955	**30**
31	252	320	564	.3868	29
32	281	315	594	.3781	28
33	309	310	624	.3694	27
34	338	304	654	.3607	26
35	.18367	.98299	.18684	5.3521	**25**
36	395	294	714	.3435	24
37	424	288	745	.3349	23
38	452	283	775	.3263	22
39	481	277	805	.3178	21
40	.18509	.98272	.18835	5.3093	**20**
41	538	267	865	.3008	19
42	567	261	895	.2924	18
43	595	256	925	.2839	17
44	624	250	955	.2755	16
45	.18652	.98245	.18986	5.2672	**15**
46	681	240	.19016	.2588	14
47	710	234	046	.2505	13
48	738	229	076	.2422	12
49	767	223	106	.2339	11
50	.18795	.98218	.19136	5.2257	**10**
51	824	212	166	.2174	9
52	852	207	197	.2092	8
53	881	201	227	.2011	7
54	910	196	257	.1929	6
55	.18938	.98190	.19287	5.1848	**5**
56	967	185	317	.1767	4
57	.18995	179	347	.1686	3
58	.19024	174	378	.1606	2
59	052	168	408	.1526	1
60	.19081	.98163	.19438	5.1446	**0**
′	cos	sin	cot	tan	′

79°

11°

′	sin	cos	tan	cot	′
0	.19081	.98163	.19438	5.1446	**60**
1	109	157	468	.1366	59
2	138	152	498	.1286	58
3	167	146	529	.1207	57
4	195	140	559	.1128	56
5	.19224	.98135	.19589	5.1049	**55**
6	252	129	619	.0970	54
7	281	124	649	.0892	53
8	309	118	680	.0814	52
9	338	112	710	.0736	51
10	.19366	.98107	.19740	5.0658	**50**
11	395	101	770	.0581	49
12	423	096	801	.0504	48
13	452	090	831	.0427	47
14	481	084	861	.0350	46
15	.19509	.98079	.19891	5.0273	**45**
16	538	073	921	.0197	44
17	566	067	952	.0121	43
18	595	061	.19982	5.0045	42
19	623	056	.20012	4.9969	41
20	.19652	.98050	.20042	4.9894	**40**
21	680	044	073	.9819	39
22	709	039	103	.9744	38
23	737	033	133	.9669	37
24	766	027	164	.9594	36
25	.19794	.98021	.20194	4.9520	**35**
26	823	016	224	.9446	34
27	851	010	254	.9372	33
28	880	.98004	285	.9298	32
29	908	.97998	315	.9225	31
30	.19937	.97992	.20345	4.9152	**30**
31	965	987	376	.9078	29
32	.19994	981	406	.9006	28
33	.20022	975	436	.8933	27
34	051	969	466	.8860	26
35	.20079	.97963	.20497	4.8788	**25**
36	108	958	527	.8716	24
37	136	952	557	.8644	23
38	165	946	588	.8573	22
39	193	940	618	.8501	21
40	.20222	.97934	.20648	4.8430	**20**
41	250	928	679	.8359	19
42	279	922	709	.8288	18
43	307	916	739	.8218	17
44	336	910	770	.8147	16
45	.20364	.97905	.20800	4.8077	**15**
46	393	899	830	.8007	14
47	421	893	861	.7937	13
48	450	887	891	.7867	12
49	478	881	921	.7798	11
50	.20507	.97875	.20952	4.7729	**10**
51	535	869	.20982	.7659	9
52	563	863	.21013	.7591	8
53	592	857	043	.7522	7
54	620	851	073	.7453	6
55	.20649	.97845	.21104	4.7385	**5**
56	677	839	134	.7317	4
57	706	833	164	.7249	3
58	734	827	195	.7181	2
59	763	821	225	.7114	1
60	.20791	.97815	.21256	4.7046	**0**
′	cos	sin	cot	tan	′

78°

12°

′	sin	cos	tan	cot	′
0	.20791	.97815	.21256	4.7046	**60**
1	820	809	286	.6979	59
2	848	803	316	.6912	58
3	877	797	347	.6845	57
4	905	791	377	.6779	56
5	.20933	.97784	.21408	4.6712	**55**
6	962	778	438	.6646	54
7	.20990	772	469	.6580	53
8	.21019	766	499	.6514	52
9	047	760	529	.6448	51
10	.21076	.97754	.21560	4.6382	**50**
11	104	748	590	.6317	49
12	132	742	621	.6252	48
13	161	735	651	.6187	47
14	189	729	682	.6122	46
15	.21218	.97723	.21712	4.6057	**45**
16	246	717	743	.5993	44
17	275	711	773	.5928	43
18	303	705	804	.5864	42
19	331	698	834	.5800	41
20	.21360	.97692	.21864	4.5736	**40**
21	388	686	895	.5673	39
22	417	680	925	.5609	38
23	445	673	956	.5546	37
24	474	667	.21986	.5483	36
25	.21502	.97661	.22017	4.5420	**35**
26	530	655	047	.5357	34
27	559	648	078	.5294	33
28	587	642	108	.5232	32
29	616	636	139	.5169	31
30	.21644	.97630	.22169	4.5107	**30**
31	672	623	200	.5045	29
32	701	617	231	.4983	28
33	729	611	261	.4922	27
34	758	604	292	.4860	26
35	.21786	.97598	.22322	4.4799	**25**
36	814	592	353	.4737	24
37	843	585	383	.4676	23
38	871	579	414	.4615	22
39	899	573	444	.4555	21
40	.21928	.97566	.22475	4.4494	**20**
41	956	560	505	.4434	19
42	.21985	553	536	.4373	18
43	.22013	547	567	.4313	17
44	041	541	597	.4253	16
45	.22070	.97534	.22628	4.4194	**15**
46	098	528	658	.4134	14
47	126	521	689	.4075	13
48	155	515	719	.4015	12
49	183	508	750	.3956	11
50	.22212	.97502	.22781	4.3897	**10**
51	240	496	811	.3838	9
52	268	489	842	.3779	8
53	297	483	872	.3721	7
54	325	476	903	.3662	6
55	.22353	.97470	.22934	4.3604	**5**
56	382	463	964	.3546	4
57	410	457	.22995	.3488	3
58	438	450	.23026	.3430	2
59	467	444	056	.3372	1
60	.22495	.97437	.23087	4.3315	**0**
′	cos	sin	cot	tan	′

77°

13°

′	sin	cos	tan	cot	′
0	.22495	.97437	.23087	4.3315	**60**
1	523	430	117	.3257	59
2	552	424	148	.3200	58
3	580	417	179	.3143	57
4	608	411	209	.3086	56
5	.22637	.97404	.23240	4.3029	**55**
6	665	398	271	.2972	54
7	693	391	301	.2916	53
8	722	384	332	.2859	52
9	750	378	363	.2803	51
10	.22778	.97371	.23393	4.2747	**50**
11	807	365	424	.2691	49
12	835	358	455	.2635	48
13	863	351	485	.2580	47
14	892	345	516	.2524	46
15	.22920	.97338	.23547	4.2468	**45**
16	948	331	578	.2413	44
17	.22977	325	608	.2358	43
18	.23005	318	639	.2303	42
19	033	311	670	.2248	41
20	.23062	.97304	.23700	4.2193	**40**
21	090	298	731	.2139	39
22	118	291	762	.2084	38
23	146	284	793	.2030	37
24	175	278	823	.1976	36
25	.23203	.97271	.23854	4.1922	**35**
26	231	264	885	.1868	34
27	260	257	916	.1814	33
28	288	251	946	.1760	32
29	316	244	.23977	.1706	31
30	.23345	.97237	.24008	4.1653	**30**
31	373	230	039	.1600	29
32	401	223	069	.1547	28
33	429	217	100	.1493	27
34	458	210	131	.1441	26
35	.23486	.97203	.24162	4.1388	**25**
36	514	196	193	.1335	24
37	542	189	223	.1282	23
38	571	182	254	.1230	22
39	599	176	285	.1178	21
40	.23627	.97169	.24316	4.1126	**20**
41	656	162	347	.1074	19
42	684	155	377	.1022	18
43	712	148	408	.0970	17
44	740	141	439	.0918	16
45	.23769	.97134	.24470	4.0867	**15**
46	797	127	501	.0815	14
47	825	120	532	.0764	13
48	853	113	562	.0713	12
49	882	106	593	.0662	11
50	.23910	.97100	.24624	4.0611	**10**
51	938	093	655	.0560	9
52	966	086	686	.0509	8
53	.23995	079	717	.0459	7
54	.24023	072	747	.0408	6
55	.24051	.97065	.24778	4.0358	**5**
56	079	058	809	.0308	4
57	108	051	840	.0257	3
58	136	044	871	.0207	2
59	164	037	902	.0158	1
60	.24192	.97030	.24933	4.0108	**0**
′	cos	sin	cot	tan	′

76°

14°

/	sin	cos	tan	cot	/
0	.24192	.97030	.24933	4.0108	**60**
1	220	023	964	.0058	59
2	249	015	.24995	4.0009	58
3	277	008	.25026	3.9959	57
4	305	.97001	056	.9910	56
5	.24333	.96994	.25087	3.9861	**55**
6	362	987	118	.9812	54
7	390	980	149	.9763	53
8	418	973	180	.9714	52
9	446	966	211	.9665	51
10	.24474	.96959	.25242	3.9617	**50**
11	503	952	273	.9568	49
12	531	945	304	.9520	48
13	559	937	335	.9471	47
14	587	930	366	.9423	46
15	.24615	.96923	.25397	3.9375	**45**
16	644	916	428	.9327	44
17	672	909	459	.9279	43
18	700	902	490	.9232	42
19	728	894	521	.9184	41
20	.24756	.96887	.25552	3.9136	**40**
21	784	880	583	.9089	39
22	813	873	614	.9042	38
23	841	866	645	.8995	37
24	869	858	676	.8947	36
25	.24897	.96851	.25707	3.8900	**35**
26	925	844	738	.8854	34
27	954	837	769	.8807	33
28	.24982	829	800	.8760	32
29	.25010	822	831	.8714	31
30	.25038	.96815	.25862	3.8667	**30**
31	066	807	893	.8621	29
32	094	800	924	.8575	28
33	122	793	955	.8528	27
34	151	786	.25986	.8482	26
35	.25179	.96778	.26017	3.8436	**25**
36	207	771	048	.8391	24
37	235	764	079	.8345	23
38	263	756	110	.8299	22
39	291	749	141	.8254	21
40	.25320	.96742	.26172	3.8208	**20**
41	348	734	203	.8163	19
42	376	727	235	.8118	18
43	404	719	266	.8073	17
44	432	712	297	.8028	16
45	.25460	.96705	.26328	3.7983	**15**
46	488	697	359	.7938	14
47	516	690	390	.7893	13
48	545	682	421	.7848	12
49	573	675	452	.7804	11
50	.25601	.96667	.26483	3.7760	**10**
51	629	660	515	.7715	9
52	657	653	546	.7671	8
53	685	645	577	.7627	7
54	713	638	608	.7583	6
55	.25741	.96630	.26639	3.7539	**5**
56	769	623	670	.7495	4
57	798	615	701	.7451	3
58	826	608	733	.7408	2
59	854	600	764	.7364	1
60	.25882	.96593	.26795	3.7321	**0**
/	cos	sin	cot	tan	/

75°

15°

/	sin	cos	tan	cot	/
0	.25882	.96593	.26795	3.7321	**60**
1	910	585	826	.7277	59
2	938	578	857	.7234	58
3	966	570	888	.7191	57
4	.25994	562	920	.7148	56
5	.26022	.96555	.26951	3.7105	**55**
6	050	547	.26982	.7062	54
7	079	540	.27013	.7019	53
8	107	532	044	.6976	52
9	135	524	076	.6933	51
10	.26163	.96517	.27107	3.6891	**50**
11	191	509	138	.6848	49
12	219	502	169	.6806	48
13	247	494	201	.6764	47
14	275	486	232	.6722	46
15	.26303	.96479	.27263	3.6680	**45**
16	331	471	294	.6638	44
17	359	463	326	.6596	43
18	387	456	357	.6554	42
19	415	448	388	.6512	41
20	.26443	.96440	.27419	3.6470	**40**
21	471	433	451	.6429	39
22	500	425	482	.6387	38
23	528	417	513	.6346	37
24	556	410	545	.6305	36
25	.26584	.96402	.27576	3.6264	**35**
26	612	394	607	.6222	34
27	640	386	638	.6181	33
28	668	379	670	.6140	32
29	696	371	701	.6100	31
30	.26724	.96363	.27732	3.6059	**30**
31	752	355	764	.6018	29
32	780	347	795	.5978	28
33	808	340	826	.5937	27
34	836	332	858	.5897	26
35	.26864	.96324	.27889	3.5856	**25**
36	892	316	921	.5816	24
37	920	308	952	.5776	23
38	948	301	.27983	.5736	22
39	.26976	293	.28015	.5696	21
40	.27004	.96285	.28046	3.5656	**20**
41	032	277	077	.5616	19
42	060	269	109	.5576	18
43	088	161	140	.5536	17
44	116	253	172	.5497	16
45	.27144	.96246	.28203	3.5457	**15**
46	172	238	234	.5418	14
47	200	230	266	.5379	13
48	228	222	297	.5339	12
49	256	214	329	.5300	11
50	.27284	.96206	.28360	3.5261	**10**
51	312	198	391	.5222	9
52	340	190	423	.5183	8
53	368	182	454	.5144	7
54	396	174	486	.5105	6
55	.27424	.96166	.28517	3.5067	**5**
56	452	158	549	.5028	4
57	480	150	580	.4989	3
58	508	142	612	.4951	2
59	536	134	643	.4912	1
60	.27564	.96126	.28675	3.4874	**0**
/	cos	sin	cot	tan	/

74°

16°

′	sin	cos	tan	cot	′
0	.27564	.96126	.28675	3.4874	**60**
1	592	118	706	.4836	59
2	620	110	738	.4798	58
3	648	102	769	.4760	57
4	676	094	801	.4722	56
5	.27704	.96086	.28832	3.4684	**55**
6	731	078	864	.4646	54
7	759	070	895	.4608	53
8	787	062	927	.4570	52
9	815	054	958	.4533	51
10	.27843	.96046	.28990	3.4495	**50**
11	871	037	.29021	.4458	49
12	899	029	053	.4420	48
13	927	021	084	.4383	47
14	955	013	116	.4346	46
15	.27983	.96005	.29147	3.4308	**45**
16	.28011	.95997	179	.4271	44
17	039	989	210	.4234	43
18	067	981	242	.4197	42
19	095	972	274	.4160	41
20	.28123	.95964	.29305	3.4124	**40**
21	150	956	337	.4087	39
22	178	948	368	.4050	38
23	206	940	400	.4014	37
24	234	931	432	.3977	36
25	.28262	.95923	.29463	3.3941	**35**
26	290	915	495	.3904	34
27	318	907	526	.3868	33
28	346	898	558	.3832	32
29	374	890	590	.3796	31
30	.28402	.95882	.29621	3.3759	**30**
31	429	874	653	.3723	29
32	457	865	685	.3687	28
33	485	857	716	.3652	27
34	513	849	748	.3616	26
35	.28541	.95841	.29780	3.3580	**25**
36	569	832	811	.3544	24
37	597	824	843	.3509	23
38	625	816	875	.3473	22
39	652	807	906	.3438	21
40	.28680	.95799	.29938	3.3402	**20**
41	708	791	.29970	.3367	19
42	736	782	.30001	.3332	18
43	764	774	033	.3297	17
44	792	766	065	.3261	16
45	.28820	.95757	.30097	3.3226	**15**
46	847	749	128	.3191	14
47	875	740	160	.3156	13
48	903	732	192	.3122	12
49	931	724	224	.3087	11
50	.28959	.95715	.30255	3.3052	**10**
51	.28987	707	287	.3017	9
52	.29015	698	319	.2983	8
53	042	690	351	.2948	7
54	070	681	382	.2914	6
55	.29098	.95673	.30414	3.2879	**5**
56	126	664	446	.2845	4
57	154	656	478	.2811	3
58	182	647	509	.2777	2
59	209	639	541	.2743	1
60	.29237	.95630	.30573	3.2709	**0**
′	cos	sin	cot	tan	′

73°

17°

′	sin	cos	tan	cot	′
0	.29237	.95630	.30573	3.2709	**60**
1	265	622	605	.2675	59
2	293	613	637	.2641	58
3	321	605	669	.2607	57
4	348	596	700	.2573	56
5	.29376	.95588	.30732	3.2539	**55**
6	404	579	764	.2506	54
7	432	571	796	.2472	53
8	460	562	828	.2438	52
9	487	554	860	.2405	51
10	.29515	.95545	.30891	3.2371	**50**
11	543	536	923	.2338	49
12	571	528	955	.2305	48
13	599	519	.30987	.2272	47
14	626	511	.31019	.2238	46
15	.29654	.95502	.31051	3.2205	**45**
16	682	493	083	.2172	44
17	710	485	115	.2139	43
18	737	476	147	.2106	42
19	765	467	178	.2073	41
20	.29793	.95459	.31210	3.2041	**40**
21	821	450	242	.2008	39
22	849	441	274	.1975	38
23	876	433	306	.1943	37
24	904	424	338	.1910	36
25	.29932	.95415	.31370	3.1878	**35**
26	960	407	402	.1845	34
27	.29987	398	434	.1813	33
28	.30015	389	466	.1780	32
29	043	380	498	.1748	31
30	.30071	.95372	.31530	3.1716	**30**
31	098	363	562	.1684	29
32	126	354	594	.1652	28
33	154	345	626	.1620	27
34	182	337	658	.1588	26
35	.30209	.95328	.31690	3.1556	**25**
36	237	319	722	.1524	24
37	265	310	754	.1492	23
38	292	301	786	.1460	22
39	320	293	818	.1429	21
40	.30348	.95284	.31850	3.1397	**20**
41	376	275	882	.1366	19
42	403	266	914	.1334	18
43	431	257	946	.1303	17
44	459	248	.31978	.1271	16
45	.30486	.95240	.32010	3.1240	**15**
46	514	231	042	.1209	14
47	542	222	074	.1178	13
48	570	213	106	.1146	12
49	597	204	139	.1115	11
50	.30625	.95195	.32171	3.1084	**10**
51	653	186	203	.1053	9
52	680	177	235	.1022	8
53	708	168	267	.0991	7
54	736	159	299	.0961	6
55	.30763	.95150	.32331	3.0930	**5**
56	791	142	363	.0899	4
57	819	133	396	.0868	3
58	846	124	428	.0838	2
59	874	115	460	.0807	1
60	.30902	.95106	.32492	3.0777	**0**
′	cos	sin	cot	tan	′

72°

18°

′	sin	cos	tan	cot	′
0	.30902	.95106	.32492	3.0777	**60**
1	929	097	524	.0746	59
2	957	088	556	.0716	58
3	.30985	079	588	.0686	57
4	.31012	070	621	.0655	56
5	.31040	.95061	.32653	3.0625	**55**
6	068	052	685	.0595	54
7	095	043	717	.0565	53
8	123	033	749	.0535	52
9	151	024	782	.0505	51
10	.31178	.95015	.32814	3.0475	**50**
11	206	.95006	846	.0445	49
12	233	.94997	878	.0415	48
13	261	988	911	.0385	47
14	289	979	943	.0356	46
15	.31316	.94970	.32975	3.0326	**45**
16	344	961	.33007	.0296	44
17	372	952	040	.0267	43
18	399	943	072	.0237	42
19	427	933	104	.0208	41
20	.31454	.94924	.33136	3.0178	**40**
21	482	915	169	.0149	39
22	510	906	201	.0120	38
23	537	897	233	.0090	37
24	565	888	266	.0061	36
25	.31593	.94878	.33298	3.0032	**35**
26	620	869	330	3.0003	34
27	648	860	363	2.9974	33
28	675	851	395	.9945	32
29	703	842	427	.9916	31
30	.31730	.94832	.33460	2.9887	**30**
31	758	823	492	.9858	29
32	786	814	524	.9829	28
33	813	805	557	.9800	27
34	841	795	589	.9772	26
35	.31868	.94786	.33621	2.9743	**25**
36	896	777	654	.9714	24
37	923	768	686	.9686	23
38	951	758	718	.9657	22
39	.31979	749	751	.9629	21
40	.32006	.94740	.33783	2.9600	**20**
41	034	730	816	.9572	19
42	061	721	848	.9544	18
43	089	712	881	.9515	17
44	116	702	913	.9487	16
45	.32144	.94693	.33945	2.9459	**15**
46	171	684	.33978	.9431	14
47	199	674	.34010	.9403	13
48	227	665	043	.9375	12
49	254	656	075	.9347	11
50	.32282	.94646	.34108	2.9319	**10**
51	309	637	140	.9291	9
52	337	627	173	.9263	8
53	364	618	205	.9235	7
54	392	609	238	.9208	6
55	.32419	.94599	.34270	2.9180	**5**
56	447	590	303	.9152	4
57	474	580	335	.9125	3
58	502	571	368	.9097	2
59	529	561	400	.9070	1
60	.32557	.94552	.34433	2.9042	**0**
′	cos	sin	cot	tan	′

71°

19°

′	sin	cos	tan	cot	′
0	.32557	.94552	.34433	2.9042	**60**
1	584	542	465	.9015	59
2	612	533	498	.8987	58
3	639	523	530	.8960	57
4	667	514	563	.8933	56
5	.32694	.94504	.34596	2.8905	**55**
6	722	495	628	.8878	54
7	749	485	661	.8851	53
8	777	476	693	.8824	52
9	804	466	726	.8797	51
10	.32832	.94457	.34758	2.8770	**50**
11	859	447	791	.8743	49
12	887	438	824	.8716	48
13	914	428	856	.8689	47
14	942	418	889	.8662	46
15	.32969	.94409	.34922	2.8636	**45**
16	.32997	399	954	.8609	44
17	.33024	390	.34987	.8582	43
18	051	380	.35020	.8556	42
19	079	370	052	.8529	41
20	.33106	.94361	.35085	2.8502	**40**
21	134	351	118	.8476	39
22	161	342	150	.8449	38
23	189	332	183	.8423	37
24	216	322	216	.8397	36
25	.33244	.94313	.35248	2.8370	**35**
26	271	303	281	.8344	34
27	298	293	314	.8318	33
28	326	284	346	.8291	32
29	353	274	379	.8265	31
30	.33381	.94264	.35412	2.8239	**30**
31	408	254	445	.8213	29
32	436	245	477	.8187	28
33	463	235	510	.8161	27
34	490	225	543	.8135	26
35	.33518	.94215	.35576	2.8109	**25**
36	545	206	608	.8083	24
37	573	196	641	.8057	23
38	600	186	674	.8032	22
39	627	176	707	.8006	21
40	.33655	.94167	.35740	2.7980	**20**
41	682	157	772	.7955	19
42	710	147	805	.7929	18
43	737	137	838	.7903	17
44	764	127	871	.7878	16
45	.33792	.94118	.35904	2.7852	**15**
46	819	108	937	.7827	14
47	846	098	.35969	.7801	13
48	874	088	.36002	.7776	12
49	901	078	035	.7751	11
50	.33929	.94068	.36068	2.7725	**10**
51	956	058	101	.7700	9
52	.33983	049	134	.7675	8
53	.34011	039	167	.7650	7
54	038	029	199	.7625	6
55	.34065	.94019	.36232	2.7600	**5**
56	093	.94009	265	.7575	4
57	120	.93999	298	.7550	3
58	147	989	331	.7525	2
59	175	979	364	.7500	1
60	.34202	.93969	.36397	2.7475	**0**
′	cos	sin	cot	tan	′

70°

20°

/	sin	cos	tan	cot	/
0	.34202	.93969	.36397	2.7475	**60**
1	229	959	430	.7450	59
2	257	949	463	.7425	58
3	284	939	496	.7400	57
4	311	929	529	.7376	56
5	.34339	.93919	.36562	2.7351	**55**
6	366	909	595	.7326	54
7	393	899	628	.7302	53
8	421	889	661	.7277	52
9	448	879	694	.7253	51
10	.34475	.93869	.36727	2.7228	**50**
11	503	859	760	.7204	49
12	530	849	793	.7179	48
13	557	839	826	.7155	47
14	584	829	859	.7130	46
15	.34612	.93819	.36892	2.7106	**45**
16	639	809	925	.7082	44
17	666	799	958	.7058	43
18	694	789	.36991	.7034	42
19	721	779	.37024	.7009	41
20	.34748	.93769	.37057	2.6985	**40**
21	775	759	090	.6961	39
22	803	748	123	.6937	38
23	830	738	157	.6913	37
24	857	728	190	.6889	36
25	.34884	.93718	.37223	2.6865	**35**
26	912	708	256	.6841	34
27	939	698	289	.6818	33
28	966	688	322	.6794	32
29	.34993	677	355	.6770	31
30	.35021	.93667	.37388	2.6746	**30**
31	048	657	422	.6723	29
32	075	647	455	.6699	28
33	102	637	488	.6675	27
34	130	626	521	.6652	26
35	.35157	.93616	.37554	2.6628	**25**
36	184	606	588	.6605	24
37	211	596	621	.6581	23
38	239	585	654	.6558	22
39	266	575	687	.6534	21
40	.35293	.93565	.37720	2.6511	**20**
41	320	555	754	.6488	19
42	347	544	787	.6464	18
43	375	534	820	.6441	17
44	402	524	853	.6418	16
45	.35429	.93514	.37887	2.6395	**15**
46	456	503	920	.6371	14
47	484	493	953	.6348	13
48	511	483	.37986	.6325	12
49	538	472	.38020	.6302	11
50	.35565	.93462	.38053	2.6279	**10**
51	592	452	086	.6256	9
52	619	441	120	.6233	8
53	647	431	153	.6210	7
54	674	420	186	.6187	6
55	.35701	.93410	.38220	2.6165	**5**
56	728	400	253	.6142	4
57	755	389	286	.6119	3
58	782	379	320	.6096	2
59	810	368	353	.6074	1
60	.35837	.93358	.38386	2.6051	**0**
/	cos	sin	cot	tan	/

21°

/	sin	cos	tan	cot	/
0	.35837	.93358	.38386	2.6051	**60**
1	864	348	420	.6028	59
2	891	337	453	.6006	58
3	918	327	487	.5983	57
4	945	316	520	.5961	56
5	.35973	.93306	.38553	2.5938	**55**
6	.36000	295	587	.5916	54
7	027	285	620	.5893	53
8	054	274	654	.5871	52
9	081	264	687	.5848	51
10	.36108	.93253	.38721	2.5826	**50**
11	135	243	754	.5804	49
12	162	232	787	.5782	48
13	190	222	821	.5759	47
14	217	211	854	.5737	46
15	.36244	.93201	.38888	2.5715	**45**
16	271	190	921	.5693	44
17	298	180	955	.5671	43
18	325	169	.38988	.5649	42
19	352	159	.39022	.5627	41
20	.36379	.93148	.39055	2.5605	**40**
21	406	137	089	.5583	39
22	434	127	122	.5561	38
23	461	116	156	.5539	37
24	488	106	190	.5517	36
25	.36515	.93095	.39223	2.5495	**35**
26	542	084	257	.5473	34
27	569	074	290	.5452	33
28	596	063	324	.5430	32
29	623	052	357	.5408	31
30	.36650	.93042	.39391	2.5386	**30**
31	677	031	425	.5365	29
32	704	020	458	.5343	28
33	731	.93010	492	.5322	27
34	758	.92999	526	.5300	26
35	.36785	.92988	.39559	2.5279	**25**
36	812	978	593	.5257	24
37	839	967	626	.5236	23
38	867	956	660	.5214	22
39	894	945	694	.5193	21
40	.36921	.92935	.39727	2.5172	**20**
41	948	924	761	.5150	19
42	.36975	913	795	.5129	18
43	.37002	902	829	.5108	17
44	029	892	862	.5086	16
45	.37056	.92881	.39896	2.5065	**15**
46	083	870	930	.5044	14
47	110	859	963	.5023	13
48	137	849	.39997	.5002	12
49	164	838	.40031	.4981	11
50	.37191	.92827	.40065	2.4960	**10**
51	218	816	098	.4939	9
52	245	805	132	.4918	8
53	272	794	166	.4897	7
54	299	784	200	.4876	6
55	.37326	.92773	.40234	2.4855	**5**
56	353	762	267	.4834	4
57	380	751	301	.4813	3
58	407	740	335	.4792	2
59	434	729	369	.4772	1
60	.37461	.92718	.40403	2.4751	**0**
/	cos	sin	cot	tan	/

69° **68°**

22°

/	sin	cos	tan	cot	/
0	.37461	.92718	.40403	2.4751	**60**
1	488	707	436	.4730	59
2	515	697	470	.4709	58
3	542	686	504	.4689	57
4	569	675	538	.4668	56
5	.37595	.92664	.40572	2.4648	**55**
6	622	653	606	.4627	54
7	649	642	640	.4606	53
8	676	631	674	.4586	52
9	703	620	707	.4566	51
10	.37730	.92609	.40741	2.4545	**50**
11	757	598	775	.4525	49
12	784	587	809	.4504	48
13	811	576	843	.4484	47
14	838	565	877	.4464	46
15	.37865	.92554	.40911	2.4443	**45**
16	892	543	945	.4423	44
17	919	532	.40979	.4403	43
18	946	521	.41013	.4383	42
19	973	510	047	.4362	41
20	.37999	.92499	.41081	2.4342	**40**
21	.38026	488	115	.4322	39
22	053	477	149	.4302	38
23	080	466	183	.4282	37
24	107	455	217	.4262	36
25	.38134	.92444	.41251	2.4242	**35**
26	161	432	285	.4222	34
27	188	421	319	.4202	33
28	215	410	353	.4182	32
29	241	399	387	.4162	31
30	.38268	.92388	.41421	2.4142	**30**
31	295	377	455	.4122	29
32	322	366	490	.4102	28
33	349	355	524	.4083	27
34	376	343	558	.4063	26
35	.38403	.92332	.41592	2.4043	**25**
36	430	321	626	.4023	24
37	456	310	660	.4004	23
38	483	299	694	.3984	22
39	510	287	728	.3964	21
40	.38537	.92276	.41763	2.3945	**20**
41	564	265	797	.3925	19
42	591	254	831	.3906	18
43	617	243	865	.3886	17
44	644	231	899	.3867	16
45	.38671	.92220	.41933	2.3847	**15**
46	698	209	.41968	.3828	14
47	725	198	.42002	.3808	13
48	752	186	036	.3789	12
49	778	175	070	.3770	11
50	.38805	.92164	.42105	2.3750	**10**
51	832	152	139	.3731	9
52	859	141	173	.3712	8
53	886	130	207	.3693	7
54	912	119	242	.3673	6
55	.38939	.92107	.42276	2.3654	**5**
56	966	096	310	.3635	4
57	.38993	085	345	.3616	3
58	.39020	073	379	.3597	2
59	046	062	413	.3578	1
60	.39073	.92050	.42447	2.3559	**0**
/	cos	sin	cot	tan	/

67°

23°

/	sin	cos	tan	cot	/
0	.39073	.92050	.42447	2.3559	**60**
1	100	039	482	.3539	59
2	127	028	516	.3520	58
3	153	016	551	.3501	57
4	180	.92005	585	.3483	56
5	.39207	.91994	.42619	2.3464	**55**
6	234	982	654	.3445	54
7	260	971	688	.3426	53
8	287	959	722	.3407	52
9	314	948	757	.3388	51
10	.39341	.91936	.42791	2.3369	**50**
11	367	925	826	.3351	49
12	394	914	860	.3332	48
13	421	902	894	.3313	47
14	448	891	929	.3294	46
15	.39474	.91879	.42963	2.3276	**45**
16	501	868	.42998	.3257	44
17	528	856	.43032	.3238	43
18	555	845	067	.3220	42
19	581	833	101	.3201	41
20	.39608	.91822	.43136	2.3183	**40**
21	635	810	170	.3164	39
22	661	799	205	.3146	38
23	688	787	239	.3127	37
24	715	775	274	.3109	36
25	.39741	.91764	.43308	2.3090	**35**
26	768	752	343	.3072	34
27	795	741	378	.3053	33
28	822	729	412	.3035	32
29	848	718	447	.3017	31
30	.39875	.91706	.43481	2.2998	**30**
31	902	694	516	.2980	29
32	928	683	550	.2962	28
33	955	671	585	.2944	27
34	.39982	660	620	.2925	26
35	.40008	.91648	.43654	2.2907	**25**
36	035	636	689	.2889	24
37	062	625	724	.2871	23
38	088	613	758	.2853	22
39	115	601	793	.2835	21
40	.40141	.91590	.43828	2.2817	**20**
41	168	578	862	.2799	19
42	195	566	897	.2781	18
43	221	555	932	.2763	17
44	248	543	.43966	.2745	16
45	.40275	.91531	.44001	2.2727	**15**
46	301	519	036	.2709	14
47	328	508	071	.2691	13
48	355	496	105	.2673	12
49	381	484	140	.2655	11
50	.40408	.91472	.44175	2.2637	**10**
51	434	461	210	.2620	9
52	461	449	244	.2602	8
53	488	437	279	.2584	7
54	514	425	314	.2566	6
55	.40541	.91414	.44349	2.2549	**5**
56	567	402	384	.2531	4
57	594	390	418	.2513	3
58	621	378	453	.2496	2
59	647	366	488	.2478	1
60	.40674	.91355	.44523	2.2460	**0**
/	cos	sin	cot	tan	/

66°

24°

′	sin	cos	tan	cot	′
0	.40674	.91355	.44523	2.2460	**60**
1	700	343	558	.2443	59
2	727	331	593	.2425	58
3	753	319	627	.2408	57
4	780	307	662	.2390	56
5	.40806	.91295	.44697	2.2373	**55**
6	833	283	732	.2355	54
7	860	272	767	.2338	53
8	886	260	802	.2320	52
9	913	248	837	.2303	51
10	.40939	.91236	.44872	2.2286	**50**
11	966	224	907	.2268	49
12	.40992	212	942	.2251	48
13	.41019	200	.44977	.2234	47
14	045	188	.45012	.2216	46
15	.41072	.91176	.45047	2.2199	**45**
16	098	164	082	.2182	44
17	125	152	117	.2165	43
18	151	140	152	.2148	42
19	178	128	187	2130	41
20	.41204	.91116	.45222	2.2113	**40**
21	231	104	257	.2096	39
22	257	092	292	.2079	38
23	284	080	327	.2062	37
24	310	068	362	.2045	36
25	.41337	.91056	.45397	2.2028	**35**
26	363	044	432	.2011	34
27	390	032	467	.1994	33
28	416	020	502	.1977	32
29	443	.91008	538	.1960	31
30	.41469	.90996	.45573	2.1943	**30**
31	496	984	608	.1926	29
32	522	972	643	.1909	28
33	549	960	678	.1892	27
34	575	948	713	.1876	26
35	.41602	.90936	.45748	2.1859	**25**
36	628	924	784	.1842	24
37	655	911	819	.1825	23
38	681	899	854	.1808	22
39	707	887	889	.1792	21
40	.41734	.90875	.45924	2.1775	**20**
41	760	863	960	.1758	19
42	787	851	.45995	.1742	18
43	813	839	.46030	.1725	17
44	840	826	065	.1708	16
45	.41866	.90814	.46101	2.1692	**15**
46	892	802	136	.1675	14
47	919	790	171	.1659	13
48	945	778	206	.1642	12
49	972	766	242	.1625	11
50	.41998	.90753	.46277	2.1609	**10**
51	.42024	741	312	.1592	9
52	051	729	348	.1576	8
53	077	717	383	.1560	7
54	104	704	418	.1543	6
55	.42130	.90692	.46454	2.1527	**5**
56	156	680	489	.1510	4
57	183	668	525	.1494	3
58	209	655	560	.1478	2
59	235	643	595	.1461	1
60	.42262	.90631	.46631	2.1445	**0**
′	cos	sin	cot	tan	′

65°

25°

′	sin	cos	tan	cot	′
0	.42262	.90631	.46631	2.1445	**60**
1	288	618	666	.1429	59
2	315	606	702	.1413	58
3	341	594	737	.1396	57
4	367	582	772	.1380	56
5	.42394	.90569	.46808	2.1364	**55**
6	420	557	843	.1348	54
7	446	545	879	.1332	53
8	473	532	914	.1315	52
9	499	520	950	.1299	51
10	.42525	.90507	.46985	2.1283	**50**
11	552	495	.47021	.1267	49
12	578	483	056	.1251	48
13	604	470	092	.1235	47
14	631	458	128	.1219	46
15	.42657	.90446	.47163	2.1203	**45**
16	683	433	199	.1187	44
17	709	421	234	.1171	43
18	736	408	270	.1155	42
19	762	396	305	.1139	41
20	.42788	.90383	.47341	2.1123	**40**
21	815	371	377	.1107	39
22	841	358	412	.1092	38
23	867	346	448	.1076	37
24	894	334	483	.1060	36
25	.42920	.90321	.47519	2.1044	**35**
26	946	309	555	.1028	34
27	972	296	590	.1013	33
28	.42999	284	626	.0997	32
29	.43025	271	662	.0981	31
30	.43051	.90259	.47698	2.0965	**30**
31	077	246	733	.0950	29
32	104	233	769	.0934	28
33	130	221	805	.0918	27
34	156	208	840	.0903	26
35	.43182	.90196	.47876	2.0887	**25**
36	209	183	912	.0872	24
37	235	171	948	.0856	23
38	261	158	.47984	.0840	22
39	287	146	.48019	.0825	21
40	.43313	.90133	.48055	2.0809	**20**
41	340	120	091	.0794	19
42	366	108	127	.0778	18
43	392	095	163	.0763	17
44	418	082	198	.0748	16
45	.43445	.90070	.48234	2.0732	**15**
46	471	057	270	.0717	14
47	497	045	306	.0701	13
48	523	032	342	.0686	12
49	549	019	378	.0671	11
50	.43575	.90007	.48414	2.0655	**10**
51	602	.89994	450	.0640	9
52	628	981	486	.0625	8
53	654	968	521	.0609	7
54	680	956	557	.0594	6
55	.43706	.89943	.48593	2.0579	**5**
56	733	930	629	.0564	4
57	759	918	665	.0549	3
58	785	905	701	.0533	2
59	811	892	737	.0518	1
60	.43837	.89879	.48773	2.0503	**0**
′	cos	sin	cot	tan	′

64°

26°

/	sin	cos	tan	cot	/
0	.43837	.89879	.48773	2.0503	**60**
1	863	867	809	.0488	59
2	889	854	845	.0473	58
3	916	841	881	.0458	57
4	942	828	917	.0443	56
5	.43968	.89816	.48953	2.0428	**55**
6	.43994	803	.48989	.0413	54
7	.44020	790	.49026	.0398	53
8	046	777	062	.0383	52
9	072	764	098	.0368	51
10	.44098	.89752	.49134	2.0353	**50**
11	124	739	170	.0338	49
12	151	726	206	.0323	48
13	177	713	242	.0308	47
14	203	700	278	.0293	46
15	.44229	.89687	.49315	2.0278	**45**
16	255	674	351	.0263	44
17	281	662	387	.0248	43
18	307	649	423	.0233	42
19	333	636	459	.0219	41
20	.44359	.89623	.49495	2.0204	**40**
21	385	610	532	.0189	39
22	411	597	568	.0174	38
23	437	584	604	.0160	37
24	464	571	640	.0145	36
25	.44490	.89558	.49677	2.0130	**35**
26	516	545	713	.0115	34
27	542	532	749	.0101	33
28	568	519	786	.0086	32
29	594	506	822	.0072	31
30	.44620	.89493	.49858	2.0057	**30**
31	646	480	894	.0042	29
32	672	467	931	.0028	28
33	698	454	.49967	2.0013	27
34	724	441	.50004	1.9999	26
35	.44750	.89428	.50040	1.9984	**25**
36	776	415	076	.9970	24
37	802	402	113	.9955	23
38	828	389	149	.9941	22
39	854	376	185	.9926	21
40	.44880	.89363	.50222	1.9912	**20**
41	906	350	258	.9897	19
42	932	337	295	.9883	18
43	958	324	331	.9868	17
44	.44984	311	368	.9854	16
45	.45010	.89298	.50404	1.9840	**15**
46	036	285	441	.9825	14
47	062	272	477	.9811	13
48	088	259	514	.9797	12
49	114	245	550	.9782	11
50	.45140	.89232	.50587	1.9768	**10**
51	166	219	623	.9754	9
52	192	206	660	.9740	8
53	218	193	696	.9725	7
54	243	180	733	.9711	6
55	.45269	.89167	.50769	1.9697	**5**
56	295	153	806	.9683	4
57	321	140	843	.9669	3
58	347	127	879	.9654	2
59	373	114	916	.9640	1
60	.45399	.89101	.50953	1.9626	**0**
/	cos	sin	cot	tan	/

27°

/	sin	cos	tan	cot	/
0	.45399	.89101	.50953	1.9626	**60**
1	425	087	.50989	.9612	59
2	451	074	.51026	.9598	58
3	477	061	063	.9584	57
4	503	048	099	.9570	56
5	.45529	.89035	.51136	1.9556	**55**
6	554	021	173	.9542	54
7	580	.89008	209	.9528	53
8	606	.88995	246	.9514	52
9	632	981	283	.9500	51
10	.45658	.88968	.51319	1.9486	**50**
11	684	955	356	.9472	49
12	710	942	393	.9458	48
13	736	928	430	.9444	47
14	762	915	467	.9430	46
15	.45787	.88902	.51503	1.9416	**45**
16	813	888	540	.9402	44
17	839	875	577	.9388	43
18	865	862	614	.9375	42
19	891	848	651	.9361	41
20	.45917	.88835	.51688	1.9347	**40**
21	942	822	724	.9333	39
22	968	808	761	.9319	38
23	.45994	795	798	.9306	37
24	.46020	782	835	.9292	36
25	.46046	.88768	.51872	1.9278	**35**
26	072	755	909	.9265	34
27	097	741	946	.9251	33
28	123	728	.51983	.9237	32
29	149	715	.52020	.9223	31
30	.46175	.88701	.52057	1.9210	**30**
31	201	688	094	.9196	29
32	226	674	131	.9183	28
33	252	661	168	.9169	27
34	278	647	205	.9155	26
35	.46304	.88634	.52242	1.9142	**25**
36	330	620	279	.9128	24
37	355	607	316	.9115	23
38	381	593	353	.9101	22
39	407	580	390	.9088	21
40	.46433	.88566	.52427	1.9074	**20**
41	458	553	464	.9061	19
42	484	539	501	.9047	18
43	510	526	538	.9034	17
44	536	512	575	.9020	16
45	.46561	.88499	.52613	1.9007	**15**
46	587	485	650	.8993	14
47	613	472	687	.8980	13
48	639	458	724	.8967	12
49	664	445	761	.8953	11
50	.46690	.88431	.52798	1.8940	**10**
51	716	417	836	.8927	9
52	742	404	873	.8913	8
53	767	390	910	.8900	7
54	793	377	947	.8887	6
55	.46819	.88363	.52985	1.8873	**5**
56	844	349	.53022	.8860	4
57	870	336	059	.8847	3
58	896	322	096	.8834	2
59	921	308	134	.8820	1
60	.46947	.88295	.53171	1.8807	**0**
/	cos	sin	cot	tan	/

63° 62°

28°

/	sin	cos	tan	cot	/
0	.46947	.88295	.53171	1.8807	**60**
1	973	281	208	.8794	59
2	.46999	267	246	.8781	58
3	.47024	254	283	.8768	57
4	050	240	320	.8755	56
5	.47076	.88226	.53358	1.8741	**55**
6	101	213	395	.8728	54
7	127	199	432	.8715	53
8	153	185	470	.8702	52
9	178	172	507	.8689	51
10	.47204	.88158	.53545	1.8676	**50**
11	229	144	582	.8663	49
12	255	130	620	.8650	48
13	281	117	657	.8637	47
14	306	103	694	.8624	46
15	.47332	.88089	.53732	1.8611	**45**
16	358	075	769	.8598	44
17	383	062	807	.8585	43
18	409	048	844	.8572	42
19	434	034	882	.8559	41
20	.47460	.88020	.53920	1.8546	**40**
21	486	.88006	957	.8533	39
22	511	.87993	.53995	.8520	38
23	537	979	.54032	.8507	37
24	562	965	070	.8495	36
25	.47588	.87951	.54107	1.8482	**35**
26	614	937	145	.8469	34
27	639	923	183	8456	33
28	665	909	220	.8443	32
29	690	896	258	.8430	31
30	.47716	.87882	.54296	1.8418	**30**
31	741	868	333	.8405	29
32	767	854	371	.8392	28
33	793	840	409	.8379	27
34	818	826	446	.8367	26
35	.47844	.87812	.54484	1.8354	**25**
36	869	798	522	.8341	24
37	895	784	560	.8329	23
38	920	770	597	.8316	22
39	946	756	635	.8303	21
40	.47971	.87743	.54673	1.8291	**20**
41	.47997	729	711	.8278	19
42	.48022	715	748	.8265	18
43	048	701	786	.8253	17
44	073	687	824	.8240	16
45	.48099	.87673	.54862	1.8228	**15**
46	124	659	900	.8215	14
47	150	645	938	.8202	13
48	175	631	.54975	.8190	12
49	201	617	.55013	.8177	11
50	.48226	.87603	.55051	1.8165	**10**
51	252	589	089	.8152	9
52	277	575	127	.8140	8
53	303	561	165	.8127	7
54	328	546	203	.8115	6
55	.48354	.87532	.55241	1.8103	**5**
56	379	518	279	.8090	4
57	405	504	317	.8078	3
58	430	490	355	.8065	2
59	456	476	393	.8053	1
60	.48481	.87462	.55431	1.8040	**0**
/	cos	sin	cot	tan	/

29°

/	sin	cos	tan	cot	/
0	.48481	.87462	.55431	1.8040	**60**
1	506	448	469	.8028	59
2	532	434	507	.8016	58
3	557	420	545	.8003	57
4	583	406	583	.7991	56
5	.48608	.87391	.55621	1.7979	**55**
6	634	377	659	.7966	54
7	659	363	697	.7954	53
8	684	349	736	.7942	52
9	710	335	774	.7930	51
10	.48735	.87321	.55812	1.7917	**50**
11	761	306	850	.7905	49
12	786	292	888	.7893	48
13	811	278	926	.7881	47
14	837	264	.55964	.7868	46
15	.48862	.87250	.56003	1.7856	**45**
16	888	235	041	.7844	44
17	913	221	079	.7832	43
18	938	207	117	.7820	42
19	964	193	156	.7808	41
20	.48989	.87178	.56194	1.7796	**40**
21	.49014	164	232	.7783	39
22	040	150	270	.7771	38
23	065	136	309	.7759	37
24	090	121	347	.7747	36
25	.49116	.87107	.56385	1.7735	**35**
26	141	093	424	.7723	34
27	166	079	462	.7711	33
28	192	064	501	.7699	32
29	217	050	539	.7687	31
30	.49242	.87036	.56577	1.7675	**30**
31	268	021	616	.7663	29
32	293	.87007	654	.7651	28
33	318	.86993	693	.7639	27
34	344	978	731	.7627	26
35	.49369	.86964	.56769	1.7615	**25**
36	394	949	808	.7603	24
37	419	935	846	.7591	23
38	445	921	885	.7579	22
39	470	906	923	.7567	21
40	.49495	.86892	.56962	1.7556	**20**
41	521	878	.57000	.7544	19
42	546	863	039	.7532	18
43	571	849	078	.7520	17
44	596	834	116	.7508	16
45	.49622	.86820	.57155	1.7496	**15**
46	647	805	193	.7485	14
47	672	791	232	.7473	13
48	697	777	271	.7461	12
49	723	762	309	.7449	11
50	.49748	.86748	.57348	1.7437	**10**
51	773	733	386	.7426	9
52	798	719	425	.7414	8
53	824	704	464	.7402	7
54	849	690	503	.7391	6
55	.49874	.86675	.57541	1.7379	**5**
56	899	661	580	.7367	4
57	924	646	619	.7355	3
58	950	632	657	.7344	2
59	.49975	617	696	.7332	1
60	.50000	.86603	.57735	1.7321	**0**
/	cos	sin	cot	tan	/

61°

60°

30°

′	sin	cos	tan	cot	′
0	.50000	.86603	.57735	1.7321	**60**
1	025	588	774	.7309	59
2	050	573	813	.7297	58
3	076	559	851	.7286	57
4	101	544	890	.7274	56
5	.50126	.86530	.57929	1.7262	**55**
6	151	515	.57968	.7251	54
7	176	501	.58007	.7239	53
8	201	486	046	.7228	52
9	227	471	085	.7216	51
10	.50252	.86457	.58124	1.7205	**50**
11	277	442	162	.7193	49
12	302	427	201	.7182	48
13	327	413	240	.7170	47
14	352	398	279	.7159	46
15	.50377	.86384	.58318	1.7147	**45**
16	403	369	357	.7136	44
17	428	354	396	.7124	43
18	453	340	435	.7113	42
19	478	325	474	.7102	41
20	.50503	.86310	.58513	1.7090	**40**
21	528	295	552	.7079	39
22	553	281	591	.7067	38
23	578	266	631	.7056	37
24	603	251	670	.7045	36
25	.50628	.86237	.58709	1.7033	**35**
26	654	222	748	.7022	34
27	679	207	787	.7011	33
28	704	192	826	.6999	32
29	729	178	865	.6988	31
30	.50754	.86163	.58905	1.6977	**30**
31	779	148	944	.6965	29
32	804	133	.58983	.6954	28
33	829	119	.59022	.6943	27
34	854	104	061	.6932	26
35	.50879	.86089	.59101	1.6920	**25**
36	904	074	140	.6909	24
37	929	059	179	.6898	23
38	954	045	218	.6887	22
39	.50979	030	258	.6875	21
40	.51004	.86015	.59297	1.6864	**20**
41	029	.86000	336	.6853	19
42	054	.85985	376	.6842	18
43	079	970	415	.6831	17
44	104	956	454	.6820	16
45	.51129	.85941	.59494	1.6808	**15**
46	154	926	533	.6797	14
47	179	911	573	.6786	13
48	204	896	612	.6775	12
49	229	881	651	.6764	11
50	.51254	.85866	.59691	1.6753	**10**
51	279	851	730	.6742	9
52	304	836	770	.6731	8
53	329	821	809	.6720	7
54	354	806	849	.6709	6
55	.51379	.85792	.59888	1.6698	**5**
56	404	777	928	.6687	4
57	429	762	.59967	.6676	3
58	454	747	.60007	.6665	2
59	479	732	046	.6654	1
60	.51504	.85717	.60086	1.6643	**0**
′	cos	sin	cot	tan	′

59°

31°

′	sin	cos	tan	cot	′
0	.51504	.85717	.60086	1.6643	**60**
1	529	702	126	.6632	59
2	554	687	165	.6621	58
3	579	672	205	.6610	57
4	604	657	245	.6599	56
5	.51628	.85642	.60284	1.6588	**55**
6	653	627	324	.6577	54
7	678	612	364	.6566	53
8	703	597	403	.6555	52
9	728	582	443	.6545	51
10	.51753	.85567	.60483	1.6534	**50**
11	778	551	522	.6523	49
12	803	536	562	.6512	48
13	828	521	602	.6501	47
14	852	506	642	.6490	46
15	.51877	.85491	.60681	1.6479	**45**
16	902	476	721	.6469	44
17	927	461	761	.6458	43
18	952	446	801	.6447	42
19	.51977	431	841	.6436	41
20	.52002	.85416	.60881	1.6426	**40**
21	026	401	921	.6415	39
22	051	385	.60960	.6404	38
23	076	370	.61000	.6393	37
24	101	355	040	.6383	36
25	.52126	.85340	.61080	1.6372	**35**
26	151	325	120	.6361	34
27	175	310	160	.6351	33
28	200	294	200	.6340	32
29	225	279	240	.6329	31
30	.52250	.85264	.61280	1.6319	**30**
31	275	249	320	.6308	29
32	299	234	360	.6297	28
33	324	218	400	.6287	27
34	349	203	440	.6276	26
35	.52374	.85188	.61480	1.6265	**25**
36	399	173	520	.6255	24
37	423	157	561	.6244	23
38	448	142	601	.6234	22
39	473	127	641	.6223	21
40	.52498	.85112	.61681	1.6212	**20**
41	522	096	721	.6202	19
42	547	081	761	.6191	18
43	572	066	801	.6181	17
44	597	051	842	.6170	16
45	.52621	.85035	.61882	1.6160	**15**
46	646	020	922	.6149	14
47	671	.85005	.61962	.6139	13
48	696	.84989	.62003	.6128	12
49	720	974	043	.6118	11
50	.52745	.84959	.62083	1.6107	**10**
51	770	943	124	.6097	9
52	794	928	164	.6087	8
53	819	913	204	.6076	7
54	844	897	245	.6066	6
55	.52869	.84882	.62285	1.6055	**5**
56	893	866	325	.6045	4
57	918	851	366	.6034	3
58	943	836	406	.6024	2
59	967	820	446	.6014	1
60	.52992	.84805	.62487	1.6003	**0**
′	cos	sin	cot	tan	′

58°

32°

′	sin	cos	tan	cot	′
0	.52992	.84805	.62487	1.6003	**60**
1	.53017	789	527	.5993	59
2	041	774	568	.5983	58
3	066	759	608	.5972	57
4	091	743	649	.5962	56
5	.53115	.84728	.62689	1.5952	**55**
6	140	712	730	.5941	54
7	164	697	770	.5931	53
8	189	681	811	.5921	52
9	214	666	852	.5911	51
10	.53238	.84650	.62892	1.5900	**50**
11	263	635	933	.5890	49
12	288	619	.62973	.5880	48
13	312	604	.63014	.5869	47
14	337	588	055	.5859	46
15	.53361	.84573	.63095	1.5849	**45**
16	386	557	136	.5839	44
17	411	542	177	.5829	43
18	435	526	217	.5818	42
19	460	511	258	.5808	41
20	.53484	.84495	.63299	1.5798	**40**
21	509	480	340	.5788	39
22	534	464	380	.5778	38
23	558	448	421	.5768	37
24	583	433	462	.5757	36
25	.53607	.84417	.63503	1.5747	**35**
26	632	402	544	.5737	34
27	656	386	584	.5727	33
28	681	370	625	.5717	32
29	705	355	666	.5707	31
30	.53730	.84339	.63707	1.5697	**30**
31	754	324	748	.5687	29
32	779	308	789	.5677	28
33	804	292	830	.5667	27
34	828	277	871	.5657	26
35	.53853	.84261	.63912	1.5647	**25**
36	877	245	953	.5637	24
37	902	230	.63994	.5627	23
38	926	214	.64035	.5617	22
39	951	198	076	.5607	21
40	.53975	.84182	.64117	1.5597	**20**
41	.54000	167	158	.5587	19
42	024	151	199	.5577	18
43	049	135	240	.5567	17
44	073	120	281	.5557	16
45	.54097	.84104	.64322	1.5547	**15**
46	122	088	363	.5537	14
47	146	072	404	.5527	13
48	171	057	446	.5517	12
49	195	041	487	.5507	11
50	.54220	.84025	.64528	1.5497	**10**
51	244	.84009	569	.5487	9
52	269	.83994	610	.5477	8
53	293	978	652	.5468	7
54	317	962	693	.5458	6
55	.54342	.83946	.64734	1.5448	**5**
56	366	930	775	.5438	4
57	391	915	817	.5428	3
58	415	899	858	.5418	2
59	440	883	899	.5408	1
60	.54464	.83867	.64941	1.5399	**0**
′	cos	sin	cot	tan	′

57°

33°

′	sin	cos	tan	cot	′
0	.54464	.83867	.64941	1.5399	**60**
1	488	851	.64982	.5389	59
2	513	835	.65024	.5379	58
3	537	819	065	.5369	57
4	561	804	106	.5359	56
5	.54586	.83788	.65148	1.5350	**55**
6	610	772	189	.5340	54
7	635	756	231	.5330	53
8	659	740	272	.5320	52
9	683	724	314	.5311	51
10	.54708	.83708	.65355	1.5301	**50**
11	732	692	397	.5291	49
12	756	676	438	.5282	48
13	781	660	480	.5272	47
14	805	645	521	.5262	46
15	.54829	.83629	.65563	1.5253	**45**
16	854	613	604	.5243	44
17	878	597	646	.5233	43
18	902	581	688	.5224	42
19	927	565	729	.5214	41
20	.54951	.83549	.65771	1.5204	**40**
21	975	533	813	.5195	39
22	.54999	517	854	.5185	38
23	.55024	501	896	.5175	37
24	048	485	938	.5166	36
25	.55072	.83469	.65980	1.5156	**35**
26	097	453	.66021	.5147	34
27	121	437	063	.5137	33
28	145	421	105	.5127	32
29	169	405	147	.5118	31
30	.55194	.83389	.66189	1.5108	**30**
31	218	373	230	.5099	29
32	242	356	272	.5089	28
33	266	340	314	.5080	27
34	291	324	356	.5070	26
35	.55315	.83308	.66398	1.5061	**25**
36	339	292	440	.5051	24
37	363	276	482	.5042	23
38	388	260	524	.5032	22
39	412	244	566	.5023	21
40	.55436	.83228	.66608	1.5013	**20**
41	460	212	650	.5004	19
42	484	195	692	.4994	18
43	509	179	734	.4985	17
44	533	163	776	.4975	16
45	.55557	.83147	.66818	1.4966	**15**
46	581	131	860	.4957	14
47	605	115	902	.4947	13
48	630	098	944	.4938	12
49	654	082	.66986	.4928	11
50	.55678	.83066	.67028	1.4919	**10**
51	702	050	071	.4910	9
52	726	034	113	.4900	8
53	750	017	155	.4891	7
54	775	.83001	197	.4882	6
55	.55799	.82985	.67239	1.4872	**5**
56	823	969	282	.4863	4
57	847	953	324	.4854	3
58	871	936	366	.4844	2
59	895	920	409	.4835	1
60	.55919	.82904	.67451	1.4826	**0**
′	cos	sin	cot	tan	′

56°

34°

/	sin	cos	tan	cot	/
0	.55919	.82904	.67451	1.4826	**60**
1	943	887	493	.4816	59
2	968	871	536	.4807	58
3	.55992	855	578	.4798	57
4	.56016	839	620	.4788	56
5	.56040	.82822	.67663	1.4779	**55**
6	064	806	705	.4770	54
7	088	790	748	.4761	53
8	112	773	790	.4751	52
9	136	757	832	.4742	51
10	.56160	.82741	.67875	1.4733	**50**
11	184	724	917	.4724	49
12	208	708	.67960	.4715	48
13	232	692	.68002	.4705	47
14	256	675	045	.4696	46
15	.56280	.82659	.68088	1.4687	**45**
16	305	643	130	.4678	44
17	329	626	173	.4669	43
18	353	610	215	.4659	42
19	377	593	258	.4650	41
20	.56401	.82577	.68301	1.4641	**40**
21	425	561	343	.4632	39
22	449	544	386	.4623	38
23	473	528	429	.4614	37
24	497	511	471	.4605	36
25	.56521	.82495	.68514	1.4596	**35**
26	545	478	557	.4586	34
27	569	462	600	.4577	33
28	593	446	642	.4568	32
29	617	429	685	.4559	31
30	.56641	.82413	.68728	1.4550	**30**
31	665	396	771	.4541	29
32	689	380	814	.4532	28
33	713	363	857	.4523	27
34	736	347	900	.4514	26
35	.56760	.82330	.68942	1.4505	**25**
36	784	314	.68985	.4496	24
37	808	297	.69028	.4487	23
38	832	281	071	.4478	22
39	856	264	114	.4469	21
40	.56880	.82248	.69157	1.4460	**20**
41	904	231	200	.4451	19
42	928	214	243	.4442	18
43	952	198	286	.4433	17
44	.56976	181	329	.4424	16
45	.57000	.82165	.69372	1.4415	**15**
46	024	148	416	.4406	14
47	047	132	459	.4397	13
48	071	115	502	.4388	12
49	095	098	545	.4379	11
50	.57119	.82082	.69588	1.4370	**10**
51	143	065	631	.4361	9
52	167	048	675	.4352	8
53	191	032	718	.4344	7
54	215	.82015	761	.4335	6
55	.57238	.81999	.69804	1.4326	**5**
56	262	982	847	.4317	4
57	286	965	891	.4308	3
58	310	949	934	.4299	2
59	334	932	.69977	.4290	1
60	.57358	.81915	.70021	1.4281	**0**
/	cos	sin	cot	tan	/

55°

35°

/	sin	cos	tan	cot	/
0	.57358	.81915	.70021	1.4281	**60**
1	381	899	064	.4273	59
2	405	882	107	.4264	58
3	429	865	151	.4255	57
4	453	848	194	.4246	56
5	.57477	.81832	.70238	1.4237	**55**
6	501	815	281	.4229	54
7	524	798	325	.4220	53
8	548	782	368	.4211	52
9	572	765	412	.4202	51
10	.57596	.81748	.70455	1.4193	**50**
11	619	731	499	.4185	49
12	643	714	542	.4176	48
13	667	698	586	.4167	47
14	691	681	629	.4158	46
15	.57715	.81664	.70673	1.4150	**45**
16	738	647	717	.4141	44
17	762	631	760	.4132	43
18	786	614	804	.4124	42
19	810	597	848	.4115	41
20	.57833	.81580	.70891	1.4106	**40**
21	857	563	935	.4097	39
22	881	546	.70979	.4089	38
23	904	530	.71023	.4080	37
24	928	513	066	.4071	36
25	.57952	.81496	.71110	1.4063	**35**
26	976	479	154	.4054	34
27	.57999	462	198	.4045	33
28	.58023	445	242	.4037	32
29	047	428	285	.4028	31
30	.58070	.81412	.71329	1.4019	**30**
31	094	395	373	.4011	29
32	118	378	417	.4002	28
33	141	361	461	.3994	27
34	165	344	505	.3985	26
35	.58189	.81327	.71549	1.3976	**25**
36	212	310	593	.3968	24
37	236	293	637	.3959	23
38	260	276	681	.3951	22
39	283	259	725	.3942	21
40	.58307	.81242	.71769	1.3934	**20**
41	330	225	813	.3925	19
42	354	208	857	.3916	18
43	378	191	901	.3908	17
44	401	174	946	.3899	16
45	.58425	.81157	.71990	1.3891	**15**
46	449	140	.72034	.3882	14
47	472	123	078	.3874	13
48	496	106	122	.3865	12
49	519	089	167	.3857	11
50	.58543	.81072	.72211	1.3848	**10**
51	567	055	255	.3840	9
52	590	038	299	.3831	8
53	614	021	344	.3823	7
54	637	.81004	388	.3814	6
55	.58661	.80987	.72432	1.3806	**5**
56	684	970	477	.3798	4
57	708	953	521	.3789	3
58	731	936	565	.3781	2
59	755	919	610	.3772	1
60	.58779	.80902	.72654	1.3764	**0**
/	cos	sin	cot	tan	/

54°

36°

′	sin	cos	tan	cot	′
0	.58779	.80902	.72654	1.3764	**60**
1	802	885	699	.3755	59
2	826	867	743	.3747	58
3	849	850	788	.3739	57
4	873	833	832	.3730	56
5	.58896	.80816	.72877	1.3722	**55**
6	920	799	921	.3713	54
7	943	782	.72966	.3705	53
8	967	765	.73010	.3697	52
9	.58990	748	055	.3688	51
10	.59014	.80730	.73100	1.3680	**50**
11	037	713	144	.3672	49
12	061	696	189	.3663	48
13	084	679	234	.3655	47
14	108	662	278	.3647	46
15	.59131	.80644	.73323	1.3638	**45**
16	154	627	368	.3630	44
17	178	610	413	.3622	43
18	201	593	457	.3613	42
19	225	576	502	.3605	41
20	.59248	.80558	.73547	1.3597	**40**
21	272	541	592	.3588	39
22	295	524	637	.3580	38
23	318	507	681	.3572	37
24	342	489	726	.3564	36
25	.59365	.80472	.73771	1.3555	**35**
26	389	455	816	.3547	34
27	412	438	861	.3539	33
28	436	420	906	.3531	32
29	459	403	951	.3522	31
30	.59482	.80386	.73996	1.3514	**30**
31	506	368	.74041	.3506	29
32	529	351	086	.3498	28
33	552	334	131	.3490	27
34	576	316	176	.3481	26
35	.59599	.80299	.74221	1.3473	**25**
36	622	282	267	.3465	24
37	646	264	312	.3457	23
38	669	247	357	.3449	22
39	693	230	402	.3440	21
40	.59716	.80212	.74447	1.3432	**20**
41	739	195	492	.3424	19
42	763	178	538	.3416	18
43	786	160	583	.3408	17
44	809	143	628	.3400	16
45	.59832	.80125	.74674	1.3392	**15**
46	856	108	719	.3384	14
47	879	091	764	.3375	13
48	902	073	810	.3367	12
49	926	056	855	.3359	11
50	.59949	.80038	.74900	1.3351	**10**
51	972	021	946	.3343	9
52	.59995	.80003	.74991	.3335	8
53	.60019	.79986	.75037	.3327	7
54	042	968	082	.3319	6
55	.60065	.79951	.75128	1.3311	**5**
56	089	934	173	.3303	4
57	112	916	219	.3295	3
58	135	899	264	.3287	2
59	158	881	310	.3278	1
60	.60182	.79864	.75355	1.3270	**0**
′	cos	sin	cot	tan	′

53°

37°

′	sin	cos	tan	cot	′
0	.60182	.79864	.75355	1.3270	**60**
1	205	846	401	.3262	59
2	228	829	447	.3254	58
3	251	811	492	.3246	57
4	274	793	538	.3238	56
5	.60298	.79776	.75584	1.3230	**55**
6	321	758	629	.3222	54
7	344	741	675	.3214	53
8	367	723	721	.3206	52
9	390	706	767	.3198	51
10	.60414	.79688	.75812	1.3190	**50**
11	437	671	858	.3182	49
12	460	653	904	.3175	48
13	483	635	950	.3167	47
14	506	618	.75996	.3159	46
15	.60529	.79600	.76042	1.3151	**45**
16	553	583	088	.3143	44
17	576	565	134	.3135	43
18	599	547	180	.3127	42
19	622	530	226	.3119	41
20	.60645	.79512	.76272	1.3111	**40**
21	668	494	318	.3103	39
22	691	477	364	.3095	38
23	714	459	410	.3087	37
24	738	441	456	.3079	36
25	.60761	.79424	.76502	1.3072	**35**
26	784	406	548	.3064	34
27	807	388	594	.3056	33
28	830	371	640	.3048	32
29	853	353	686	.3040	31
30	.60876	.79335	.76733	1.3032	**30**
31	899	318	779	.3024	29
32	922	300	825	.3017	28
33	945	282	871	.3009	27
34	968	264	918	.3001	26
35	.60991	.79247	.76964	1.2993	**25**
36	.61015	229	.77010	.2985	24
37	038	211	057	.2977	23
38	061	193	103	.2970	22
39	084	176	149	.2962	21
40	.61107	.79158	.77196	1.2954	**20**
41	130	140	242	.2946	19
42	153	122	289	.2938	18
43	176	105	335	.2931	17
44	199	087	382	.2923	16
45	.61222	.79069	.77428	1.2915	**15**
46	245	051	475	.2907	14
47	268	033	521	.2900	13
48	291	.79016	568	.2892	12
49	314	.78998	615	.2884	11
50	.61337	.78980	.77661	1.2876	**10**
51	360	962	708	.2869	9
52	383	944	754	.2861	8
53	406	926	801	.2853	7
54	429	908	848	.2846	6
55	.61451	.78891	.77895	1.2838	**5**
56	474	873	941	.2830	4
57	497	855	.77988	.2822	3
58	520	837	.78035	.2815	2
59	543	819	082	.2807	1
60	.61566	.78801	.78129	1.2799	**0**
′	cos	sin	cot	tan	′

52°

38°

/	sin	cos	tan	cot	/
0	.61566	.78801	.78129	1.2799	**60**
1	589	783	175	.2792	59
2	612	765	222	.2784	58
3	635	747	269	.2776	57
4	658	729	316	.2769	56
5	.61681	.78711	.78363	1.2761	**55**
6	704	694	410	.2753	54
7	726	676	457	.2746	53
8	749	658	504	.2738	52
9	772	640	551	.2731	51
10	.61795	.78622	.78598	1.2723	**50**
11	818	604	645	.2715	49
12	841	586	692	.2708	48
13	864	568	739	.2700	47
14	887	550	786	.2693	46
15	.61909	.78532	.78834	1.2685	**45**
16	932	514	881	.2677	44
17	955	496	928	.2670	43
18	.61978	478	.78975	.2662	42
19	.62001	460	.79022	.2655	41
20	.62024	.78442	.79070	1.2647	**40**
21	046	424	117	.2640	39
22	069	405	164	.2632	38
23	092	387	212	.2624	37
24	115	369	259	.2617	36
25	.62138	.78351	.79306	1.2609	**35**
26	160	333	354	.2602	34
27	183	315	401	.2594	33
28	206	297	449	.2587	32
29	229	279	496	.2579	31
30	.62251	.78261	.79544	1.2572	**30**
31	274	243	591	.2564	29
32	297	225	639	.2557	28
33	320	206	686	.2549	27
34	342	188	734	.2542	26
35	.62365	.78170	.79781	1.2534	**25**
36	388	152	829	.2527	24
37	411	134	877	.2519	23
38	433	116	924	.2512	22
39	456	098	.79972	.2504	21
40	.62479	.78079	.80020	1.2497	**20**
41	502	061	067	.2489	19
42	524	043	115	.2482	18
43	547	025	163	.2475	17
44	570	.78007	211	.2467	16
45	.62592	.77988	.80258	1.2460	**15**
46	615	970	306	.2452	14
47	638	952	354	.2445	13
48	660	934	402	.2437	12
49	683	916	450	.2430	11
50	.62706	.77897	.80498	1.2423	**10**
51	728	879	546	.2415	9
52	751	861	594	.2408	8
53	774	843	642	.2401	7
54	796	824	690	.2393	6
55	.62819	.77806	.80738	1.2386	**5**
56	842	788	786	.2378	4
57	864	769	834	.2371	3
58	887	751	882	.2364	2
59	909	733	930	.2356	1
60	.62932	.77715	.80978	1.2349	**0**
/	cos	sin	cot	tan	/

39°

/	sin	cos	tan	cot	/
0	.62932	.77715	.80978	1.2349	**60**
1	955	696	.81027	.2342	59
2	.62977	678	075	.2334	58
3	.63000	660	123	.2327	57
4	022	641	171	.2320	56
5	.63045	.77623	.81220	1.2312	**55**
6	068	605	268	.2305	54
7	090	586	316	.2298	53
8	113	568	364	.2290	52
9	135	550	413	.2283	51
10	.63158	.77531	.81461	1.2276	**50**
11	180	513	510	.2268	49
12	203	494	558	.2261	48
13	225	476	606	.2254	47
14	248	458	655	.2247	46
15	.63271	.77439	.81703	1.2239	**45**
16	293	421	752	.2232	44
17	316	402	800	.2225	43
18	338	384	849	.2218	42
19	361	366	898	.2210	41
20	.63383	.77347	.81946	1.2203	**40**
21	406	329	.81995	.2196	39
22	428	310	.82044	.2189	38
23	451	292	092	.2181	37
24	473	273	141	.2174	36
25	.63496	.77255	.82190	1.2167	**35**
26	518	236	238	.2160	34
27	540	218	287	.2153	33
28	563	199	336	.2145	32
29	585	181	385	.2138	31
30	.63608	.77162	.82434	1.2131	**30**
31	630	144	483	.2124	29
32	653	125	531	.2117	28
33	675	107	580	.2109	27
34	698	088	629	.2102	26
35	.63720	.77070	.82678	1.2095	**25**
36	742	051	727	.2088	24
37	765	033	776	.2081	23
38	787	.77014	825	.2074	22
39	810	.76996	874	.2066	21
40	.63832	.76977	.82923	1.2059	**20**
41	854	959	.82972	.2052	19
42	877	940	.83022	.2045	18
43	899	921	071	.2038	17
44	922	903	120	.2031	16
45	.63944	.76884	.83169	1.2024	**15**
46	966	866	218	.2017	14
47	.63989	847	268	.2009	13
48	.64011	828	317	.2002	12
49	033	810	366	.1995	11
50	.64056	.76791	.83415	1.1988	**10**
51	078	772	465	.1981	9
52	100	754	514	.1974	8
53	123	735	564	.1967	7
54	145	717	613	.1960	6
55	.64167	.76698	.83662	1.1953	**5**
56	190	679	712	.1946	4
57	212	661	761	.1939	3
58	234	642	811	.1932	2
59	256	623	860	.1925	1
60	.64279	.76604	.83910	1.1918	**0**
/	cos	sin	cot	tan	/

50°

51°

40°

′	sin	cos	tan	cot	′
0	.64279	.76604	.83910	1.1918	**60**
1	301	586	.83960	.1910	59
2	323	567	.84009	.1903	58
3	346	548	059	.1896	57
4	368	530	108	.1889	56
5	.64390	.76511	.84158	1.1882	**55**
6	412	492	208	.1875	54
7	435	473	258	.1868	53
8	457	455	307	.1861	52
9	479	436	357	.1854	51
10	.64501	.76417	.84407	1.1847	**50**
11	524	398	457	.1840	49
12	546	380	507	.1833	48
13	568	361	556	.1826	47
14	590	342	606	.1819	46
15	.64612	.76323	.84656	1.1812	**45**
16	635	304	706	.1806	44
17	657	286	756	.1799	43
18	679	267	806	.1792	42
19	701	248	856	.1785	41
20	.64723	.76229	.84906	1.1778	**40**
21	746	210	.84956	.1771	39
22	768	192	.85006	.1764	38
23	790	173	057	.1757	37
24	812	154	107	.1750	36
25	.64834	.76135	.85157	1.1743	**35**
26	856	116	207	.1736	34
27	878	097	257	.1729	33
28	901	078	308	.1722	32
29	923	059	358	.1715	31
30	.64945	.76041	.85408	1.1708	**30**
31	967	022	458	.1702	29
32	.64989	.76003	509	.1695	28
33	.65011	.75984	559	.1688	27
34	033	965	609	.1681	26
35	.65055	.75946	.85660	1.1674	**25**
36	077	927	710	.1667	24
37	100	908	761	.1660	23
38	122	889	811	.1653	22
39	144	870	862	.1647	21
40	.65166	.75851	.85912	1.1640	**20**
41	188	832	.85963	.1633	19
42	210	813	.86014	.1626	18
43	232	794	064	.1619	17
44	254	775	115	.1612	16
45	.65276	.75756	.86166	1.1606	**15**
46	298	738	216	.1599	14
47	320	719	267	.1592	13
48	342	700	318	.1585	12
49	364	680	368	.1578	11
50	.65386	.75661	.86419	1.1571	**10**
51	408	642	470	.1565	9
52	430	623	521	.1558	8
53	452	604	572	.1551	7
54	474	585	623	.1544	6
55	.65496	.75566	.86674	1.1538	**5**
56	518	547	725	.1531	4
57	540	528	776	.1524	3
58	562	509	827	.1517	2
59	584	490	878	.1510	1
60	.65606	.75471	.86929	1.1504	**0**
′	cos	sin	cot	tan	′

41°

′	sin	cos	tan	cot	′
0	.65606	.75471	.86929	1.1504	**60**
1	628	452	.86980	.1497	59
2	650	433	.87031	.1490	58
3	672	414	082	.1483	57
4	694	395	133	.1477	56
5	.65716	.75375	.87184	1.1470	**55**
6	738	356	236	.1463	54
7	759	337	287	.1456	53
8	781	318	338	.1450	52
9	803	299	389	.1443	51
10	.65825	.75280	.87441	1.1436	**50**
11	847	261	492	.1430	49
12	869	241	543	.1423	48
13	891	222	595	.1416	47
14	913	203	646	.1410	46
15	.65935	.75184	.87698	1.1403	**45**
16	956	165	749	.1396	44
17	.65978	146	801	.1389	43
18	.66000	126	852	.1383	42
19	022	107	904	.1376	41
20	.66044	.75088	.87955	1.1369	**40**
21	066	069	.88007	.1363	39
22	088	050	059	.1356	38
23	109	030	110	.1349	37
24	131	.75011	162	.1343	36
25	.66153	.74992	.88214	1.1336	**35**
26	175	973	265	.1329	34
27	197	953	317	.1323	33
28	218	934	369	.1316	32
29	240	915	421	.1310	31
30	.66262	.74896	.88473	1.1303	**30**
31	284	876	524	.1296	29
32	306	857	576	.1290	28
33	327	838	628	.1283	27
34	349	818	680	.1276	26
35	.66371	.74799	.88732	1.1270	**25**
36	393	780	784	.1263	24
37	414	760	836	.1257	23
38	436	741	888	.1250	22
39	458	722	940	.1243	21
40	.66480	.74703	.88992	1.1237	**20**
41	501	683	.89045	.1230	19
42	523	664	097	.1224	18
43	545	644	149	.1217	17
44	566	625	201	.1211	16
45	.66588	.74606	.89253	1.1204	**15**
46	610	586	306	.1197	14
47	632	567	358	.1191	13
48	653	548	410	.1184	12
49	675	528	463	.1178	11
50	.66697	.74509	.89515	1.1171	**10**
51	718	489	567	.1165	9
52	740	470	620	.1158	8
53	762	451	672	.1152	7
54	783	431	725	.1145	6
55	.66805	.74412	.89777	1.1139	**5**
56	827	392	830	.1132	4
57	848	373	883	.1126	3
58	870	353	935	.1119	2
59	891	334	.89988	.1113	1
60	.66913	.74314	.90040	1.1106	**0**
′	cos	sin	cot	tan	′

49°

48°

42°

′	sin	cos	tan	cot	′
0	.66913	.74314	.90040	1.1106	**60**
1	935	295	093	.1100	59
2	956	276	146	.1093	58
3	978	256	199	.1087	57
4	.66999	237	251	.1080	56
5	.67021	.74217	.90304	1.1074	**55**
6	043	198	357	.1067	54
7	064	178	410	.1061	53
8	086	159	463	.1054	52
9	107	139	516	.1048	51
10	.67129	.74120	90569	1.1041	**50**
11	151	100	621	.1035	49
12	172	080	674	.1028	48
13	194	061	727	.1022	47
14	215	041	781	.1016	46
15	.67237	.74022	.90834	1.1009	**45**
16	258	.74002	887	.1003	44
17	280	.73983	940	.0996	43
18	301	963	.90993	.0990	42
19	323	944	.91046	0983	41
20	.67344	.73924	.91099	1.0977	**40**
21	366	904	153	.0971	39
22	387	885	206	.0964	38
23	409	865	259	.0958	37
24	430	846	313	.0951	36
25	.67452	.73826	.91366	1.0945	**35**
26	473	806	419	.0939	34
27	495	787	473	.0932	33
28	516	767	526	0926	32
29	538	747	580	.0919	31
30	.67559	.73728	.91633	1.0913	**30**
31	580	708	687	.0907	29
32	602	688	740	.0900	28
33	623	669	794	.0894	27
34	645	649	847	0888	26
35	.67666	.73629	.91901	1.0881	**25**
36	688	610	.91955	.0875	24
37	709	590	.92008	.0869	23
38	730	570	062	.0862	22
39	752	551	116	.0856	21
40	.67773	.73531	.92170	1.0850	**20**
41	795	511	224	.0843	19
42	816	491	277	.0837	18
43	837	472	331	.0831	17
44	859	452	385	.0824	16
45	.67880	.73432	.92439	1.0818	**15**
46	901	413	493	.0812	14
47	923	393	547	.0805	13
48	944	373	601	.0799	12
49	965	353	655	.0793	11
50	.67987	.73333	.92709	1.0786	**10**
51	.68008	314	763	.0780	9
52	029	294	817	.0774	8
53	051	274	872	.0768	7
54	072	254	926	.0761	6
55	.68093	.73234	.92980	1.0755	**5**
56	115	215	.93034	.0749	4
57	136	195	088	.0742	3
58	157	175	143	.0736	2
59	179	155	197	.0730	1
60	.68200	.73135	.93252	1.0724	**0**
′	cos	sin	cot	tan	′

43°

′	sin	cos	tan	cot	′
0	.68200	.73135	.93252	1.0724	**60**
1	221	116	306	.0717	59
2	242	096	360	.0711	58
3	264	076	415	.0705	57
4	285	056	469	.0699	56
5	.68306	.73036	.93524	1.0692	**55**
6	327	.73016	578	.0686	54
7	349	.72996	633	.0680	53
8	370	976	688	.0674	52
9	391	957	742	.0668	51
10	.68412	.72937	.93797	1.0661	**50**
11	434	917	852	.0655	49
12	455	897	906	.0649	48
13	476	877	.93961	.0643	47
14	497	857	.94016	.0637	46
15	.68518	.72837	.94071	1.0630	**45**
16	539	817	125	.0624	44
17	561	797	180	.0618	43
18	582	777	235	.0612	42
19	603	757	290	0606	41
20	.68624	.72737	.94345	1.0599	**40**
21	645	717	400	.0593	39
22	666	697	455	.0587	38
23	688	677	510	.0581	37
24	709	657	565	.0575	36
25	.68730	.72637	.94620	1.0569	**35**
26	751	617	676	.0562	34
27	772	597	731	.0556	33
28	793	577	786	.0550	32
29	814	557	841	.0544	31
30	.68835	.72537	.94896	1.0538	**30**
31	857	517	.94952	.0532	29
32	878	497	.95007	.0526	28
33	899	477	062	.0519	27
34	920	457	118	.0513	26
35	.68941	.72437	.95173	1.0507	**25**
36	962	417	229	.0501	24
37	.68983	397	284	.0495	23
38	.69004	377	340	.0489	22
39	025	357	395	.0483	21
40	.69046	.72337	.95451	1.0477	**20**
41	067	317	506	.0470	19
42	088	297	562	.0464	18
43	109	277	618	.0458	17
44	130	257	673	.0452	16
45	.69151	.72236	.95729	1.0446	**15**
46	172	216	785	.0440	14
47	193	196	841	.0434	13
48	214	176	897	.0428	12
49	235	156	.95952	.0422	11
50	.69256	.72136	.96008	1.0416	**10**
51	277	116	064	.0410	9
52	298	095	120	.0404	8
53	319	075	176	.0398	7
54	340	055	232	.0392	6
55	.69361	.72035	.96288	1.0385	**5**
56	382	.72015	344	.0379	4
57	403	.71995	400	.0373	3
58	424	974	457	.0367	2
59	445	954	513	.0361	1
60	.69466	.71934	.96569	1.0355	**0**
′	cos	sin	cot	tan	′

47°

46°

44°

′	sin	cos	tan	cot	′
0	.69466	.71934	.96569	1.0355	**60**
1	487	914	625	.0349	59
2	508	894	681	.0343	58
3	529	873	738	.0337	57
4	549	853	794	.0331	56
5	.69570	.71833	.96850	1.0325	**55**
6	591	813	907	.0319	54
7	612	792	.96963	.0313	53
8	633	772	.97020	.0307	52
9	654	752	076	.0301	51
10	.69675	.71732	.97133	1.0295	**50**
11	696	711	189	.0289	49
12	717	691	246	.0283	48
13	737	671	302	.0277	47
14	758	650	359	.0271	46
15	.69779	.71630	.97416	1.0265	**45**
16	800	610	472	.0259	44
17	821	590	529	.0253	43
18	842	569	586	.0247	42
19	862	549	643	.0241	41
20	.69883	.71529	.97700	1.0235	**40**
21	904	508	756	.0230	39
22	925	488	813	.0224	38
23	946	468	870	.0218	37
24	966	447	927	.0212	36
25	.69987	.71427	.97984	1.0206	**35**
26	.70008	407	.98041	.0200	34
27	029	386	098	.0194	33
28	049	366	155	.0188	32
29	070	345	213	.0182	31
30	.70091	.71325	.98270	1.0176	**30**
31	112	305	327	.0170	29
32	132	284	384	.0164	28
33	153	264	441	.0158	27
34	174	243	499	.0152	26
35	.70195	.71223	.98556	1.0147	**25**
36	215	203	613	.0141	24
37	236	182	671	.0135	23
38	257	162	728	.0129	22
39	277	141	786	.0123	21
40	.70298	.71121	.98843	1.0117	**20**
41	319	100	901	.0111	19
42	339	080	.98958	.0105	18
43	360	059	.99016	.0099	17
44	381	039	073	.0094	16
45	.70401	.71019	.99131	1.0088	**15**
46	422	.70998	189	.0082	14
47	443	978	247	.0076	13
48	463	957	304	.0070	12
49	484	937	362	.0064	11
50	.70505	.70916	.99420	1.0058	**10**
51	525	896	478	.0052	9
52	546	875	536	.0047	8
53	567	855	594	.0041	7
54	587	834	652	.0035	6
55	.70608	.70813	.99710	1.0029	**5**
56	628	793	768	.0023	4
57	649	772	826	.0017	3
58	670	752	884	.0012	2
59	690	731	.99942	.0006	1
60	.70711	.70711	1.0000	1.0000	**0**
′	cos	sin	cot	tan	′

45°

TABLE V

POWERS, ROOTS, RECIPROCALS

Table V gives the squares, square roots, cubes, cube roots, and reciprocals of numbers from 1 to 10 at intervals of 0.1.

The following rules regarding the position of the decimal point are to be noted:

1. Moving the point k places in n moves the point $2k$ places in n^2 in the same direction.

EXAMPLE. $(3.8)^2 = 14.44$; $(380)^2 = 144{,}400$; $(0.38)^2 = 0.1444$; etc.

2. Moving the point $2k$ places in n moves the point k places in $\sqrt{n}$ and $\sqrt{10\,n}$ in the same direction.

EXAMPLES. $\sqrt{3.8} = 1.94936$; $\sqrt{380} = 19.4936$.

$\sqrt{38} = \sqrt{10 \cdot 3.8} = 6.16441$; $\sqrt{3800} = 61.6441$.

3. Moving the point k places in n moves the point $3k$ places in n^3 in the same direction.

EXAMPLE. $(3.8)^3 = 54.872$; $(0.38)^3 = 0.054872$.

4. Moving the point $3k$ places in n moves the point k places in $\sqrt[3]{n}$, $\sqrt[3]{10\,n}$, and $\sqrt[3]{100\,n}$ in the same direction.

EXAMPLES. $\sqrt[3]{3.8} = 1.56049$; $\sqrt[3]{3800} = 15.6049$.

$\sqrt[3]{38} = \sqrt[3]{10 \cdot 3.8} = 3.36198$; $\sqrt[3]{0.038} = 0.336198$.

$\sqrt[3]{380} = \sqrt[3]{100 \cdot 3.8} = 7.24316$; $\sqrt[3]{0.38} = 0.724316$.

5. Moving the decimal point k places in n moves the point k places in $\frac{1}{n}$ in the *opposite* direction.

EXAMPLE. $\frac{1}{3.8} = 0.26316$; $\frac{1}{380} = 0.0026316$; $\frac{1}{0.38} = 2.6316$.

Interpolation will usually give four-figure accuracy except in the columns for n^2 and n^3.

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
1.0	1.00	1.00 000	3.16 228	1.000	1.00 000	2.15 443	4.64 159	1.00 000
1.1	1.21	1.04 881	3.31 662	1.331	1.03 228	2.22 398	4.79 142	.90 909
1.2	1.44	1.09 545	3.46 410	1.728	1.06 266	2.28 943	4.93 242	.83 333
1.3	1.69	1.14 018	3.60 555	2.197	1.09 139	2.35 133	5.06 580	.76 923
1.4	1.96	1.18 322	3.74 166	2.744	1.11 869	2.41 014	5.19 249	.71 429
1.5	2.25	1.22 474	3.87 298	3.375	1.14 471	2.46 621	5.31 329	.66 667
1.6	2.56	1.26 491	4.00 000	4.096	1.16 961	2.51 984	5.42 884	.62 500
1.7	2.89	1.30 384	4.12 311	4.913	1.19 348	2.57 128	5.53 966	.58 824
1.8	3.24	1.34 164	4.24 264	5.832	1.21 644	2.62 074	5.64 622	.55 556
1.9	3.61	1.37 840	4.35 890	6.859	1.23 856	2.66 840	5.74 890	.52 632
2.0	4.00	1.41 421	4.47 214	8.000	1.25 992	2.71 442	5.84 804	.50 000
2.1	4.41	1.44 914	4.58 258	9.261	1.28 058	2.75 892	5.94 392	.47 619
2.2	4.84	1.48 324	4.69 042	10.648	1.30 059	2.80 204	6.03 681	.45 455
2.3	5.29	1.51 658	4.79 583	12.167	1.32 001	2.84 387	6.12 693	.43 478
2.4	5.76	1.54 919	4.89 898	13.824	1.33 887	2.88 450	6.21 447	.41 667
2.5	6.25	1.58 114	5.00 000	15.625	1.35 721	2.92 402	6.29 961	.40 000
2.6	6.76	1.61 245	5.09 902	17.576	1.37 507	2.96 250	6.38 250	.38 462
2.7	7.29	1.64 317	5.19 615	19.683	1.39 248	3.00 000	6.46 330	.37 037
2.8	7.84	1.67 332	5.29 150	21.952	1.40 946	3.03 659	6.54 213	.35 714
2.9	8.41	1.70 294	5.38 516	24.389	1.42 604	3.07 232	6.61 911	.34 483
3.0	9.00	1.73 205	5.47 723	27.000	1.44 225	3.10 723	6.69 433	.33 333
3.1	9.61	1.76 068	5.56 776	29.791	1.45 810	3.14 138	6.76 790	.32 258
3.2	10.24	1.78 885	5.65 685	32.768	1.47 361	3.17 480	6.83 990	.31 250
3.3	10.89	1.81 659	5.74 456	35.937	1.48 881	3.20 753	6.91 042	.30 303
3.4	11.56	1.84 391	5.83 095	39.304	1.50 369	3.23 961	6.97 953	.29 412
3.5	12.25	1.87 083	5.91 608	42.875	1.51 829	3.27 107	7.04 730	.28 571
3.6	12.96	1.89 737	6.00 000	46.656	1.53 262	3.30 193	7.11 379	.27 778
3.7	13.69	1.92 354	6.08 276	50.653	1.54 668	3.33 222	7.17 905	.27 027
3.8	14.44	1.94 936	6.16 441	54.872	1.56 049	3.36 198	7.24 316	.26 316
3.9	15.21	1.97 484	6.24 500	59.319	1.57 406	3.39 121	7.30 614	.25 641
4.0	16.00	2.00 000	6.32 456	64.000	1.58 740	3.41 995	7.36 806	.25 000
4.1	16.81	2.02 485	6.40 312	68.921	1.60 052	3.44 822	7.42 896	.24 390
4.2	17.64	2.04 939	6.48 074	74.088	1.61 343	3.47 603	7.48 887	.23 810
4.3	18.49	2.07 364	6.55 744	79.507	1.62 613	3.50 340	7.54 784	.23 256
4.4	19.36	2.09 762	6.63 325	85.184	1.63 864	3.53 035	7.60 590	.22 727
4.5	20.25	2.12 132	6.70 820	91.125	1.65 096	3.55 689	7.66 309	.22 222
4.6	21.16	2.14 476	6.78 233	97.336	1.66 310	3.58 305	7.71 944	.21 739
4.7	22.09	2.16 795	6.85 565	103.823	1.67 507	3.60 883	7.77 498	.21 277
4.8	23.04	2.19 089	6.92 820	110.592	1.68 687	3.63 424	7.82 974	.20 833
4.9	24.01	2.21 359	7.00 000	117.649	1.69 850	3.65 931	7.88 374	.20 408
5.0	25.00	2.23 607	7.07 107	125.000	1.70 998	3.68 403	7.93 701	.20 000
5.1	26.01	2.25 832	7.14 143	132.651	1.72 130	3.70 843	7.98 957	.19 608
5.2	27.04	2.28 035	7.21 110	140.608	1.73 248	3.73 251	8.04 145	.19 231
5.3	28.09	2.30 217	7.28 011	148.877	1.74 351	3.75 629	8.09 267	.18 868
5.4	29.16	2.32 379	7.34 847	157.464	1.75 441	3.77 976	8.14 325	.18 519
5.5	30.25	2.34 521	7.41 620	166.375	1.76 517	3.80 295	8.19 321	.18 182
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
5.5	30.25	2.34 521	7.41 620	166.375	1.76 517	3.80 295	8.19 321	.18 182
5.6	31.36	2.36 643	7.48 331	175.616	1.77 581	3.82 586	8.24 257	.17 857
5.7	32.49	2.38 747	7.54 983	185.193	1.78 632	3.84 850	8.29 134	.17 544
5.8	33.64	2.40 832	7.61 577	195.112	1.79 670	3.87 088	8.33 955	.17 241
5.9	34.81	2.42 899	7.68 115	205.379	1.80 697	3.89 300	8.38 721	.16 949
6.0	36.00	2.44 949	7.74 597	216.000	1.81 712	3.91 487	8.43 433	.16 667
6.1	37.21	2.46 982	7.81 025	226.981	1.82 716	3.93 650	8.48 093	.16 393
6.2	38.44	2.48 998	7.87 401	238.328	1.83 709	3.95 789	8.52 702	.16 129
6.3	39.69	2.50 998	7.93 725	250.047	1.84 691	3.97 906	8.57 262	.15 873
6.4	40.96	2.52 982	8.00 000	262.144	1.85 664	4.00 000	8.61 774	.15 625
6.5	42.25	2.54 951	8.06 226	274.625	1.86 626	4.02 073	8.66 239	.15 385
6.6	43.56	2.56 905	8.12 404	287.496	1.87 578	4.04 124	8.70 659	.15 152
6.7	44.89	2.58 844	8.18 535	300.763	1.88 520	4.06 155	8.75 034	.14 925
6.8	46.24	2.60 768	8.24 621	314.432	1.89 454	4.08 166	8.79 366	.14 706
6.9	47.61	2.62 679	8.30 662	328.509	1.90 378	4.10 157	8.83 656	.14 493
7.0	49.00	2.64 575	8.36 660	343.000	1.91 293	4.12 129	8.87 904	.14 286
7.1	50.41	2.66 458	8.42 615	357.911	1.92 200	4.14 082	8.92 112	.14 085
7.2	51.84	2.68 328	8.48 528	373.248	1.93 098	4.16 017	8.96 281	.13 889
7.3	53.29	2.70 185	8.54 400	389.017	1.93 988	4.17 934	9.00 411	.13 699
7.4	54.76	2.72 029	8.60 233	405.224	1.94 870	4.19 834	9.04 504	.13 514
7.5	56.25	2.73 861	8.66 025	421.875	1.95 743	4.21 716	9.08 560	.13 333
7.6	57.76	2.75 681	8.71 780	438.976	1.96 610	4.23 582	9.12 581	.13 158
7.7	59.29	2.77 489	8.77 496	456.533	1.97 468	4.25 432	9.16 566	.12 987
7.8	60.84	2.79 285	8.83 176	474.552	1.98 319	4.27 266	9.20 516	.12 821
7.9	62.41	2.81 069	8.88 819	493.039	1.99 163	4.29 084	9.24 434	.12 658
8.0	64.00	2.82 843	8.94 427	512.000	2.00 000	4.30 887	9.28 318	.12 500
8.1	65.61	2.84 605	9.00 000	531.441	2.00 830	4.32 675	9.32 170	.12 346
8.2	67.24	2.86 356	9.05 539	551.368	2.01 653	4.34 448	9.35 990	.12 195
8.3	68.89	2.88 097	9.11 043	571.787	2.02 469	4.36 207	9.39 780	.12 048
8.4	70.56	2.89 828	9.16 515	592.704	2.03 279	4.37 952	9.43 539	.11 905
8.5	72.25	2.91 548	9.21 954	614.125	2.04 083	4.39 683	9.47 268	.11 765
8.6	73.96	2.93 258	9.27 362	636.056	2.04 880	4.41 400	9.50 969	.11 628
8.7	75.69	2.94 958	9.32 738	658.503	2.05 671	4.43 105	9.54 640	.11 494
8.8	77.44	2.96 648	9.38 083	681.472	2.06 456	4.44 796	9.58 284	.11 364
8.9	79.21	2.98 329	9.43 398	704.969	2.07 235	4.46 475	9.61 900	.11 236
9.0	81.00	3.00 000	9.48 683	729.000	2.08 008	4.48 140	9.65 489	.11 111
9.1	82.81	3.01 662	9.53 939	753.571	2.08 776	4.49 794	9.69 052	.10 989
9.2	84.64	3.03 315	9.59 166	778.688	2.09 538	4.51 436	9.72 589	.10 870
9.3	86.49	3.04 959	9.64 365	804.357	2.10 294	4.53 065	9.76 100	.10 753
9.4	88.36	3.06 594	9.69 536	830.584	2.11 045	4.54 684	9.79 586	.10 638
9.5	90.25	3.08 221	9.74 679	857.375	2.11 791	4.56 290	9.83 048	.10 526
9.6	92.16	3.09 839	9.79 796	884.736	2.12 532	4.57 886	9.86 485	.10 417
9.7	94.09	3.11 448	9.84 886	912.673	2.13 267	4.59 470	9.89 898	.10 309
9.8	96.04	3.13 050	9.89 949	941.192	2.13 997	4.61 044	9.93 288	.10 204
9.9	98.01	3.14 643	9.94 987	970.299	2.14 723	4.62 607	9.96 655	.10 101
10.0	100.00	3.16 228	10.00 000	1000.000	2.15 443	4.64 159	10.00 000	.10 000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

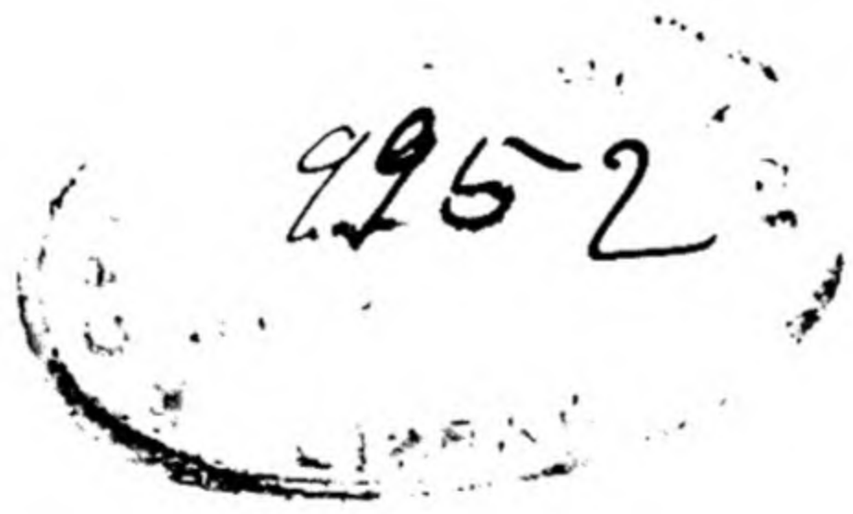
TABLE VI

PROPORTIONAL PARTS OF DIFFERENCES

Table VI contains the proportional parts of differences from 1 to 100. The differences are by integers and the parts by tenths.

For example, if the difference between two numbers is 13, $\frac{4}{10}$ of this difference is found from the table by looking opposite 13 and under 4. Thus $\frac{4}{10}$ of 13 is 5.2.

A difference which is a fractional part of 10 is found by a similar procedure. For example, to find $\frac{45}{67}$ of 10, look opposite 67 for the number nearest 45. This is 46.9 and is under 7. Hence, to the nearest integer, $\frac{45}{67}$ of 10 is 7.



D	1	2	3	4	5	6	7	8	9
1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
2	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
3	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7
4	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
5	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
6	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4
7	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3
8	0.8	1.6	2.4	3.2	4.0	4.8	5.6	6.4	7.2
9	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1
10	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
11	1.1	2.2	3.3	4.4	5.5	6.6	7.7	8.8	9.9
12	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6	10.8
13	1.3	2.6	3.9	5.2	6.5	7.8	9.1	10.4	11.7
14	1.4	2.8	4.2	5.6	7.0	8.4	9.8	11.2	12.6
15	1.5	3.0	4.5	6.0	7.5	9.0	10.5	12.0	13.5
16	1.6	3.2	4.8	6.4	8.0	9.6	11.2	12.8	14.4
17	1.7	3.4	5.1	6.8	8.5	10.2	11.9	13.6	15.3
18	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4	16.2
19	1.9	3.8	5.7	7.6	9.5	11.4	13.3	15.2	17.1
20	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0
21	2.1	4.2	6.3	8.4	10.5	12.6	14.7	16.8	18.9
22	2.2	4.4	6.6	8.8	11.0	13.2	15.4	17.6	19.8
23	2.3	4.6	6.9	9.2	11.5	13.8	16.1	18.4	20.7
24	2.4	4.8	7.2	9.6	12.0	14.4	16.8	19.2	21.6
25	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5
26	2.6	5.2	7.8	10.4	13.0	15.6	18.2	20.8	23.4
27	2.7	5.4	8.1	10.8	13.5	16.2	18.9	21.6	24.3
28	2.8	5.6	8.4	11.2	14.0	16.8	19.6	22.4	25.2
29	2.9	5.8	8.7	11.6	14.5	17.4	20.3	23.2	26.1
30	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0
31	3.1	6.2	9.3	12.4	15.5	18.6	21.7	24.8	27.9
32	3.2	6.4	9.6	12.8	16.0	19.2	22.4	25.6	28.8
33	3.3	6.6	9.9	13.2	16.5	19.8	23.1	26.4	29.7
34	3.4	6.8	10.2	13.6	17.0	20.4	23.8	27.2	30.6
35	3.5	7.0	10.5	14.0	17.5	21.0	24.5	28.0	31.5
36	3.6	7.2	10.8	14.4	18.0	21.6	25.2	28.8	32.4
37	3.7	7.4	11.1	14.8	18.5	22.2	25.9	29.6	33.3
38	3.8	7.6	11.4	15.2	19.0	22.8	26.6	30.4	34.2
39	3.9	7.8	11.7	15.6	19.5	23.4	27.3	31.2	35.1
40	4.0	8.0	12.0	16.0	20.0	24.0	28.0	32.0	36.0
41	4.1	8.2	12.3	16.4	20.5	24.6	28.7	32.8	36.9
42	4.2	8.4	12.6	16.8	21.0	25.2	29.4	33.6	37.8
43	4.3	8.6	12.9	17.2	21.5	25.8	30.1	34.4	38.7
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
45	4.5	9.0	13.5	18.0	22.5	27.0	31.5	36.0	40.5
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
47	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	42.3
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2
49	4.9	9.8	14.7	19.6	24.5	29.4	34.3	39.2	44.1
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

D	1	2	3	4	5	6	7	8	9
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
53	5.3	10.6	15.9	21.2	26.5	31.8	37.1	42.4	47.7
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6
55	5.5	11.0	16.5	22.0	27.5	33.0	38.5	44.0	49.5
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
59	5.9	11.8	17.7	23.6	29.5	35.4	41.3	47.2	53.1
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0
61	6.1	12.2	18.3	24.4	30.5	36.6	42.7	48.8	54.9
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8
63	6.3	12.6	18.9	25.2	31.5	37.8	44.1	50.4	56.7
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
66	6.6	13.2	19.8	26.4	33.0	39.6	46.2	52.8	59.4
67	6.7	13.4	20.1	26.8	33.5	40.2	46.9	53.6	60.3
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
69	6.9	13.8	20.7	27.6	34.5	41.4	48.3	55.2	62.1
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
71	7.1	14.2	21.3	28.4	35.5	42.6	49.7	56.8	63.9
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
73	7.3	14.6	21.9	29.2	36.5	43.8	51.1	58.4	65.7
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
77	7.7	15.4	23.1	30.8	38.5	46.2	53.9	61.6	69.3
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
79	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
81	8.1	16.2	24.3	32.4	40.5	48.6	56.7	64.8	72.9
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
83	8.3	16.6	24.9	33.2	41.5	49.8	58.1	66.4	74.7
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.6
85	8.5	17.0	25.5	34.0	42.5	51.0	59.5	68.0	76.5
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2
89	8.9	17.8	26.7	35.6	44.5	53.4	62.3	71.2	80.1
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
91	9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.9
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
93	9.3	18.6	27.9	37.2	46.5	55.8	65.1	74.4	83.7
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
95	9.5	19.0	28.5	38.0	47.5	57.0	66.5	76.0	85.5
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
97	9.7	19.4	29.1	38.8	48.5	58.2	67.9	77.6	87.3
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
99	9.9	19.8	29.7	39.6	49.5	59.4	69.3	79.2	89.1
100	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
	1	2	3	4	5	6	7	8	9

CONVERSION TABLES

VII. DEGREES TO RADIANS

°	Radians	°	Radians	°	Radians
1	0.01745 83	10	0.17453 29	100	1.74532 93
2	0.03490 66	20	0.34906 59	110	1.91986 22
3	0.05235 99	30	0.52359 88	120	2.09439 51
4	0.06981 32	40	0.69813 17	130	2.26892 80
5	0.08726 65	50	0.87266 46	140	2.44346 10
6	0.10471 98	60	1.04719 76	150	2.61799 39
7	0.12217 30	70	1.22173 05	160	2.79252 68
8	0.13962 63	80	1.39626 34	170	2.96705 97
9	0.15707 96	90	1.57079 63	180	3.14159 27

VIII. MINUTES TO RADIANS

′	Radians	′	Radians	′	Radians
1	0.00029 09	6	0.00174 53	20	0.00581 78
2	0.00058 18	7	0.00203 62	30	0.00872 66
3	0.00087 27	8	0.00232 71	40	0.01163 55
4	0.00116 36	9	0.00261 80	50	0.01454 44
5	0.00145 44	10	0.00290 89	60	0.01745 33

To change tenths of a minute to radians move the decimal point one place to the left in the value given for the corresponding minute.

IX. RADIANS TO DEGREES AND MINUTES

n	Radians		Tenths		Hundredths		Thousandths
	°	′	°	′	°	′	′
1	57	17.75	5	43.77	0	34.38	3.44
2	114	35.49	11	27.55	1	8.75	6.88
3	171	53.24	17	11.32	1	43.13	10.31
4	229	10.99	22	55.10	2	17.51	13.75
5	286	28.73	28	38.87	2	51.89	17.19
6	343	46.48	34	22.65	3	26.26	20.63
7	401	4.23	40	6.42	4	0.64	24.06
8	458	21.97	45	50.20	4	35.02	27.50
9	515	39.72	51	33.97	5	9.40	30.94

X. MINUTES TO DECIMAL PARTS OF A DEGREE

′	Degrees	′	Degrees	′	Degrees
1	0.01667	6	0.10000	20	0.33333
2	0.03333	7	0.11667	30	0.50000
3	0.05000	8	0.13333	40	0.66667
4	0.06667	9	0.15000	50	0.83333
5	0.08333	10	0.16667	60	1.00000

TABLE XI

NATURAL HAVERSINES AND THEIR LOGARITHMS

Table XI gives the natural and logarithmic values of haversines of angles from 0° to 180° at intervals of 10 minutes. Interpolation will give the values for angles to the nearest minute. Decimal points have been omitted from the values of the haversines as well as from the mantissas of the logarithms.

On the first page of the table the characteristics of log haversines of angles greater than 1° 0′ are obtained in each case by inspection of the corresponding natural value. Thus the characteristic of log hav 45° 10′ is -1, since hav 45° 10′ = .1475; and the characteristic of log hav 7° 40′ is -3, since hav 7° 40′ = .0045.

The characteristic of log hav 0° 10′ and log hav 0° 20′ is -6. The characteristic for each of the angles from 0° 30′ to 1° 0′, inclusive, is -5.

On the second and third pages of the table the characteristic of log hav is -1, or $9-10$, in each case. This is indicated by the 9 at the top of each column of mantissas.

Degrees	0′		10′		20′		30′		40′		50′	
	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav
	Find characteristic of log hav by inspection of nat. hav											
0	—	0000	3254	0000	9275	0000	2796	0000	5295	0000	7233	0001
1	8817	0001	0156	0001	1316	0001	2339	0002	3254	0002	4081	0003
2	4837	0003	5532	0004	6176	0004	6775	0005	7336	0005	7862	0006
3	8358	0007	8828	0008	9273	0008	9697	0009	0101	0010	0487	0011
4	0856	0012	1211	0013	1551	0014	1879	0015	2195	0017	2499	0018
5	2794	0019	3078	0020	3354	0022	3621	0023	3880	0024	4132	0026
6	4376	0027	4614	0029	4845	0031	5071	0032	5290	0034	5504	0036
7	5714	0037	5918	0039	6117	0041	6312	0043	6503	0045	6689	0047
8	6872	0049	7051	0051	7226	0053	7397	0055	7566	0057	7731	0059
9	7893	0062	8052	0064	8208	0066	8361	0069	8512	0071	8660	0073
10	8806	0076	8949	0079	9090	0081	9229	0084	9365	0086	9499	0089
11	9631	0092	9762	0095	9890	0097	0016	0100	0141	0103	0264	0106
12	0385	0109	0504	0112	0622	0115	0738	0119	0852	0122	0966	0125
13	1077	0128	1187	0131	1296	0135	1404	0138	1510	0142	1614	0145
14	1718	0149	1820	0152	1921	0156	2021	0159	2120	0163	2217	0167
15	2314	0170	2409	0174	2504	0178	2597	0182	2689	0186	2781	0190
16	2871	0194	2961	0198	3049	0202	3137	0206	3223	0210	3309	0214
17	3394	0218	3478	0223	3561	0227	3644	0231	3726	0236	3807	0240
18	3887	0245	3966	0249	4045	0254	4123	0258	4200	0263	4276	0268
19	4352	0272	4427	0277	4502	0282	4576	0287	4649	0292	4721	0297
20	4793	0302	4865	0307	4935	0312	5006	0317	5075	0322	5144	0327
21	5213	0332	5281	0337	5348	0343	5415	0348	5481	0353	5547	0359
22	5612	0364	5677	0370	5741	0375	5805	0381	5868	0386	5931	0392
23	5993	0397	6055	0403	6116	0409	6177	0415	6238	0421	6298	0426
24	6358	0432	6417	0438	6476	0444	6534	0450	6592	0456	6650	0462
25	6707	0468	6764	0475	6820	0481	6876	0487	6932	0493	6987	0500
26	7042	0506	7096	0512	7150	0519	7204	0525	7258	0532	7311	0538
27	7364	0545	7416	0552	7468	0558	7520	0565	7572	0572	7623	0578
28	7674	0585	7724	0592	7774	0599	7824	0606	7874	0613	7923	0620
29	7972	0627	8021	0634	8069	0641	8117	0648	8165	0655	8213	0663
30	8260	0670	8307	0677	8354	0684	8400	0692	8446	0699	8492	0707
31	8538	0714	8583	0722	8629	0729	8673	0737	8718	0744	8763	0752
32	8807	0760	8851	0767	8894	0775	8938	0783	8981	0791	9024	0799
33	9067	0807	9109	0815	9152	0823	9194	0831	9236	0839	9277	0847
34	9319	0855	9360	0863	9401	0871	9442	0879	9482	0888	9523	0896
35	9563	0904	9603	0913	9643	0921	9682	0929	9721	0938	9761	0946
36	9800	0955	9838	0963	9877	0972	9915	0981	9954	0989	9992	0998
37	0030	1007	0067	1016	0105	1024	0142	1033	0179	1042	0216	1051
38	0253	1060	0289	1069	0326	1078	0362	1087	0398	1096	0434	1105
39	0470	1114	0505	1123	0541	1133	0576	1142	0611	1151	0646	1160
40	0681	1170	0716	1179	0750	1189	0784	1198	0819	1207	0853	1217
41	0887	1226	0920	1236	0954	1246	0987	1255	1020	1265	1054	1275
42	1087	1284	1119	1294	1152	1304	1185	1314	1217	1323	1249	1333
43	1282	1343	1314	1353	1345	1363	1377	1373	1409	1383	1440	1393
44	1472	1403	1503	1413	1534	1424	1565	1434	1596	1444	1626	1454
45	1657	1464	1687	1475	1718	1485	1748	1495	1778	1506	1808	1516
46	1838	1527	1867	1537	1897	1548	1926	1558	1956	1569	1985	1579
47	2014	1590	2043	1601	2072	1611	2101	1622	2129	1633	2158	1644
48	2186	1654	2215	1665	2243	1676	2271	1687	2299	1698	2327	1709
49	2355	1720	2382	1731	2410	1742	2437	1753	2465	1764	2492	1775
50	2519	1786	2546	1797	2573	1808	2600	1820	2627	1831	2653	1842
51	2680	1853	2706	1865	2732	1876	2759	1887	2785	1899	2811	1910
52	2837	1922	2863	1933	2888	1945	2914	1956	2940	1968	2965	1979
53	2991	1991	3016	2003	3041	2014	3066	2026	3091	2038	3116	2049
54	3141	2061	3166	2073	3190	2085	3215	2096	3239	2108	3264	2120
55	3288	2132	3312	2144	3336	2156	3361	2168	3384	2180	3408	2192
56	3432	2204	3456	2216	3480	2228	3503	2240	3527	2252	3550	2265
57	3573	2277	3596	2289	3620	2301	3643	2314	3666	2326	3689	2338
58	3711	2350	3734	2363	3757	2375	3779	2388	3802	2400	3824	2412
59	3847	2425	3869	2437	3891	2450	3913	2462	3935	2475	3957	2487

Degrees	0′		10′		20′		30′		40′		50′	
	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav
	9.		9.		9.		9.		9.		9.	
60	3979	2500	4001	2513	4023	2525	4045	2538	4066	2551	4088	2563
61	4109	2576	4131	2589	4152	2601	4173	2614	4195	2627	4216	2640
62	4237	2653	4258	2665	4279	2678	4300	2691	4320	2704	4341	2717
63	4362	2730	4382	2743	4403	2756	4423	2769	4444	2782	4464	2795
64	4484	2808	4504	2821	4524	2834	4545	2847	4565	2861	4584	2874
65	4604	2887	4624	2900	4644	2913	4664	2927	4683	2940	4703	2953
66	4722	2966	4742	2980	4761	2993	4780	3006	4799	3020	4819	3033
67	4838	3046	4857	3060	4876	3073	4895	3087	4914	3100	4932	3113
68	4951	3127	4970	3140	4989	3154	5007	3167	5026	3181	5044	3195
69	5063	3208	5081	3222	5099	3235	5117	3249	5136	3263	5154	3276
70	5172	3290	5190	3304	5208	3317	5226	3331	5244	3345	5261	3358
71	5279	3372	5297	3386	5314	3400	5332	3413	5349	3427	5367	3441
72	5384	3455	5402	3469	5419	3483	5436	3496	5454	3510	5471	3524
73	5488	3538	5505	3552	5522	3566	5539	3580	5556	3594	5572	3608
74	5589	3622	5606	3636	5623	3650	5639	3664	5656	3678	5672	3692
75	5689	3706	5705	3720	5722	3734	5738	3748	5754	3762	5771	3776
76	5787	3790	5803	3805	5819	3819	5835	3833	5851	3847	5867	3861
77	5883	3875	5899	3889	5915	3904	5930	3918	5946	3932	5962	3946
78	5977	3960	5993	3975	6009	3989	6024	4003	6039	4017	6055	4032
79	6070	4046	6086	4060	6101	4075	6116	4089	6131	4103	6146	4117
80	6161	4132	6176	4146	6191	4160	6206	4175	6221	4189	6236	4203
81	6251	4218	6266	4232	6280	4247	6295	4261	6310	4275	6324	4290
82	6339	4304	6353	4319	6368	4333	6382	4347	6397	4362	6411	4376
83	6425	4391	6440	4405	6454	4420	6468	4434	6482	4448	6496	4463
84	6510	4477	6524	4492	6538	4506	6552	4521	6566	4535	6580	4550
85	6594	4564	6607	4579	6621	4593	6635	4608	6648	4622	6662	4637
86	6676	4651	6689	4666	6703	4680	6716	4695	6730	4709	6743	4724
87	6756	4738	6770	4753	6783	4767	6796	4782	6809	4796	6822	4811
88	6835	4826	6848	4840	6862	4855	6875	4869	6887	4884	6900	4898
89	6913	4913	6926	4927	6939	4942	6952	4956	6964	4971	6977	4985
90	6990	5000	7002	5015	7015	5029	7027	5044	7040	5058	7052	5073
91	7065	5087	7077	5102	7090	5116	7102	5131	7114	5145	7126	5160
92	7139	5174	7151	5189	7163	5204	7175	5218	7187	5233	7199	5247
93	7211	5262	7223	5276	7235	5291	7247	5305	7259	5320	7271	5334
94	7283	5349	7294	5363	7306	5378	7318	5392	7329	5407	7341	5421
95	7353	5436	7364	5450	7376	5465	7387	5479	7399	5494	7410	5508
96	7421	5523	7433	5537	7444	5552	7455	5566	7467	5580	7478	5595
97	7489	5609	7500	5624	7511	5638	7523	5653	7534	5667	7545	5681
98	7556	5696	7567	5710	7577	5725	7588	5739	7599	5753	7610	5768
99	7621	5782	7632	5797	7642	5811	7653	5825	7664	5840	7674	5854
100	7685	5868	7696	5883	7706	5897	7717	5911	7727	5925	7738	5940
101	7748	5954	7759	5968	7769	5983	7779	5997	7790	6011	7800	6025
102	7810	6040	7820	6054	7830	6068	7841	6082	7851	6096	7861	6111
103	7871	6125	7881	6139	7891	6153	7901	6167	7911	6181	7921	6195
104	7931	6210	7940	6224	7950	6238	7960	6252	7970	6266	7980	6280
105	7989	6294	7999	6308	8009	6322	8018	6336	8028	6350	8037	6364
106	8047	6378	8056	6392	8066	6406	8075	6420	8085	6434	8094	6448
107	8104	6462	8113	6476	8122	6490	8131	6504	8141	6517	8150	6531
108	8159	6545	8168	6559	8177	6573	8187	6587	8196	6600	8205	6614
109	8214	6628	8223	6642	8232	6655	8241	6669	8250	6683	8258	6696
110	8267	6710	8276	6724	8285	6737	8294	6751	8302	6765	8311	6778
111	8320	6792	8329	6805	8337	6819	8346	6833	8354	6846	8363	6860
112	8371	6873	8380	6887	8388	6900	8397	6913	8405	6927	8414	6940
113	8422	6954	8430	6967	8439	6980	8447	6994	8455	7007	8464	7020
114	8472	7034	8480	7047	8488	7060	8496	7073	8504	7087	8513	7100
115	8521	7113	8529	7126	8537	7139	8545	7153	8553	7166	8561	7179
116	8568	7192	8576	7205	8584	7218	8592	7231	8600	7244	8608	7257
117	8615	7270	8623	7283	8631	7296	8638	7309	8646	7322	8654	7335
118	8661	7347	8669	7360	8676	7373	8684	7386	8691	7399	8699	7411
119	8706	7424	8714	7437	8721	7449	8729	7462	8736	7475	8743	7487

Degrees	0′		10′		20′		30′		40′		50′	
	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav	log hav	nat. hav
	9.		9.		9.		9.		9.		9.	
120	8751	7500	8758	7513	8765	7525	8772	7538	8780	7550	8787	7563
121	8794	7575	8801	7588	8808	7600	8815	7612	8822	7625	8829	7637
122	8836	7650	8843	7662	8850	7674	8857	7686	8864	7699	8871	7711
123	8878	7723	8885	7735	8892	7748	8898	7760	8905	7772	8912	7784
124	8919	7796	8925	7808	8932	7820	8939	7832	8945	7844	8952	7856
125	8959	7868	8965	7880	8972	7892	8978	7904	8985	7915	8991	7927
126	8998	7939	9004	7951	9010	7962	9017	7974	9023	7986	9030	7997
127	9036	8009	9042	8021	9048	8032	9055	8044	9061	8055	9067	8067
128	9073	8078	9079	8090	9085	8101	9092	8113	9098	8124	9104	8135
129	9110	8147	9116	8158	9122	8169	9128	8180	9134	8192	9140	8203
130	9146	8214	9151	8225	9157	8236	9163	8247	9169	8258	9175	8269
131	9180	8280	9186	8291	9192	8302	9198	8313	9203	8324	9209	8335
132	9215	8346	9220	8356	9226	8367	9231	8378	9237	8389	9242	8399
133	9248	8410	9253	8421	9259	8431	9264	8442	9270	8452	9275	8463
134	9281	8473	9286	8484	9291	8494	9297	8505	9302	8515	9307	8525
135	9312	8536	9318	8546	9323	8556	9328	8566	9333	8576	9338	8587
136	9343	8597	9348	8607	9353	8617	9359	8627	9364	8637	9369	8647
137	9374	8657	9379	8667	9383	8677	9388	8686	9393	8696	9398	8706
138	9403	8716	9408	8725	9413	8735	9417	8745	9422	8754	9427	8764
139	9432	8774	9436	8783	9441	8793	9446	8802	9450	8811	9455	8821
140	9460	8830	9464	8840	9469	8849	9473	8858	9478	8867	9482	8877
141	9487	8886	9491	8895	9496	8904	9500	8913	9505	8922	9509	8931
142	9513	8940	9518	8949	9522	8958	9526	8967	9531	8976	9535	8984
143	9539	8993	9543	9002	9548	9011	9552	9019	9556	9028	9560	9037
144	9564	9045	9568	9054	9572	9062	9576	9071	9580	9079	9584	9087
145	9588	9096	9592	9104	9596	9112	9600	9121	9604	9129	9608	9137
146	9612	9145	9616	9153	9620	9161	9623	9169	9627	9177	9631	9185
147	9635	9193	9638	9201	9642	9209	9646	9217	9650	9225	9653	9233
148	9657	9240	9660	9248	9664	9256	9668	9263	9671	9271	9675	9278
149	9678	9286	9682	9293	9685	9301	9689	9308	9692	9316	9695	9323
150	9699	9330	9702	9337	9706	9345	9709	9352	9712	9359	9716	9366
151	9719	9373	9722	9380	9725	9387	9729	9394	9732	9401	9735	9408
152	9738	9415	9741	9422	9744	9428	9747	9435	9751	9442	9754	9448
153	9757	9455	9760	9462	9763	9468	9766	9475	9769	9481	9772	9488
154	9774	9494	9777	9500	9780	9507	9783	9513	9786	9519	9789	9525
155	9792	9532	9794	9538	9797	9544	9800	9550	9803	9556	9805	9562
156	9808	9568	9811	9574	9813	9579	9816	9585	9819	9591	9821	9597
157	9824	9603	9826	9608	9829	9614	9831	9619	9834	9625	9836	9630
158	9839	9636	9841	9641	9844	9647	9846	9652	9849	9657	9851	9663
159	9853	9668	9856	9673	9858	9678	9860	9683	9863	9688	9865	9693
160	9867	9698	9869	9703	9871	9708	9874	9713	9876	9718	9878	9723
161	9880	9728	9882	9732	9884	9737	9886	9742	9888	9746	9890	9751
162	9892	9755	9894	9760	9896	9764	9898	9769	9900	9773	9902	9777
163	9904	9782	9906	9786	9908	9790	9910	9794	9911	9798	9913	9802
164	9915	9806	9917	9810	9919	9814	9920	9818	9922	9822	9924	9826
165	9925	9830	9927	9833	9929	9837	9930	9841	9932	9844	9933	9848
166	9935	9851	9937	9855	9938	9858	9940	9862	9941	9865	9943	9869
167	9944	9872	9945	9875	9947	9878	9948	9881	9950	9885	9951	9888
168	9952	9891	9954	9894	9955	9897	9956	9900	9957	9903	9959	9905
169	9960	9908	9961	9911	9962	9914	9963	9916	9965	9919	9966	9921
170	9967	9924	9968	9927	9969	9929	9970	9931	9971	9934	9972	9936
171	9973	9938	9974	9941	9975	9943	9976	9945	9977	9947	9978	9949
172	9979	9951	9980	9953	9981	9955	9981	9957	9982	9959	9983	9961
173	9984	9963	9985	9964	9985	9966	9986	9968	9987	9969	9987	9971
174	9988	9973	9989	9974	9989	9976	9990	9977	9991	9978	9991	9980
175	9992	9981	9992	9982	9993	9983	9993	9985	9994	9986	9994	9987
176	9995	9988	9995	9989	9996	9990	9996	9991	9996	9992	9997	9992
177	9997	9993	9997	9994	9998	9995	9998	9995	9998	9996	9998	9996
178	9999	9997	9999	9997	9999	9998	9999	9998	9999	9999	9999	9999
179	9999	9999	9999	9999	0000	1.00	0000	1.00	0000	1.00	0000	1.00
180	0000	1.00										